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The ψ-particles in an SU_4 scheme with anomalous currents

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Abstract. The recently discovered narrow peaks (the ψ-particles) in e^+e^- system at 3·105 and 3·695 GeV are interpreted as hadrons in a broken SU_4 symmetry scheme. A new additional additive quantum number, paracharge Z, is combined with the usual SU_3 quantum numbers in the group SU_4. The ψ(3·1) is assigned to a near ideally mixed $\underline{15} \oplus \underline{1}$ multiplet of vector mesons (containing the ρ) as the $I = Y = 0$, charge conjugation $C = -$ combination of $Z = \pm 1$ members. The ψ(3·7) is assigned correspondingly to another mixed $\underline{15} \oplus \underline{1}$ multiplet containing the ρ'(1600). The hadronic electromagnetic interactions are modified by the addition of (non-minimal) anomalous pieces that can change Z. The decays of the ψ-particles are discussed. New enlarged SU_4 multiplets of other hadrons are proposed. Tests of our scheme are put forward. The most crucial test will be the observation of two rather broad resonances in e^+e^- collisions with masses around 4·2 GeV and 5·1 GeV. Another prediction is the presence of energetic photons in the decays of the ψ-particles. Important results concerning the recently observed phenomena in the process $e^+e^- \to$ hadrons follow in this scheme.

Keywords. ψ-particles; paracharge; SU_4 symmetry; anomalous currents; e^+e^- annihilation.

1. Introduction

From the recent unexpected discovery (Aubert *et al* 1974 *a*, Augustin *et al* 1974, Bacci *et al* 1974, Abrams *et al* 1974) of the particles ψ (3·1) of mass 3·105 GeV, and ψ (3·7), of mass 3·695 GeV, as extremely sharp resonance peaks in the e^+e^- system in the processes $e^+e^- \to e^+e^-$ (Bhabha scattering), $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to$ hadrons, and $pp \to e^+e^- +$ anything*, it appears that we might be witnessing the opening up of a new era in particle spectroscopy of yet another dimension.

While it may very well be that the new particle states depart from the class of hadrons, we shall describe in the present paper a phenomenological scheme that treats them simply as hadrons†. In general, however, the decay width of a typical hadron having a mass of a few GeV is expected to be quite sizable, unless forbidden by some selection rules. It is, therefore, natural to introduce a new additive

* ψ (3·7) has not been seen in $pp \to e^+e^- +$ anything, in spite of a reported search (Aubert *et al* 1974 *b*). This could be because of the small branching ratio into e^+e^- and a larger total width.

† A brief account was given in TIFR preprint (Das *et al* 1974), *Phys Rev. Lett.* (1975).

quantum number, which we name‡ the "paracharge" Z, in order to understand the narrow decay widths of these new particles. All the earlier known particles will be assumed to have $Z = 0$, while the new particles, the ψ (3·1) and the ψ (3·7), will be assumed to have $Z \neq 0$. While pure strong interactions must conserve this new quantum number Z, the electromagnetic and weak interactions, in general, could allow Z violating decays of the particles with $Z \neq 0$ and account for the observed small width of the ψ (3·1). The Z quantum number will allow the decay of the ψ (3·7) into the ψ (3·1) plus normal hadrons; but will be shown to be suppressed for reasons to be discussed. Thus the narrow width of the ψ (3·7) will also be accounted for.

The observation of the ψ (3·1) and the ψ (3·7) in Bhabha scattering will be taken to proceed directly through the electromagnetic current (figure 1). This has the merit that the charged leptons are not required to have any new unconventional interaction and the usual quantum electrodynamics of the leptons is completely adhered to. On this picture, these particles will have *spin-parity* $J^P = \underline{1}^-$ *and charge-conjugation* $C = -$. We are thus led quite naturally to postulate new additional anomalous (non-minimal) pieces in the phenomenological *hadronic* electromagnetic currents. These new pieces, besides having other attributes, must be capable of changing Z in order to produce the ψ particles. We may also have new additional pieces in the weak interaction currents.

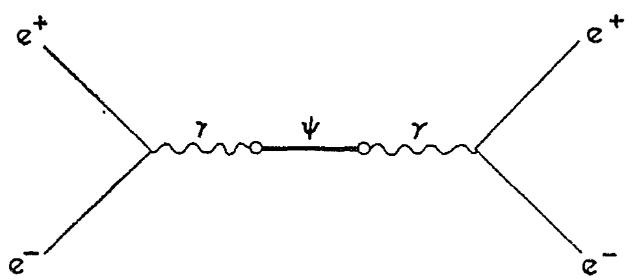

Figure 1. Mechanism for the observation of the ψ particles in Bhabha scattering.

The simplest way to incorporate the new "paracharge" quantum number Z in a larger strong interaction symmetry group, which also includes the conventional SU_3 symmetry, is to use the SU_4 group. Like the SU_3 symmetry, the SU_4 symmetry too is assumed to be a regularly broken symmetry according to the chain

$$SU_4 \supset SU_3 \otimes U_1(Z) \supset SU_2(I) \otimes U_1(Y) \otimes U_1(Z).$$

This scheme can be neatly pictured in terms of a quartet of phenomenological fields, mathematical or real, $\xi = (p, n, \lambda, \chi)$, transforming as the basic representation of SU_4. The triplet (p, n, λ) is comprised of the usual SU_3 quarks, assigned $Z = 0$, while χ is a fourth, much heavier SU_3 singlet quark with $Z = +1$. The

‡ The name "paracharge" (Z) [in company with hypercharge (Y)] is being used here to distinguish our scheme from older SU_4 schemes with an additional additive quantum number, since the detailed physical content of our scheme is very different from those schemes (even though the same mathematical framework of the SU_4 group is employed). We must also emphasize right at the outset that we are taking a strictly phenomenological approach in the present discussion and do not have any "fundamental" gauge theory in mind. For older SU_4 schemes, *see, e.g.*, Gaillard *et al* (1974).

conventional (lower energy) hadron spectroscopy is strictly adhered to by constructing the mesons as $\bar{\xi} \otimes \xi$ and the baryons as $\xi \otimes \xi \otimes \xi$, making suitable assignments of the physical quantum numbers. The usual modifications, like adding colour or applying parastatistics to the quarks, etc., can be easily incorporated but will not be gone into here.

The ψ (3·1) has the quantum numbers $I = Y = 0$ and is the $C = -$ mixture of $Z = \pm 1$ states of a near ideally mixed $\underline{15} \oplus \underline{1}$ multiplet containing the vector mesons of the ρ-family. A new vector state P with $I = Y = Z = 0$ and $C = -$ is also expected at around 4·3 GeV in this multiplet on the basis of SU_4 mass formulae. The ψ (3·7) has the same quantum numbers as the ψ (3·1); but belongs to another mixed $\underline{15} \oplus \underline{1}$ multiplet which contains the ρ' (1600). Correspondingly, another state P', with the same quantum numbers as the P, is predicted in this multiplet at around 5 GeV.

The scheme we propose here has the merit of making clearly testable predictions for the decays of the ψ-particles. Specially to be noted is the fact that the dominant decays will be through the emission of extremely energetic photons leading to final states of η, η', f' and ϕ mesons. This helps, at least qualitatively, in resolving the so-called "energy crisis" in the process $e^+e^- \to$ hadrons.

In section 2 we describe the symmetry scheme. The assignment of quantum numbers to the ψ-particles is given in section 3. The phenomenological electromagnetic and weak currents are introduced in section 4. Section 5 is devoted to the breaking of the SU_4 symmetry and discussing the mass-formula. The general mass-formula and SU_4 multiplets of other hadrons are discussed in section 6. Section 7 is for the present the crucial section discussing the mechanisms for the decays of the new particles. This section is subdivided into five subsections dealing with (1) general considerations, (2) selection rules for the decays of the ψ (3·1), (3) estimates of the partial decay widths of the ψ (3·1), (4) decays of the ψ (3·7) and (5) decays of other new particles. In section 8 we discuss the implication of the anomalous current in $e^+e^- \to$ hadrons, and finally in section 9 we make some concluding comments, indicating further lines of work under way. In Appendix 1 we collect a number of useful results on the SU_4 group.

2. The symmetry scheme

We shall be guided, as stated earlier, in the assignment of quantum numbers in our symmetry scheme by adhering strictly to the conventional lower energy hadron spectroscopy.

We would like to accommodate the lower lying particle ψ (3·1) in the regular 15 dimensional representation of SU_4 containing the usual SU_3 octet of vector mesons of the ρ-family. This requirement, together with the quantum numbers $Q = Y = 0$, $Z \neq 0$ for the ψ (3·1), *uniquely* specifies the expressions for Q, I_3, Y and Z in terms of the generators of the SU_4 symmetry group.

The physical multiplets are most simply constructed as $\bar{\xi} \otimes \xi$ (mesons) and as $\xi \otimes \xi \otimes \xi$ (baryons) by combining the basic quartets

$\bar{\xi}\,(\sim 4^*) = (\bar{p}, \bar{n}, \bar{\lambda}, \bar{\chi})$.

Here (p, n, λ) is the conventional SU_3 quark triplet assigned $Z = 0$ and χ is an additional SU_3 singlet paracharged quark with $Z = 1$. We note the Q, I_3, Y, assignments for the quark quartet (note the values given to the χ):

$$Q \text{ (electric charge)} = \text{diag}\left(\tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}\right)$$

$$I_3 = \text{diag}\left(\tfrac{1}{2}, -\tfrac{1}{2}, 0, 0\right)$$

$$Y \text{ (hypercharge)} = 2Q - 2I_3 = \text{diag}\left(\tfrac{1}{3}, \tfrac{1}{3}, -\tfrac{2}{3}, -\tfrac{2}{3}\right)$$

$$Z \text{ (paracharge)} = \text{diag}\,(0, 0, 0, 1).$$

The 4×4 diagonal matrices representing the "physical charges" Q, Y, Z for the basic quartet are not traceless. They are thus themselves not to be identified with the traceless diagonal (mathematical) generators of the group SU_4. The three diagonal generators of SU_4 in the basic representation are (using an obvious extension of the conventional SU_3 notation, see Appendix 1):

$$F_3 = I_3 = \tfrac{1}{2}\lambda_3 = \text{diag}\left(+\frac{1}{2}, -\frac{1}{2}, 0, 0\right),$$

$$F_8 = \frac{1}{\sqrt{3}}\lambda_8 = \text{diag}\left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}, 0\right),$$

$$F_{15} = \sqrt{\frac{3}{8}}\,\lambda_{15} = \text{diag}\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{3}{4}\right).$$

The remaining 12 non-diagonal "shift" generators may be constructed in the standard way (see Appendix 1). Making also the baryon number assignment of the quartet:

$$B = \text{diag}\left(\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\right)$$

we may now express the (physical) "charges" in terms of the (mathematical) generators of SU_4 and the baryon number B as follows:

$$Q = F_3 + \tfrac{1}{2}F_8 + \tfrac{1}{3}F_{15} - \tfrac{1}{4}B$$

$$Y = F_8 + \tfrac{2}{3}F_{15} - \tfrac{1}{2}B = F_8 - \tfrac{2}{3}Z$$

$$Z = \tfrac{3}{4}B - F_{15}.$$

These expressions are of course valid for all the representations of SU (4). The construction of representations and determination of the quantum numbers now follow on completely standard lines from the above choice of the basic representation ξ and its conjugate $\bar{\xi}$.

3. Assignments of the ψ (3·1) and ψ (3·7) to 15-plets

As mentioned earlier we regard both the ψ (3·1) and the ψ (3·7) to be vector mesons. We have a well established SU_3 nonet of vector mesons $(\rho, \omega_8, K^*, \bar{K}^*) \oplus \omega_1$. We will extend this SU_3 nonet to an SU_4 representation, $\underline{15} \oplus \underline{1}$, forming a U_4 16-plet. The old ω_1 will be identified now with the SU_4 singlet. The $SU_3 \otimes U_1(Z)$ content of the 15-plet of SU_4 is displayed below.

$$SU_4 \qquad SU_3 \otimes U_1(Z)$$

$$\underline{15} \to \underline{8}\,(Z = 0) \oplus \underline{3}\,(Z = -1) \oplus \underline{3}^*\,(Z = +1) \oplus \underline{1}\,(Z = 0).$$

The new SU$_3$ multiplets (besides the usual, $Z = 0$, SU$_3$ octet) have the (I, Y, Z) contents:

$$\begin{array}{lll} \underline{3}\,(Z=-1) \xrightarrow[\mathrm{SU}_2(I)\otimes \mathrm{U}_1(Y)]{} & \left\{\begin{array}{ll} (\mathrm{D}^+, \mathrm{D}^0): & I=\tfrac{1}{2}\,(Q=1,0),\ Y=1,\ Z=-1, \\ \mathrm{S} \qquad : & I=0\,(Q=0),\ Y=0,\ Z=-1; \end{array}\right. \\ \underline{3}^*\,(Z=+1) \longrightarrow & \left\{\begin{array}{ll} (\bar{\mathrm{D}}^0, \mathrm{D}^-): & I=\tfrac{1}{2}\,(Q=0,-1),\ Y=-1,\ Z=+1 \\ \bar{S} \qquad : & I=0\,(Q=0),\ Y=0,\ Z=+1 \end{array}\right. \\ \underline{1}\,(Z=0) \longrightarrow & \ \ \mathrm{P}_1 \qquad : \ I=0\,(Q=0),\ Y=0,\ Z=0. \end{array}$$

Thus, in addition to the old nonet, we have seven new vector mesons: $(\mathrm{D}^+, \mathrm{D}^0)$, $(\overline{\mathrm{D}}{}^0, \mathrm{D}^-)$. S, $\bar{S}$ and P_1. The $\underline{3}^*$ $(Z = +1)$ and $\underline{3}$ $(Z = -1)$ mesons form mutually charge conjugate sets. The quark contents can be read off from the orthonormal states of the 15-plet given in Appendix 1.

The ψ (3·1), which is a charge-conjugation $C = -$ state, can thus be identified with*

$$\psi\,(3\cdot 1) = \frac{\mathrm{S} - \bar{\mathrm{S}}}{\sqrt{2}} \equiv S^0_-.$$

Obviously the ψ (3.1) is a linear combination of $Z = +1$ and $Z = -1$ states and is thus clearly stable under pure strong interactions if there are no lower lying hadrons with $Z \neq 0$. We will show later that a 0^- particle with the same SU$_4$ assignment and slightly lower in mass does not change this conclusion.

The only other $C = -$ states belonging to a 16-plet which can be seen in Bhabha scattering (apart from the ρ^0), according to figure 1, have $I = Y = Z = 0$. There are three such physical states in the 16-plet of U$_4$ and these, in general, would be some linear combinations of ω_1, ω_8 and P_1. Two of these can be identified with the ω (790) and the ϕ (1020) while the third one we shall denote by P.

We consider the ψ (3·7) to be the $\mathrm{S}^0_-{}'$ state belonging to another SU$_4$ 15-plet, the one containing the ρ' (1600), etc. Correspondingly, there will also be the P$'$ state with $I = Y = Z = 0$.

4. The hadronic electromagnetic and weak currents

The logic of our scheme as developed so far forces us to make a departure from the conventional description of the phenomenological electromagnetic interactions of the hadrons. To be able to produce, for example, the S^0_-, it is clear then that the usual electromagnetic current must be modified by the addition of an anomalous piece that enables transitions with $\triangle Z = \pm 1$, $\triangle I = 0$, $\triangle Y = 0$. This additional piece is thus anomalous in terms of the quantum numbers. Besides, it must be anomalous in another important sense. Since the electric charge carried by the conserved electromagnetic current is still given by $Q = I_3 + \frac{1}{2}Y$, and since Q is a superselection quantum number, the "charge" carried by the new additional piece *must* vanish identically.

We postulate that the anomalous current transforms as a member of a 15-plet tensor operator $\mathcal{G}^i_\mu$ (x), $(i = 1, \ldots, 15)$, of SU$_4$. Denoting by $\mathcal{F}^i_\mu$ (x), $(i = 0.$

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* We denote by $\mathrm{S}^0_\pm = (\mathrm{S} \pm \bar{\mathrm{S}})/\sqrt{2}$ the $C = \pm$ eigenstates analogous to the usual K^0_1, K^0_2 mixtures of K^0 and $\bar{\mathrm{K}}{}^0$.

1, ..., 15), the normal currents whose "charges" $F_i \equiv \int \mathcal{F}_0^i \, d^3 x$ are essentially the generators of U_4, we write for our phenomenological hadronic electromagnetic current density operator the expression

$$J_\mu^{em}(x) = \mathcal{F}_\mu^3(x) + \tfrac{1}{2}\mathcal{F}_\mu^Y(x) + \mathcal{G}_\mu^{em}(x),$$

$$\mathcal{F}_\mu^Y = \mathcal{F}_\mu^8 + \tfrac{2}{3}\mathcal{F}_\mu^{15} - \tfrac{1}{2}\mathcal{F}_\mu^B,$$

where $\mathcal{F}_\mu^B = (\sqrt{2/3})\,\mathcal{F}_\mu^0$ is the baryonic current. The anomalous (non-minimal, current $\mathcal{G}_\mu^{em}$ must have $C = -$ and contains, in geneial, the components $i = 0$) 3, 8, 13, 15 of $\mathcal{G}_\mu^i(x)$. In particular, it is the component $\mathcal{G}_\mu^{13}$ that transforms like the S_-^0 and enables transitions with $\triangle Z = \pm 1$, $\triangle I = \triangle Y = 0$. The strengths of the various pieces will have to be fixed through a phenomenological analysis, in this framework, of suitable experiments.

As with the hadronic electromagnetic current, so also with the weak hadronic currents may we have to contend with the addition of new anomalous pieces, since we like to retain the idea that all these currents have close family relationships. The anomalous weak current $\mathcal{G}_\mu^{weak}$ (vector and axial-vector) will then be taken also to transform according to 15-plets of SU_4. There will thus be present, in particular, pieces having the quantum numbers of the $D^\pm$, $\bar{D}^0$, D^0 and P_1. We may specially note the neutral pieces which will induce transitions with $|\triangle I| = \frac{1}{2}$ $|\triangle Y| = 1$, $|\triangle Z| = 1$. These will *not* contribute, *e.g.*, to $K_L \to \mu\bar{\mu}$, $K^+ \to \pi^+ l\bar{l}$ in the lowest order. They will, however, give rise to $Z \neq 0$, $Y \neq 0$ hadrons in $\nu + p \to \nu +$ hadrons, for instance.

As an example of the anomalous currents, it is useful to note the form they will have in terms of the quartet quark fields:

$$\mathcal{G}_\mu^i(x) = \beta \frac{1}{2M} \partial_\nu \left(\bar{\xi} \frac{\lambda^i}{2} \sigma_{\mu\nu} \xi \right)$$

where M is a suitable mass parameter and β is a strength parameter. These currents may thus be viewed as generalizations of an intrinsic magnetic moment (Pauli) current of the quarks.

In the picture so far drawn, it is clear that the $C = +$ state S_+^0 can be produced singly in the lowest order of electromagnetism. This, as well as the states S, $\bar{S}$, $D^\pm$, D^0, $\overline{D^0}$ (and the corresponding primed states), can be produced in Z-conserving associated productions involving strong interactions. They will also be produced singly in weak interaction processes (see, however, section 9) and in the decays of heavier particles having the appropriate quantum numbers.

5. The SU_4 symmetry breaking and mass relations of the vector mesons

We shall now consider the mass formula for the SU_4 symmetry breaking down to $SU_3 \otimes U_1(Z)$. To begin with we postulate (as in the well-known case of SU_3 symmetry breaking) that most of the effective symmetry breaking in the mass-squared matrix of the mesons transforms as the F_{15}. This leads to the following mass formula relating the masses of the SU_3-multiplets in the SU_4 15-plet (for outlines of the derivation see Appendix 1) assuming it does not mix with any nearby multiplet:

$$m^2(\underset{-}{3}) = \tfrac{1}{3} m^2(\underset{-}{8}) + \tfrac{2}{3} m^2(\underset{-}{1}).$$

This formula as such should not be used for the vector mesons, since it is well known that mixing of the neutral states is quite crucial. Let us start from the ideal situation of an entire U_4 degenerate 16-plet and calculate the mass splitting taking account of the maximal mixing* of the neutral mesons ω_8, P_1 and ω_1. We then have to diagonalize the 3×3 mass-squared matrix in the $(\omega_8, P_1, \omega_1)$-space. To describe the symmetry breaking $U_4 \to SU_3 \to SU_2$ we may picture the *effective* Hamiltonian in the quark field language as

$$H = H_0 + \mu_\lambda \bar{\lambda}\lambda + \mu_\chi \bar{\chi}\chi.$$

The quark contents of the orthonormal (degenerate) eigenstates of H_0, namely ω_8, P_1 and ω_1, are given in Appendix 1. Using these we may write the mass-squared matrix in terms of the symmetry breaking parameters (the quark mass differences) μ_λ and μ_χ. The diagonalization leads to the orthonormal physical eigenstates of H, with quark contents as follows:

$$\omega = \frac{1}{\sqrt{2}}(\,|\,p\bar{p}\,\rangle + |\,n\bar{n}\,\rangle), \quad \phi = |\,\lambda\bar{\lambda}\,\rangle, \quad P = |\,\chi\bar{\chi}\,\rangle.$$

The mass-relations are:

$$m_\rho^2 = m_\omega^2\,; \quad 2m_{K^*}^2 = m_\omega^2 + m_\phi^2\,;$$

$$m_{K^*}^2 - m_\omega^2 = m_S^2 - m_D^2\,;$$

$$2m_D^2 = m_\omega^2 + m_P^2.$$

These relations predict $m_P > m_S$ and $m_D \simeq m_S$. Further, taking $m_S = 3{\cdot}1$ GeV, we obtain $m_P \simeq 4{\cdot}3$ GeV. Similarly, if we take $m_{S'} = 3{\cdot}7$ GeV, we obtain $m_{P'} = 5{\cdot}0$ GeV, with $m_{\rho'} = 1{\cdot}6$ GeV. Since the mass of the ρ' is rather uncertain on account of its large width, the mass of the P′ is also uncertain to the same extent. Also the mass formulae are expected to hold only to within $\simeq 5\%$. Both the P and the P′ should be produced in e^+e^- collisions at the appropriate energies.

It is of particular interest to note that the quark contents of the conventional (SU_3) maximally mixed ω and ϕ states follow here quite automatically. Also to be noted is the $|\,\chi\bar{\chi}\,\rangle$ structure of the P and the P′. Of course, we must remember that maximal mixing could itself be of approximate validity.

A measure of the SU_3 *versus* the SU_4 symmetry breakings is provided by the ratio $(m_{K^*}^2 - m_\rho^2) : (m_P^2 - m_S^2) \simeq 1 : 30$.

6. Other multiplets and the general mass formula

Once the paracharge has been unfrozen with the energies now becoming available we, of course, expect the enlargement of all the conventional SU_3 multiplets already known. Observation of the new expected states will require the use of a variety of experimental techniques.

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* Our treatment here and below parallels that for the SU_3 case given by Okubo (1963). When we use "quark rules" below it is equivalent to the use of the SU_4 generalization of the nonet ansatz of Okubo. In particular the ansatz (absence of the trace of a single $\underset{-}{16}$-plet in couplings) allows the decay $S' \to S + 2\pi$ while the Zweig rule (only planar connected quark diagrams allowed) forbids it.

We expect the enlargements into SU_4-families of the $J^P = 0^-$ nonet (π, K, $\bar{K}$, η, η'), of the $J^P = 2^+$ nonet, of the baryon octet, of the baryon decimet, etc. The additional members of these families will be produced in strong interaction processes *via* associated production (conserving Z), and also in weak and electromagnetic processes. Decays of heavier particles with appropriate quantum numbers will be another source of new particles.

The baryons in our scheme are obtained through the construction $\xi \otimes \xi \otimes \xi$ (see Appendix 1):

$$\underline{4} \otimes \underline{4} \otimes \underline{4} = \underline{20}_s \oplus \underline{20} \oplus \underline{20} \oplus \underline{4}^*$$

The SU_3-octet of baryons (with $Z = 0$) belongs to

$$\underline{20} \xrightarrow{SU_4 \to SU_3} \underline{8}\,(Z = 0) \oplus \underline{6}\,(Z = 1) \oplus \underline{3}^*\,(Z = 1) \oplus \underline{3}\,(Z = 2)$$

The (I, Y) subfamilies within each of the additional SU_3-families with different non-zero Z-values are as follows:

$$\underline{6}\ \ (Z = 1) \to (1, 0) \oplus (\tfrac{1}{2}, -1) \oplus (0, -2),$$

$$\underline{3}^*\,(Z = 1) \to (0, 0) \oplus (\tfrac{1}{2}, -1)$$

$$\underline{3}\ \ (Z = 2) \to (\tfrac{1}{2}, -1) \oplus (0, -2)$$

The SU_3 decimet (with $Z = 0$) occurs in the SU_4 family:

$$\underline{20}_s \xrightarrow{SU_4 \to SU_3} \underline{10}\,(Z = 0) \oplus \underline{6}\,(Z = 1) \oplus \underline{3}\,(Z = 2) \oplus \underline{1}\,(Z = 3).$$

The (I, Y) contents of the additional SU_3-families (with $Z \neq 0$) in the above $\underline{20}$ is as follows:

$$\underline{6}\,(Z = 1) \to (1, 0) \oplus (\tfrac{1}{2}, -1) + (0, -2)$$

$$\underline{3}\,(Z = 2) \to (\tfrac{1}{2}, -1) \oplus (0, -2)$$

$$\underline{1}\,(Z = 3) \to (0, -2)$$

We can clearly also have an SU_4 multiplet $\underline{4}^*$ of baryons with (SU_3, Z) contents:

$$\underline{4}^* = \underline{3}^*\,(Z = \underline{1}) \oplus 1\,(Z = 0),$$

where the (I, Y) contents are

$$\underline{3}^*\,(Z = 1) \to (0, 0) \oplus (\tfrac{1}{2}, -1)$$

$$\underline{1}\,(Z = 0) \quad \to \quad (0, 0).$$

We have thus, in the $\underline{20}$, in addition to the $Z = 0$ octet, one Λ-like particle (with the same I and Y and so the same Q) with $Z = 1$ which we denote by Λ_1, and similarly one Σ_1 triplet, two Ξ_1 doublets, one Ξ_2 doublet, one Ω_1 and one Ω_2. In the $\underline{20}_s$, we have one each of Σ_1^*, Ξ_1^*, Ξ_2^*, Ω_1^*, Ω_2^* and Ω_3^*. The particle symbols refer to I and Y, whereas the subscripts give the Z values.

The general mass formula (applicable to representations $\underset{\sim}{R}$ of SU_4 such that $\underset{\sim}{R} \otimes \underset{\sim}{15}$ does not contain $\underset{\sim}{15}$ more than twice - quite enough for the situations of interest here) for the symmetry breakings $SU_4 \to SU_3 \otimes U_1(Z) \to SU_2(I) \otimes U_1(Y) \otimes U_1(Z)$ is as follows [where (p, q) denotes an SU_3-multiplet in the highest weight notation]:

$$m = m_0 + m_1 Z + m_2 [C(p, q) - Z^2] + m_3 F_8 + m_4 [I(I+1) - \tfrac{1}{4}(F_8)^2 - \tfrac{1}{9} C(p, q)],$$

$$C(p, q) \equiv p^2 + pq + q^2 + 3(p + q).$$

Some relevant points concerning the derivation are given in Appendix 1.

In the interesting case of $\underset{\sim}{20}$ $(B = 1)$, we have the relation among the SU_3-symmetric masses

$$\tfrac{3}{2} m(\underset{\sim}{3}^*) + \tfrac{1}{2} m(\underset{\sim}{6}) = m(\underset{\sim}{8}) + m(\underset{\sim}{3}),$$

while in the case $\underset{\sim}{20}_s$ $(B = 1)$ we have the equal spacing rule:

$$m(\underset{\sim}{10}) - m(\underset{\sim}{6}) = m(\underset{\sim}{6}) - m(\underset{\sim}{3}) = m(\underset{\sim}{3}) - m(\underset{\sim}{1}).$$

7. Decay systematics of the ψ-particle

7.1 *General considerations*

Before discussing the detailed decay pattern of the ψ-particles it is useful to discuss some general features of the experimental information so far available. The present experimental setup at SLAC detects only the modes involving at least two charged particles. The neutral particle modes (*i.e.*, those in which all the ultimately detected decay products are neutral) are thus really unknown. For this reason the various partial decay widths extracted from experiments are to be taken only as lower bounds.

It is possible to derive certain bounds on the parameter x defined as

$$x \equiv \frac{\Gamma_{ch}}{\Gamma_t},$$

where Γ_{ch} stands for the partial width of decays in which at least two charged particles are detected, and Γ_t is the total width. Let us define (in obvious notation):

$$\Gamma^2_{e\bar{e}} \equiv A\Gamma_t, \quad (\Gamma_{e\bar{e}} = \Gamma_{\mu\bar{\mu}}),$$

$$\Gamma_{ch}\Gamma_{e\bar{e}} \equiv B\Gamma_t,$$

$$\Gamma_h \equiv C\Gamma_{e\bar{e}},$$

where Γ_h is the partial width for decays into exclusively hadronic channels (charged or neutral). Then we obtain

$$\Gamma_{e\bar{e}} = \Gamma_{\mu\bar{\mu}} = \Gamma_{ch}\frac{A}{B},$$

Now we have

$$\Gamma_t = \Gamma_{e\bar{e}} + \Gamma_{\mu\bar{\mu}} + \Gamma_h + \sum_i \Gamma_i,$$

$$\Gamma_{ch} = \Gamma_{e\bar{e}} + \Gamma_{\mu\bar{\mu}} + x_h\Gamma_h + \sum_i x_i\Gamma_i,$$

where Γ_i stands for any mode involving photons (or perhaps) lepton pairs in addition to hadrons. x_h and x_i are the x-parameters for the individual channels. We then obtain

$$\sum_i x_i\Gamma_i = \Gamma_{ch}\left\{1 - \frac{A}{B}(2 + x_h C)\right\},$$

$$\sum_i \Gamma_i = \Gamma_{ch}\left\{\Gamma_{ch}\frac{A}{B^2} - \frac{A}{B}(2 + C)\right\}.$$

Clearly.

$$\rho = \operatorname*{Max}_i \{x_i\} \geqslant \frac{1 - \frac{A}{B}(2 + x_h C)}{\Gamma_{ch}\frac{A}{B^2} - \frac{A}{B}(2 + C)} \geqslant \operatorname*{Min}_i \{x_i\} \equiv \sigma.$$

Since $x = B^2/A\Gamma_{ch}$, we find from the above inequalities,

$$x_{max} = \frac{\rho}{1 - \frac{A}{B}[2(1-\rho) + C(x_h - \rho)]}$$

$$x_{min} = \frac{\sigma}{1 - \frac{A}{B}[2(1-\sigma) + C(x_h - \sigma)]}$$

which will clearly depend on the decay schemes of the particle concerned in any particular theoretical model. Experimental determination of x would thus be very useful for distinguishing between alternative theoretical schemes. We shall discuss our scheme below.

The present experiments on the ψ-particles (since they do not have sufficient resolution and therefore can measure reliably only the peak heights) provide values for A (from Bhabha scattering) and B (from $e^+e^- \to$ two charged particles $+ \ldots$). These numbers are still preliminary. We take $A/B \simeq 1/16$ for the $\psi(3\cdot1)$ for illustration. The value of C is known from e^+e^- annihilation experiments just below the $\psi(3\cdot1)$ peak to be $\simeq 3$, in a reasonable model discussed in the following subsection.

7.2 $\psi(3\cdot1) \equiv S^0_-$ *decays: Selection rules*

In our scheme the $\psi(3\cdot1)$ is identified with the S^0_- so that it has $Z = \pm 1$ components in a $C = -$ state with $I = Y = 0$. It clearly cannot decay through (Z-conserving) strong interactions into the usual hadrons with $Z = 0$. Of course, since we expect a 15-plet of pseudoscalar mesons also, it could decay strongly into suitable members of this along with the usual hadrons, provided that the paracharged pseudoscalars (PS) have sufficiently lower masses. These masses, however, cannot be much lower since the experimental width is known to be exceptionally small. It is interesting to note that $S^0_- \not\rightarrow S^0_-$ (PS) $+ \pi$ by isospin conservation; $S^0_- \not\rightarrow S^0_+$ (PS) $+ 2\pi$ by parity, charge conjugation and Bose statistics for the pions. $S^0_- \rightarrow S^0_-$ (PS) $+ 2\pi$ will go only in high angular momentum states [with the two pions in mutual $l = 2$ and together in $l = 2$ relative to S^0_- (PS), at least] and so will be suppressed. We then expect the strong decays to be negligible.

(i) The dominant mechanism for the decay of the S^0_- in our scheme is through the electromagnetic interaction, since the current involved can also change Z. Among these the main mode is expected to be

$$S^0_- \rightarrow \mathrm{h} + \gamma,$$

where h stands for one or more hadrons, and $|\triangle Z| = 0, 1$. Consider first the piece of the electromagnetic current $\mathcal{G}^{13}_\mu$, which effects $|\triangle I| = \triangle Y = 0$, $|\triangle Z| = 1$ transitions; then h must have $Z = I = Y = 0$ and $C = +$. Thus $S^0_- \not\rightarrow \pi^0 + \gamma$. The cases where h is a single particle (resonance) include

$$S^0_- \rightarrow \eta + \gamma, \quad \eta' + \gamma, \quad f + \gamma, \quad f' + \gamma, \text{ etc.}$$

In addition, because of the piece $\mathcal{F}^3_\mu + \frac{1}{2}\mathcal{F}^Y_\mu$, we shall have the decay

$$S^0_- \rightarrow S^0_+ (\mathrm{PS}) + \gamma,$$

assuming, of course, that the S^0_+ (PS) has a lower mass than the S^0_-. Since two body modes are available, we shall at this preliminary stage neglect more body modes, which will be relatively suppressed due to phase space limitations.

The special quark contents (*e.g.*, $S \sim |\lambda\bar{\chi}\rangle$ etc.) of the mesons will be made use of to further delimit the possible radiative decay modes by the well-known approximate rule* of using planar quark duality diagrams only (often called Zweig's rule; see for a review, Rosner 1974). Thus, since the pion has no λ or χ components, $S^0_- \rightarrow n\pi + \gamma$ will, according to this rule, be very rare. The same rule also suppresses $S^0_- \rightarrow f + \gamma$

Thus the dominant modes are:

$$S^0_- \rightarrow \eta + \gamma, \eta' + \gamma, f' + \gamma \text{ (anomalous current)},$$

$$S^0_- \rightarrow S^0_+ (\mathrm{PS}) + \gamma \quad \text{(normal current)}.$$

Of these, the $f'\gamma$ mode will be the fastest (E1) radiative transition.

(ii) Next in order will be the electromagnetic modes

$$S^0_- \rightarrow \text{hadrons (h)},$$

that can go according to the diagrams in figure 2 (a, b). Clearly h can be either with $I = 0$ or $I = 1$, with $C = -$. We expect the contribution of figure 2b to be

* Footnote (See p 111).

small in comparison with that of figure 2 *a* (as suggested by the success of a similar treatment of the usual vector meson decays, Gell-Mann *et al* (1962)). In that case, since we must have $J^P = 1^-$ for the h, for estimates on the exclusive decay channels we could use the intermediaries ρ, ω, ϕ, ρ', etc. However, we are interested here in the total $S^0_- \to$ hadrons width, which is more readily and reliably obtained from the e^+e^- annihilation experiments using the ratio equality

$$\frac{\Gamma(S^0_- \to \text{hadrons})}{\Gamma(S^0_- \to \mu^+\mu^-)} = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \simeq 3$$

at an energy just below the ψ (3·1) peak (discarding, as explained, figure 2 *b*).

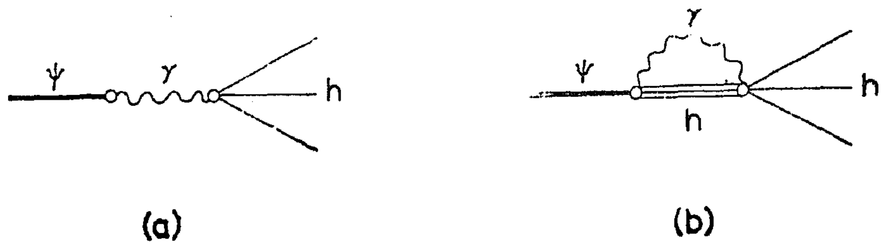

Figure 2. Mechanisms for the decays of the ψ particles into hadrons: (*a*) *via* a single photon intermediate state and (*b*) *via* an intermediate state of a photon + hadrons.

(iii) We have also the decay modes

$$S^0_- \to e^+e^- (\mu^+\mu^-),$$

$$S^0_- \to e^+e^- (\mu^+\mu^-) + \text{hadrons}.$$

We shall ignore the latter. The former mode is what determines the Bhabha scattering cross-section at the ψ (3·1) peak, as shown in figure 1.

7.3 ψ (3·1) *decays: Estimates of partials widths*

We now consider in some detail the dominant modes $S^0_- \to S^0_+$ (PS) $+ \gamma$ (*via* the normal current) and $S^0_- \to \eta + \gamma, \eta' + \gamma f' + \gamma$ (*via* the anomalous current). For illustration, consider the matrix element relevant for $S^0_- \to A + \gamma$, where A is a $J^P = 0^-$ meson, given by

$$\langle A(k) \mid e J^{em}_\mu(0) \mid S^0_- ; p, \epsilon \rangle = e \frac{G_A(q^2 = 0)}{m} \frac{\epsilon_{\mu\nu\rho\sigma}\epsilon_\nu p_\rho k_\sigma}{\sqrt{4p_0 k_0 V^2}},$$

where $q = p - k$, and m is a mass parameter so that G_A is a dimensionless form factor. The decay rate is

$$\Gamma(S^0_- \to A + \gamma) = \frac{\alpha G^2_A(0)}{24} \left(\frac{m_S}{m}\right)^2 \left(1 - \frac{m^2_A}{m^2_S}\right)^3 . m_S,$$

where m_A and m_S are the masses of the A and the S^0_-.

For the case A $= S^0_+$ (PS), the partial width is sensitive to the mass of the pseudoscalar particle. If we estimate it using the spin-SU_4 spin independent relation*

* This is the analogue of the spin-unitary (SU_3) spin independence relation $m^2(K^*) - m^2(K) = m^2(\rho) - m^2(\pi)$ (Babu 1964, Schwinger 1964).

$$m^2(S) - m^2(S(PS)) \simeq m_\rho^2 - m_\pi^2 \simeq 0{\cdot}57\ \mathrm{GeV}^2,$$

to be $\simeq 2{\cdot}9$ GeV, and if we take $m \simeq 1$ GeV (characteristic of the normal current) and $G_A^2(0) \simeq 1$, we obtain $\Gamma(S_-^0 \to S_+^0(PS) + \gamma) \simeq 20$ keV. This estimate, like others in this section has, at best, only a qualitative value, considering that small changes in the mass of $S_+^0(PS)$ can result in Γ changing by a substantial factor. It is worth mentioning that roughly the same estimate is obtained ($\simeq 30$ keV) by comparing with the known $\Gamma(\omega \to \pi^0\gamma)$ in the quark model.

In the case $A = \eta$ (or η'), the decay must go *via* the anomalous current. Since the anomalous current must have an identically zero "charge", the effective current of the particles involved has the form $\partial_\lambda T_{\lambda\mu}$, with an antisymmetric $T_{\lambda\mu}$ built out of the phenomenological fields of the particles. Then the effective form factor has the form

$$G_\eta(q^2) = \beta \frac{m}{M}\left(\frac{q.p}{m_S^2}\right) F_\eta(q^2).$$

Here the factor $q.p$ $(= \frac{1}{2}[m_S^2 - m_\eta^2 + q^2])$ takes care of the requirement of vanishing "charge" and $F_\eta(q^2)$ is supposedly a relatively smooth form factor. Taking $M \simeq m_s$ (characteristic of anomalous currents), $F_\eta(0) \simeq 1$, we obtain

$$\Gamma(S_-^0 \to \eta\gamma + \eta'\gamma) \simeq 180\,\beta^2\ \mathrm{keV}.$$

(In this estimate we have ignored the small mixing of η and η' and considered only their $\lambda\bar{\lambda}$-contents in line with the quark-rules).† An estimate of β is quite difficult at this stage. We may attempt to get a very rough idea of its magnitude from $\Gamma(S_-^0 \to e^+e^-)$. The width Γ $(S_-^0 \to e^+e^-)$ which is due to the anomalous current is expected to be of the order of $\beta^2(m_s/m_\rho)$ with respect to a typical width *via* the normal current like $\Gamma(\rho^0 \to e^+e^-)$, taking the dimensionless photon-vector meson coupling constant to be about the same in both cases. To the extent that these widths are not very different, we have $\beta^2 \simeq \frac{1}{4}$. With this estimate we obtain

$$\Gamma(S_-^0 \to \eta\gamma + \eta'\gamma) \simeq 45\ \mathrm{keV}.$$

In a similar way we estimate $\Gamma(S_-^0 \to f' + \gamma) \simeq 150$ keV. It must be emphasized that all our estimates are very preliminary at this stage, specially since the relevant electromagnetic form-factors are unknown. Also the parameters β and M may need revision in future.

These highly rough estimates are indicative only of the orders of magnitudes involved. They also emphasize the importance of the decay channels considered in relation to some of the observed features. For example, the decay $S_-^0 \to S_0^+$ (PS) $+ \gamma$ may well account for the K-mesons in the final products in the decay of the $\psi(3{\cdot}1)$ already indicated by experiments at Frascati. The K-mesons are expected, since the dominant decay mode of the S_+^0 (PS) will be $S_+^0(PS) \to \phi + \gamma$ (see Table 1) and the ϕ decays mostly into $K\bar{K}$. The f' in $S_-^0 \to f' + \gamma$ also decays mostly to $K\bar{K}$. Also from the decay modes $S_-^0 \to \eta(\eta') + \gamma$, the subsequent decays of the η and the η' give rise exclusively to charged pions and neutrals $(\gamma, \pi^0 \to 2\gamma)$ as the final decay products.

Using our estimates for the partial decay widths, and the various branching ratios for the decays of the, f' ϕ, η, η', we find

† Footnote (See p 111).

$$\Gamma_{ch} \simeq 130 \text{ keV}.$$

We also compute the parameter x from the same analysis to be

$$x \simeq 0 \cdot 6.$$

It is interesting to note here that the bounds on x discussed in section 7.1, which do not depend on a knowledge of the detailed branching ratios, are numerically

$$0 \cdot 33 \lesssim x \lesssim 0 \cdot 86.$$

If we are prepared to assume that $\Gamma(S^0_- \to \eta\gamma) = 2\Gamma(S^0_- \to \eta'\gamma)$, as suggested by the quark rule, then these bounds get further strengthened to $0 \cdot 55 \lesssim x \lesssim 0 \cdot 71$. In getting these numbers we have taken $x_h = 1$, clearly suggested by the data on $e^+ e^- \to$ hadrons just below the $\psi(3 \cdot 1)$ peak.

7.4 $\psi(3 \cdot 7)$ *decays*

The $\psi(3 \cdot 7)$ (which is the S^0_- in our scheme) has the same quantum numbers as the $\psi(3 \cdot 1)$. It can, therefore, decay strongly into the $\psi(3 \cdot 1)$ and pions (one pion is not allowed by isospin conservation). The most prominent such mode will be

$$\psi(3 \cdot 7) \to \psi(3 \cdot 1) + 2\pi \text{ (all S-waves)}.$$

However, this decay could be strongly suppressed through the Adler zero arising from the special role played by the pion in PCAC and chiral symmetry†. Another strong interaction channel is

$$\psi(3 \cdot 7) \to S^0_-(\text{PS}) + 2\pi.$$

This will also be suppressed for the same reasons as discussed for the decay of the $\psi(3 \cdot 1)$ in section 7.2. Nevertheless, the larger phase space here is likely to make this mode not entirely negligible. At the present moment, we cannot give any quantitative estimates of these widths.

The strong two body modes apparently allowed are (i) $S' \to S(\text{PS}) + \eta$, $D(\text{PS}) + \bar{K}$, $\bar{D}(\text{PS}) + K$ and (ii) $S' \to S + \eta$, $D + \bar{K}$, $\bar{D} + K$. By the SU_4 symmetry the set (i) is related to $\rho' \to \pi\pi$ (only F-type coupling allowed) and the set (ii) is related to $\rho' \to K^*\bar{K}$, $\omega\pi$ (only D-type coupling allowed). It is known that $\rho' \to 2\pi$ is absent. There is also at present no evidence for $\rho' \to K^*\bar{K}$ and $\omega\pi$. From this we conclude that the above modes of the S′ decays will be negligible compared to $S^{0\prime}_- \to S^0_- + 2\pi$, as in the case of the dominance of $\rho' \to \rho + 2\pi$ in the ρ' decays.

Concerning the radiative modes, most of the discussion of section 7.2 applies here also. Additionally, we also have the decay channel (*via* the normal current)

$$\psi(3 \cdot 7) \to S^0_+(3 \cdot 1) + \gamma$$

If we estimate $\Gamma(\psi(3 \cdot 7) \to \eta\gamma + \eta'\gamma + f'\gamma)$ and $\Gamma(\psi(3 \cdot 7) \to S^0_+(\text{PS}) + \gamma)$ by assuming the couplings involved to be the same as for the case of $\psi(3 \cdot 1)$, and correcting only for the differences in phase space, we get the rough values 350 keV and 800 keV respectively. These two modes, therefore, contribute approximately 650 keV to $\Gamma_{ch}(\psi(3 \cdot 7))$. We must emphasize once again that these

† A similar possibility in a "charmonium" scheme is discussed by J. Pasupathy (1974). We thank Dr. Pasupathy for discussions on this point. *See* also Callan *et al* (1975).

estimates are sensitive to the mass and coupling parameters involved, and are thus to be taken only as rough indications. Together with the Γ_{ch} arising from the strong interaction decays and the decay $\psi(3\cdot7) \to S^0_+(3\cdot1)+\gamma$, as well as other minor modes, we thus expect the total charged width $\Gamma_{ch}(\psi(3\cdot7))$ to be around 1 MeV.

7.5 *Decays of the other new particles*

The new expected particles, the P at around 4.3 GeV, and the P′ at around 5 GeV, both have $I = Y = Z = 0$. They can decay by strong interactions into normal hadrons. Of course, if they were strictly $|\chi\bar{\chi}\rangle$ states, as in the ideal mixing situation, and if we were to strictly follow the quark duality rule‡ (Rosner 1974) the strong decays would be forbidden. However, as is well known from the decay $\phi \to \rho\pi$, the ideal mixing is only an approximation (especially in the wave function). Also the quark rules can only be approximate. Thus we certainly expect large widths for the P and the P′, typical of strong interaction decays.

We shall not enter here into a detailed discussion of the decays of all the new particles expected in our scheme. The simplest two body decay modes of the new mesons based on quantum number selection rules, as also the quark rule, are summarised in table 1.

Table 1. Dominant decay channels of the vector meson 15-plet (containing the ρ meson) and of the pseudoscalar (PS) meson 15-plet. The second vector meson 15-plet [containing the ρ' (1600) meson] will have all these channels available as well as a few more (on account of its higher mass) some of which we have indicated in the text. We would like to stress that we have used, in this enumeration, the quark-duality selection rule† which may be quite approximate. Decay channels with more than two particles in the final state are not listed when 2-body modes are available at approximately the same energy. In all radiative modes, an asterisk*) on γ denotes a transition induced by the "normal" current $\mathcal{F}^i_\mu$; otherwise through "anomalous" current $\mathcal{G}^i_\mu$.

A. Strong decays of the vector mesons[a]

1. $P \to [P\,(PS) + \pi\pi]^b$

   $[K + \bar{K}, K + \bar{K}^*, K^* + \bar{K}, K^* + \bar{K}^*]^c$

   $[\eta + \omega, \eta' + \omega, \eta + \phi, \eta' + \phi]^c$

2. $S^0_\pm \to [S^0_\pm\,(PS) + \pi\pi]^b$

3[d]. $D^+ \to D^+\,(PS) + \pi^0$

   $D^0\,(PS) + \pi^+$

4[d]. $D^0 \to D^+\,(PS) + \pi^-$

   $D^0\,(PS) + \pi^0$

[a] Some or all the modes involving the PS mesons may actually be forbidden since their masses (which are likely to be quite close to those of the corresponding vector mesons) cannot be deduced exactly.

[b] Strongly inhibited by high angular momentum barriers and limited phase space.

[c] Allowed by the selection rules (including the quark-duality rule) since P is not expected to be a pure $|\chi\bar{\chi}\rangle$ state. Only decays arising from a $|\lambda\bar{\lambda}\rangle$ admixture are listed.

[d] The decays of $\bar{D}^0$ and D^- are obtained from these by charge conjugation.

‡† Footnote (See p 111).

Table 1. (*Contd.*)

B. Radiative decays of the vector mesons

1. $P \to S_+^0 + \gamma$	3. $S_+^0 \to \phi + \gamma$
$S_+^0\,(PS) + \gamma$	$S_-^0\,(PS) + \gamma^*$
$P\,(PS) + \gamma^*$	4[e]. $D^+ \to K^+ + \gamma$
$(\eta + \gamma^*)$[a]	$K^{*+} + \gamma$
$(\eta' + \gamma^*)$[b]	$D^+\,(PS) + \gamma^*$
2. $S_-^0 \to (\eta + \gamma)$[c]	5[e]. $D^0 \to K^0 + \gamma$
$(\eta' + \gamma)$[c]	$K^{*0} + \gamma$
$f' + \gamma$	$D^0\,(PS) + \gamma^*$
$S_+^0\,(PS) + \gamma^*$	
$(P\,(PS) + \gamma)$[d]	

[a] If the only admixture in P (which is mostly $|\chi\bar{\chi}\rangle$) is $|\lambda\bar{\lambda}\rangle$.

[b] Even if P is pure $|\chi\bar{\chi}\rangle$, this will go through because P (PS) and η' are non-ideally mixed.

[c] Quark selection rule allows both since η and η' are non-maximally mixed.

[d] If P (PS) has mass $< 3{\cdot}105$ GeV

[e] The decay modes of $\bar{D}^0$ and D^- are obtained by charge conjugation.

C. Radiative decays of the pseudoscalar mesons[a]

3[c]. $S_+^0\,(PS) \to \phi + \gamma$

4[d]. $D^+(PS) \to K^{*+} + \gamma$

5[d]. $D^0(PS) \to K^{*0} + \gamma$

$|\chi\bar{\chi}\rangle$, then the main admixture is just $|\lambda\bar{\lambda}\rangle$.

ω has no $|\lambda\bar{\lambda}\rangle$ admixture.

obtained by charge conjugation.

Qualitative aspects of the decays of the baryons (with $Z \neq 0$) are quite interesting, however. The first point to note is that none of the new baryons in the $\underline{20}_s$ can be higher than Ω_0^- (the well-known Ω^-). As for the $\underline{20}$ representation, simple quark counting (or equivalently the mass formula) tells us only that $\underline{3}$ $(Z = 2)$ is the most massive. We cannot say which of the two, $\underline{3}^*$ $(Z = 1)$ and $\underline{6}$ $(Z = 1)$, is the lighter (they are quite likely to be close in mass, much as Σ and Λ are, in the corresponding SU_3 case). So we *can* say that $M\,(6) > M\,(\Omega^-)$, otherwise Ω^- will decay rapidly into Ω_1^- (which is a member of $\underline{6}$) and a photon. The second qualitative remark is that in spite of their often high Z and Y values, *all* the new particles are unstable against radiative decay, whatever be their relative masses. For example, the most massive, Ω_3^{*-}, will cascade down by photon and pion emission to Ω_0^-, a typical chain being $\Omega_3^* \xrightarrow{\gamma} \Omega_2^* \xrightarrow{\pi\text{'s}} \Omega_2 \xrightarrow{\gamma} \Omega_1 \xrightarrow{\gamma} \Omega_0$ (for the notation, see section 6). A fair fraction of paracharged particles produced (in association) in high energy pp collisions will thus end up as Ω_0^-, and the presence of substantial numbers of these and bunches of monochromatic photons in ISR and NAL experiments is another qualitative test of our scheme.

8. $e^+ e^-$-Annihilation

Before the discovery of the ψ-particles, the experiments* on $e^+ e^-$-annihilation into hadrons had already begun to indicate some puzzles—the so-called " energy crisis ", the constancy of the cross-section, etc. Our scheme opens up at least qualitative pathways for understanding these apparent problems. Everyone of the radiative decay modes of each of the expected new particles denotes (by crossing one particle line) also a new channel in $e^+ e^-$-annihilation. We give in table 2 a catalogue of channels in $e^+ e^-$-annihilation in order of increasing threshold. As in section 7.2, we take $m\,(\mathrm{S\,(PS)}) \simeq 2.9$ GeV $\simeq m\,(\mathrm{D\,(PS)})$. It is obvious from table 2 that a very large number of channels open from around $\sqrt{s} \simeq 3{\cdot}5$ GeV.

To estimate the contribution of these channels to the annihilation cross-section, it is necessary to have an idea of the cross-section into a typical channel through the intermediary of a virtual (anomalous current) photon. For example, the cross-section for $e^+ e^- \rightarrow S^0_-\eta$ is given by

$$\sigma\,(S^0_-\eta) = \frac{\pi a^2}{24M^2}\,\beta^2 F_\eta^2\,(s)\left[\frac{s + m_S^2 - m_\eta^2}{m_S^2}\right]^2 \times$$
$$\times\,\frac{1}{s^3}\left[s^2 - 2s\,(m_S^2 + m_\eta^2) + (m_S^2 - m_\eta^2)^2\right]^{3/2},$$

where β, M and $F_\eta\,(q^2 = s)$ have been defined in section 7.2 (iii). Since $F_\eta\,(s)$ has poles at $s = m^2$ (S) and at $s = m^2$ (S′), we expect an enhancement of this cross-section just above threshold. Since the $\psi\,(3{\cdot}7)$ is our $S^{0'}_-$, this effect will be most pronounced at $\sqrt{s} = 3{\cdot}7$ GeV. With the values of M and β as estimated earlier, and taking the $\bar{S}S\eta$ coupling constant to be of the order of a typical strong interaction coupling strength, we find that it is very reasonable to have $\sigma\,(S^0_-\eta)$ of the order of $0{\cdot}1$–$0{\cdot}2$ nb. Bearing in mind that about 15 new channels open in the energy range $3{\cdot}5$ to 4 GeV, it can be safely estimated that a fraction of the order of 10% of the annihilation cross-section goes into the production of the new particles. The decays of each of these new particles, according to table 1, are dominantly into a high-energy (~ 1 GeV) photon plus hadrons. Furthermore, the final resulting hadrons are mostly either neutral or are K mesons. The large fraction of energy carried by neutrals (the energetic photons from the initial decay, as well as the much softer photons from the decays of the final π^0's) will provide in our view a simple explanation of the decrease in the fraction of the total energy

* All aspects of e^+e^- annihilation that are touched upon in this section are reviewed in the London Conference report of Richter (Richter 1974).

Table 2. New Channels in e^+e^- annihilation *via* one intermediate photon at $\sqrt{s} \gtrsim 3.5$ GeV. Channels involving particles of the primed multiplet are not indicated; they are easily worked out (the lowest such threshold will be around 4.1 GeV).

Channels	Threshold (GeV)
D^+K^-, D^-K^+, $D^0\bar{K}^0$, $\bar{D}^0K^0$	3·5
$S^0_-\eta$	3·65
D^+ (PS) K^*, D^- (PS) K^{*+}, $\bar{D}^0$ (PS) K^{*0}, D^0 (PS) $\bar{K}^{*0}$	3·9
D^+K^{*-}, D^-K^{*+}, $\bar{D}^0K^{*0}$, $D^0\bar{K}^{*0}$	3·9
(S^0_+ (PS) ϕ)[a]	4·0
$S^0_-\eta'$	4·0
$S^0_+\phi$	4·1
S^0_- (PS) f'	4·5
S^0_-f'	4·6
($P\eta$)[b]	4·8
($P\eta'$)[b]	5·2
D^+D^- (PS), D^-D^+ (PS), $D^0\bar{D}^0$ (PS), $\bar{D}^0D^0$ (PS)	5·8
$S^0_+S^0_-$ (PS), $S^0_-S^0_+$ (PS)	6·0
(S^0_-P (PS))[c]	> 6·0
etc.	

[a] There is no S^0_+ (PS) ω because here we assume ideal ω-ϕ mixing.

[b] Since P has a $|\lambda\bar{\lambda}\rangle$ component.

[c] If m (P(PS)) > 3·105 GeV.

carried away by charged particles at $\sqrt{s} = 3{\cdot}5$–5 GeV (it must be remembered that this fraction is computed by assuming all charged particles to be simply pions.) The singularity in $F_\eta(s)$ at $\sqrt{s} = m_{s'}$ due to the ψ (3·7) is sufficient to account for the pronounced dip in $\langle E_{ch} \rangle/\sqrt{s}$ observed at $\simeq 3{\cdot}7$ GeV (Richter 1974). This mechanism, peculiar to our model, is also able to account qualitatively for the fact that at $\sqrt{s} \simeq 4{\cdot}8$ GeV, the total number of K^- mesons in the final states is approximately 10% of the number of π^- mesons (Richter 1974).

We may also note that, since the dimension of the anomalous current is 4 in the free quark field theory, its contribution to the annihilation cross-section is expected to dominate over the normal current contribution and to attain constancy at asymptotic energies. It is difficult to say at what energy the asymptotic behaviour sets in. With the opening up, at energies a little beyond 3·5 GeV, of a sizable number of new particle channels, an apparent constancy of the cross-section (ignoring possible fine structures) need not be surprising. Of course, such constancy is not the asymptotic constancy.

9. Concluding comments

(a) The most important and immediately testable prediction of our scheme is the existence of two further rather broad resonances (the P and the P′) in e^+e^- collisions*, one around 4·1–4·3 GeV, and the other around 5·0–5·2 GeV.

(b) Since the dominant decays of the $\psi(3.1)$ are into $f' + \gamma, \eta + \gamma, \eta' + \gamma$, $S^0_+ (PS) + \gamma$, a very important feature is the expectation of monochromatic bunches of high energy photons both at the resonance peaks and above the thresholds of production of the new particles, many of which have still to be discovered. The new hadrons proposed in our scheme have very specific quantum numbers and other properties—in particular, all of them are electromagnetically unstable (see, *e.g.*, table 1). From table 1, it is also clear that among the decay products of the new mesons a major fraction will consist of neutrals and K mesons.

The presence of energetic bunches of monochromatic photons is also a characteristic feature of the decays of the new heavy baryons. Since among the baryons there are many with the (I, Y) values of the Ω^-, another typical expectation is the occurrence of exceptionally large numbers of Ω^-'s in very high energy pp collision products.

(c) The parameter x introduced in section 7.1 is, for the reason indicated above, a sensitive experimental parameter distinguishing between various models. We have a definite range for it from the study of the decay mechanisms (section 7). This is also an easily available experimental check on our scheme. It simply requires the detection of all neutral particles in $e^+ e^-$ annihilation at the ψ-peaks.

(d) A qualitative explanation of the somewhat surprising behaviour of the processes $e^+e^- \rightarrow$ hadrons above $\sqrt{s} \simeq 3$ GeV is available in our scheme. We may also mention that in the deep inelastic lepton-proton scattering the single production of paracharged particles is absent to the extent that the quark duality rule is strictly valid. Above the threshold ($\nu \simeq 25$ GeV) for associated production of the new particles, such channels can be expected to play a major role. This would appear as an apparent violation of scaling. Apart from this, the dimension 4 of the new anomalous currents would lead to definite violations of scaling in the *asymptotic* region.

(e) The present scheme, if it proves to be useful, opens up new lines of work. We have to examine the chiral version of the higher symmetry giving a special role to the enlarged multiplet of the pseudoscalar mesons. For purposes of hadron spectroscopy the question of spin, SU_4-spin independence is also worth considering. The detailed investigation of the new components in the weak and electromagnetic hadronic currents is expected to be fruitful. Implications of our mechanisms for generating high energy photons in stellar evolution as also in laboratory experiments are of equal interest. We hope to report our considerations on these and other related questions in future publications.

Acknowledgement

We thank Drs. V Gupta, G Rajasekaran and P V Ramanamurthy for informative correspondence. Our thanks are also due to colleagues at TIFR, especially Drs. P K Malhotra and J Pasupathy, for stimulating discussions.

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* After this work was done, we learnt that the cross section $\sigma(e^+ e^- \rightarrow$ charged particles) shows a pronounced peak at $\sqrt{s} = 4{\cdot}15$ GeV, with a width of $\simeq 250 - 300$ MeV (Augustin *et al* SLAC–PUB–1520 LBL–3621, 1975). We thank Professors P V Ramanamurthy and B V Sreekantan for bringing this promptly to our notice.

Appendix 1

Some useful results on the SU_4 group

(1) *The generators*

Let us introduce creation and annihilation operators for the basic quartets:

$$|\,p\,\rangle = a^{*1}\,|\,0\,\rangle, \quad |\,\bar{p}\,\rangle = b^{*1}\,|\,0\,\rangle,$$
$$|\,n\,\rangle = a^{*2}\,|\,0\,\rangle, \quad |\,\bar{n}\,\rangle = b^{*2}\,|\,0\,\rangle,$$
$$|\,\lambda\,\rangle = a^{*3}\,|\,0\,\rangle, \quad |\,\bar{\lambda}\,\rangle = b^{*3}\,|\,0\,\rangle,$$
$$|\,\chi\,\rangle = a^{*4}\,|\,0\,\rangle, \quad |\,\bar{\chi}\,\rangle = b^{*4}\,|\,0\,\rangle;$$
$$a_\alpha\,|\,0\,\rangle = b_\alpha\,|\,0\,\rangle = 0, \quad (\alpha = 1, 2, 3, 4); \tag{A.1}$$

with the anticommutation relations

$$[a_\alpha, a^{*\beta}]_+ = [b_\alpha, b^{*\beta}]_+ = \delta^\beta_\alpha, \ (\alpha, \beta = 1, 2, 3, 4) \tag{A.2}$$

and with all other anticommutators vanishing. Then the 15 generators of our SU_4 group are given by (note $\sum_{\gamma=1}^{4} F^\gamma_\gamma = 0$):

$$F^\alpha_\beta = (a^{*\alpha}\, a_\beta - b^{*\beta}\, b_\alpha) - \tfrac{1}{4}\, \delta^\alpha_\beta \sum_{\gamma=1}^{4} (a^{*\gamma}\, a_\gamma - b^{*\gamma}\, b_\gamma) \tag{A.3}$$

They satisfy (on account of eq. (A.2)) the correct commutation relations for the generators:

$$[F^\alpha_\beta, F^\gamma_\delta] = F^\alpha_\delta\, \delta^\gamma_\beta - F^\gamma_\beta\, \delta^\alpha_\delta. \tag{A.4}$$

For the basic quartet ξ we have, for example, because of eq. (A.1),

$$-(F^4_4)_\xi = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & -\frac{3}{4} \end{pmatrix} \tag{A.5}$$

so that the paracharge operator is given by

$$Z = \tfrac{3}{4}\, B + F^4_4. \tag{A.6}$$

The conventional SU_3 sub-group (of our SU_4) has the eight generators $f^i_j (i, j = 1, 2, 3;\ \sum_{i=1}^{3} f^i_i = 0)$ given by:

$$f^i_j = F^i_j, \quad i \neq j, \quad (i, j = 1, 2, 3);$$
$$f^i_i = F^i_i + \tfrac{1}{3}\, F^4_4, \quad (i = 1, 2, 3). \tag{A.7}$$

In the *quartet representation* ($\xi \sim \underline{4}$) we may now introduce the more conventional hermitian 4×4 representation matrices:

$$F_1 \equiv F^1_2 + F^2_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \lambda_1$$

$$F_2 \equiv \mathrm{i}\,(F_2^1 - F_1^2) = \begin{pmatrix} 0 & -\mathrm{i} & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \lambda_2$$

$$F_3 \equiv \tfrac{1}{2}\,(F_1^1 - F_2^2) = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \tfrac{1}{2}\lambda_3$$

$$F_4 \equiv (F_3^1 + F_1^3) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \lambda_4$$

$$F_5 \equiv \mathrm{i}\,(F_3^1 - F_1^3) = \begin{pmatrix} 0 & 0 & -\mathrm{i} & 0 \\ 0 & 0 & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \lambda_5$$

$$F_6 \equiv (F_3^2 + F_2^3) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \lambda_6$$

$$F_7 \equiv \mathrm{i}\,(F_3^2 - F_2^3) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} & 0 \\ 0 & \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \lambda_7$$

$$F_8 \equiv -\,(F_3^3 + \tfrac{1}{3}\,F_4^4) = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{\lambda_8}{\sqrt{3}}$$

$$F_9 \equiv (F_4^1 + F_1^4) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} = \lambda_9$$

$$F_{10} \equiv \mathrm{i}\,(F_4^1 - F_1^4) = \begin{pmatrix} 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \end{pmatrix} = \lambda_{10}$$

$$F_{11} \equiv (F_4^2 + F_2^4) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \lambda_{11}$$

$$F_{12} \equiv \mathrm{i}\,(F^2_4 - F^4_2) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & 0 & 0 \\ 0 & \mathrm{i} & 0 & 0 \end{pmatrix} = \lambda_{12}$$

$$F_{13} \equiv (F^3_4 + F^4_3) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \lambda_{13}$$

$$F_{14} \equiv \mathrm{i}\,(F^3_4 - F^4_3) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & \mathrm{i} & 0 \end{pmatrix} = \lambda_{14}$$

$$F_{15} \equiv -F^4_4 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & -\frac{3}{4} \end{pmatrix} = \sqrt{\tfrac{3}{8}}\,\lambda_{15} \tag{A.8}$$

We also introduce

$$B = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} = \frac{\sqrt{2}}{3}\,\lambda_0 \tag{A.9}$$

The λ-matrices have the normalization: $T_r\,(\lambda_i)^2 = 2$.

(2) *The* 15-*plet*

It is useful to write the quark contents of the orthonormalized states in the 15-plet with their SU_3, I, Y, Z contents:

$$\underline{8}\,(Z=0): \begin{cases} -\,|\,\mathrm{p}\bar{\mathrm{n}}\,\rangle,\ \frac{1}{\sqrt{2}}(|\,\mathrm{p}\bar{\mathrm{p}}\,\rangle - |\,\mathrm{n}\bar{\mathrm{n}}\,\rangle),\ |\,\mathrm{n}\bar{\mathrm{p}}\,\rangle: \\ \quad I = 1\ (I_3 = +1, 0, -1),\ Y = 0; \\ |\,\mathrm{p}\bar{\lambda}\,\rangle,\ |\,\mathrm{n}\bar{\lambda}\,\rangle:\ \ I = \frac{1}{2}\,(I_3 = +\frac{1}{2}, -\frac{1}{2}),\ Y = +1; \\ -\,|\,\lambda\bar{\mathrm{n}}\,\rangle,\ |\,\lambda\bar{\mathrm{p}}\,\rangle:\ \ I = \frac{1}{2}\,(I_3 = +\frac{1}{2}, -\frac{1}{2}),\ Y = -1; \\ \frac{1}{\sqrt{6}}(|\,\mathrm{p}\bar{\mathrm{p}}\,\rangle + |\,\mathrm{n}\bar{\mathrm{n}}\,\rangle - 2\,|\,\lambda\bar{\lambda}\,\rangle):\ I = Y = 0; \end{cases}$$

$$\underline{1}\,(Z=0):\ \frac{1}{2\sqrt{3}}(|\,\mathrm{p}\bar{\mathrm{p}}\,\rangle + |\,\mathrm{n}\bar{\mathrm{n}}\,\rangle + |\,\lambda\bar{\lambda}\,\rangle) - \frac{\sqrt{3}}{2}\,|\,\chi\bar{\chi}\,\rangle:\ \ I = Y = 0.$$

$$\underline{3}\,(Z=-1): \begin{cases} |\,\mathrm{p}\bar{\chi}\rangle,\ |\,\mathrm{n}\bar{\chi}\rangle:\ \ I = \frac{1}{2}\,(I_3 = +\frac{1}{2}, -\frac{1}{2}),\ Y = +1; \\ |\,\lambda\bar{\chi}\rangle:\ \ I = Y = 0. \end{cases}$$

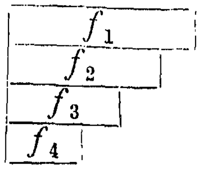
$$\underline{3}^*\,(Z=+1): \begin{cases} -\,|\,\chi\bar{\mathrm{n}}\,\rangle,\ |\,\chi\bar{\mathrm{p}}\,\rangle:\ I = \frac{1}{2}\,(I_3 = +\frac{1}{2}, -\frac{1}{2}),\ \ Y = -1; \\ |\,\chi\bar{\lambda}\rangle:\ \ I = Y = 0. \end{cases} \tag{A.10}$$

Also we have for the

$$\text{SU}_4\text{-Singlet:} \;\; \tfrac{1}{2}\left(|\,p\bar{p}\,\rangle + |\,n\bar{n}\rangle + |\,\lambda\bar{\lambda}\,\rangle + |\,\chi\bar{\chi}\,\rangle\right): I = Y = Z = 0. \tag{A.11}$$

(3) *General representations and their dimensions*

The U_4 irreducible representations are given by tensors with the symmetry of the Young tableaux with four rows with lengths $f_1, f_2, f_3, f_4; f_1 \geqslant f_2 \geqslant f_3 \geqslant f_4$:



$$\begin{aligned} \lambda_1 &\equiv f_1 - f_2 \\ \lambda_2 &\equiv f_2 - f_3 \\ \lambda_3 &\equiv f_3 - f_4 \\ \lambda_4 &\equiv f_4 \end{aligned} \tag{A.12}$$

For SU_4 irreducible representations we have to take only three rowed tableaux so that $\lambda_4 = 0$. The dimension of the representation is given by

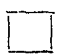
$$d = \frac{1}{12}(\lambda_1 + 1)(\lambda_2 + 1)(\lambda_3 + 1)\,[(\lambda_1 + \lambda_2 + 2)(\lambda_2 + \lambda_3 + 2)] \times [(\lambda_1 + \lambda_2 + \lambda_3) + 3]. \tag{A.13}$$

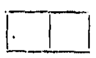
For only purely symmetric tensors:

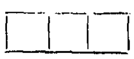
$$f_1 = \lambda, f_2 = f_3 = f_4 = 0;$$
$$\lambda_1 = \lambda,\ \lambda_2 = \lambda_3 = 0;$$
$$d = \tfrac{1}{6}(\lambda + 1)(\lambda + 2)(\lambda + 3). \tag{A.14}$$

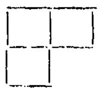
Examples:

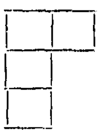
 : $\lambda_1 = \lambda_2 = \lambda_3 = 0 : d = 1$

 : $\lambda_1 = 1,\ \lambda_2 = \lambda_3 = 0\ ;\ d = 4$

 : $\lambda_1 = 2,\ \lambda_2 = \lambda_3 = 0 : d = 10$

 : $\lambda_1 = 3,\ \lambda_2 = \lambda_3 = 0 : d = 20$: $(\underline{20_s})$

 : $\lambda_1 = 1,\ \lambda_2 = 1,\ \lambda_3 = 0 : d = 20$

 : $\lambda_1 = 1,\ \lambda_2 = 0,\ \lambda_3 = 1 : d = 15$

 : $\lambda_1 = 0,\ \lambda_2 = 1,\ \lambda_3 = 0 : d = 6$

 : $\lambda_1 = 0,\ \lambda_2 = 0,\ \lambda_3 = 1 : d = 4 : (\underline{4^*})$ (A.15)

To find the physical contents, (SU_3-dimension, I, Y, Z)-values, it is important to remember that Y and Z are not the generators of our SU_4. The generators are F_8 and F_{15}, where the physical quantum numbers are

$$Y = F_8^8 - \tfrac{2}{3} Z; \quad Z = \tfrac{3}{4} B - F_{15} \tag{A.16}$$

depending crucially on the baryon number B. We, of course, still have strangeness $S = Y - B$. The quarks (p, n) have $S = 0$ and (λ, χ) have $S = -1$. We note below a few examples of the contents (SU_3-dim.; I, Y, Z):

$$\begin{aligned}
\underline{15}\,(B = 0) = {} & (\underline{8}; \tfrac{1}{2}, 1, 0) \oplus (\underline{8}; 1, 0, 0) \oplus (\underline{8}; 0, 0, 0) \oplus (\underline{8}; \tfrac{1}{2}, -1, 0) \\
& \oplus (\underline{3}; \tfrac{1}{2}, 1, -1) \oplus (\underline{3}; 0, 0, -1) \\
& \oplus (\underline{3}^*; \tfrac{1}{2}, -1, +1) \oplus (\underline{3}^*; 0, 0, +1) \oplus (\underline{1}; 0, 0, 0). \\
4^*\,(B = 1) = {} & (\underline{3}^*; \tfrac{1}{2}, -1, 1) \oplus (\underline{3}^*; 0, 0, 1) \\
& \oplus (\underline{1}; 0, 0, 0). \\
\underline{20}\,(B = 1) = {} & (\underline{8}; \tfrac{1}{2}, 1, 0) \oplus (\underline{8}; 1, 0, 0) \oplus (\underline{8}; 0, 0, 0) \oplus (\underline{8}; \tfrac{1}{2}, -1, 0) \\
& \oplus (\underline{6}; 1, 0, 1) \oplus (\underline{6}; \tfrac{1}{2}, -1, 1) \oplus (\underline{6}; 0, -2, 1) \\
& \oplus (\underline{3}^*; 0, 0, 1) \oplus (\underline{3}^*; \tfrac{1}{2}, -1, 1) \\
& \oplus (\underline{3}; \tfrac{1}{2}, -1, 2) \oplus (\underline{3}; 0, -2, 2). \\
\underline{20}_s\,(B = 1) = {} & (\underline{10}; \tfrac{3}{2}, 1, 0) \oplus (\underline{10}; 1, 0, 0) \oplus (\underline{10}; \tfrac{1}{2}, -1, 0) \\
& \oplus (\underline{10}; 0, -2, 0) \\
& \oplus (\underline{6}; 1, 0, 1) \oplus (\underline{6}; \tfrac{1}{2}, -1, 1) \oplus (\underline{6}; 0, -2, 1) \\
& \oplus (\underline{3}; \tfrac{1}{2}, -1, 2) \oplus (\underline{3}; 0, -2, 2) \\
& \oplus (\underline{1}, 0, -2, 3).
\end{aligned} \tag{A.15}$$

The construction of direct products of irreducible representations and the working out of the reduction into irreducible components follow completely standard lines (Singh 1966, Pais 1966, Itzykson and Nauenberg 1966). We simply quote an important result (Singh 1966) useful for our general mass formula (see the next subsection):

<u>Theorem</u>. The direct product $\underline{R} \otimes \underline{R}^* \otimes \underline{15}$ contains $\underline{1}$ n times,

$$n = \theta(\lambda_1 - 1) + \theta(\lambda_2 - 1) + \theta(\lambda_3 - 1)$$

where θ denotes the usual step function ($\theta(x) = 1, x \geqslant 0; = 0$ otherwise) and $(\lambda_1, \lambda_2, \lambda_3)$ signifies the irreducible representation R. (Note: the conjugate representation $\underline{R}^*$ is denoted by $(\lambda_3, \lambda_2, \lambda_1)$).

(4) *The mass formula*

Let us assume that the symmetry breaking terms in the effective Hamiltonian transform as irreducible tensor components A_8^8 (SU_3-breaking) and B_4^4 (SU_4-breaking). Now, for any regular tensor operator T_β^α (transforming as the $\underline{15}$-representation of SU_4) we have, due to the Wigner-Eckart theorem, the result that *within a given irreducible representation* $\underline{R}$,

$$T^{\alpha}_{\beta} = aF^{\alpha}_{\beta} + b \sum_{\gamma=1}^{4} \{[F^{\alpha}_{\gamma}, F^{\gamma}_{\beta}]_{+} - \text{the trace}\} +$$

$$+ c \sum_{\gamma, \delta=1}^{4} \{[F^{\alpha}_{\gamma}, [F^{\gamma}_{\delta}, F^{\delta}_{\beta}]_{+}]_{+} - \text{the trace}\} \qquad (A.16)$$

where a, b, c are the reduced matrix elements for the irreducible representation $\underline{R}$. In the special case of $\underline{R}$ being such that $\underline{R} \otimes \underline{15}$ contains $\underline{R}$ at most twice, we may drop the last term (*i.e.*, $c = 0$). Then we have *within* $\underline{R}$ (dropping the symmetrical contribution):

$$B^4_4 = M_1 Z + M_2 [C(p, q) - Z^2] \qquad (A.17)$$

$$A^3_3 = M_3 F_8 + M_4 [I(I+1) - \tfrac{1}{4} F^2_8 - \tfrac{1}{9} C(p, q)] \qquad (A.18)$$

where

$$C(p, q) \equiv p^2 + pq + q^2 + 3(p + q), \qquad (A.19)$$

and (p, q) denotes SU_3 irreducible representations in $\underline{R}$ in the highest weight notation. Thus the mass operator for $\underline{R}$ is given by

$$M = M_0 + B^4_4 + A^3_3 \qquad (A.20)$$

in terms of B^4_4, A^3_3 given above.

As examples, ignore A^3_3 (SU_3-breaking); then we have the baryon mass formulae:

$$\underline{20}\ (B = 1):\ \tfrac{3}{2} M(\underline{3}^*) + \tfrac{1}{2} M(\underline{6}) = M(\underline{8}) + M(\underline{3})$$

$$\underline{20}_S\ (B = 1):\ M(\underline{10}) - M(\underline{6}) = M(\underline{6}) - M(\underline{3}) = M(\underline{3}) - M(\underline{1}) \qquad (A.21)$$

The latter is in the form of an equal spacing rule.

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