Neutral currents in alternative U₃(W)-gauge models of weak and electromagnetic interactions

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Abstract. Two alternative U₃(W)-gauge models are presented. Both agree with the recent Abbott-Barnett fits to the neutrino-nucleon neutral-current data, and with the SLAC measurement of the asymmetry parameter for longitudinally polarised electron-deuteron inelastic scattering. Results for σ(μ⁺e⁻), σ(μ⁻e⁺) are also found in agreement with the latest measurements. The models differ in the parameter Qₚ(Z, N) characterising parity-violation in heavy atoms for which, however, the experimental situation is still unclear.

Keywords. Neutral currents; U₃(W)-gauge theory; neutrino interaction; polarised electron deuteron scattering asymmetry.

1. Introduction

In the present work, we discuss phenomena relating to weak neutral currents in the context of two new alternative models, based on the framework of a spontaneously broken U₃(W)—gauge theory (Pandit 1976), unifying weak and electromagnetic interactions. In contrast to the earlier model, the new alternatives proposed here agree very well with (i) the recent stringent determinations of the neutrino-quark and neutrino-electron neutral-current interaction parameters from the analyses of relatively new data on neutrino interactions, and (ii) the more recent crucial measurement at SLAC of the asymmetry parameter (AₑD) for the inelastic scattering of longitudinally polarised electrons off deuteron.

The standard minimal SU₂ ⊗ U₁ model (Weinberg 1967, Salam 1968, Glashow et al 1970) is already very well supported by the experimental determinations referred to above. Why then go into a more elaborate model, is a natural question to ask. Our answer is that these agreements are only sufficient, but not necessary, conditions for the validity of the model; hence, it is, at least, of some academic interest to present other models that work as well. Furthermore, in view of discoveries of a new heavy lepton (the τ) and of new heavy hadrons (the Υ's) it appears to us that it might turn out to be premature to foreclose all options other than that of the simplest standard model. In the U₃(W)-gauge theory we are forced to introduce in a critical manner new heavy quark flavours beyond the standard four (u, d, s, c), as well as new heavy leptons. As higher and higher energies become available in the future, more and

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more leptons and quarks might very well enter particle physics, and these might not enter in the simple sequential manner offered by the standard model.

We limit ourselves in this work to the common neutral current phenomena in order to establish a prima facie case for taking interest in the models proposed here.

2. General résumé of the $U_3(W)$-gauge scheme

Following (Pandit 1976), we denote by $G_a, a=1, 2, \ldots, 8$, the hermitian generators of $SU_3(W)$, and by $G_6$ the generator of $U_1(W)$, where $U_6(W) = SU_3(W) \otimes U_1(W)$. The different models we speak of are obtained by choosing fermions (leptons and quarks) to belong to different suitable representations of this group. The electric charge operator is given by

$$Q = (G_3 + \frac{1}{\sqrt{3}} G_8) + G_6. \quad (1)$$

$U_6(W)$-gauge invariant Lagrangian is constructed in the standard manner by the introduction of eight hermitian vector gauge fields coupled to the currents of $G_a, a=1, \ldots, 8$, with a single coupling constant $f$, and a ninth vector gauge field coupled to the current of $G_6$ with a coupling constant $f'$.

The gauge symmetry is spontaneously broken through the introduction (cf. Pandit 1976 and 1977) of three $SU_3(W)$ triplets of complex Higgs scalar fields

$$\phi^{(i)} = (\phi_1^{(i)}, \phi_2^{(i)}, \phi_3^{(i)}), \quad i=1, 2, 3, \quad \text{with } G_6 = -2/3 \text{ for } \phi^{(1)}$$

and $G_6 = +\frac{1}{3}$ for $\phi^{(2)}$ and $\phi^{(3)}$.

The 'vacuum' is then arranged to be such (by a suitable choice of the potential function) that $\phi_1^{(1)}, \phi_2^{(1)}$ and $\phi_3^{(1)}$ attain non-zero vacuum expectation values:

$$\langle \phi_2^{(2)} \rangle_0 = \langle \phi_3^{(3)} \rangle_0 = \eta \neq 0 \text{ and } \langle \phi_1^{(1)} \rangle_0 = \eta' \neq 0.$$  

This choice, allowing $\eta \neq \eta'$ in general, is made to meet the stringent requirement (necessary in any gauge theory seeking to unify weak and electromagnetic interactions) that only the gauge vector boson coupled to the electromagnetic current remains massless, while all the other eight (weak interaction) vector bosons become massive.

The vector boson fields diagonalising the mechanical mass-matrix generated by the above spontaneous breaking of the gauge symmetry, and their interactions are specified as follows: coupled with strength $f/\sqrt{2}$ are $W^\pm_\mu$ to the currents of $(G_1 \pm iG_2), R^\pm_\mu$ to the currents of $(G_1 \pm iG_3), H_\mu$ to the current of $(G_6 + iG_7)$ and $H^\pm_\mu$ to the current of $(G_6 - iG_7)$. Besides these non-hermitian fields, we have three hermitian neutral vector boson fields $A_\mu, Z_{1\mu}$ and $Z_2\mu$. $A_\mu$ couples to the electromagnetic current of $Q$ with strength $e/\sin \chi$. $Z_{1\mu}$ couples to the

$$(\phi_1 - \sin^2 \chi \cdot Q)\text{--current, with strength } \frac{e}{\sin \chi \cos \chi}. \quad (2)$$
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and $Z_{2\mu}$ couples to the

$$g_2-\text{current, with strength } -\frac{e}{\sqrt{3} \sin \chi},$$

(3)

where we have employed the notation:

$$g_1 = g_3 + \frac{1}{\sqrt{3}} g_8; \quad g_2 = - g_3 + \sqrt{3} g_8;$$

(4)

$$\tan \chi = (2f'/\sqrt{3}f).$$

(5)

The non-zero masses of the vector bosons are:

$$m^2(W^\pm) = m^2(R^\pm) = f^2 \eta^2(1 + \frac{1}{\delta});$$

$$m^2(Z_2) = f^2 \eta_2^2; \quad m^2(Z_1) = \frac{1}{\cos^2 \chi} \cdot f^2 \eta_1^2(1 + \frac{3}{\delta});$$

(6)

where the parameter $\delta$ is defined by

$$(\eta'/\eta_1)^2 = 1 + \delta, \quad (\delta > -1).$$

(7)

The limiting value $\delta = -1$ will not be allowed as we must have both $\eta$ and $\eta' \neq 0$ to generate all the fermion mechanical masses. In terms of the conventional four-fermion Fermi coupling constant $G$ we have the normalisation:

$$G = \frac{f^2}{4\sqrt{2} m^2 (W^\pm)}.$$  

(8)

Weak (diagonal) neutral current phenomena will thus be described by the $Z_1$ and $Z_2$ mediated interactions in terms of the two parameters $\sin^2 \chi$ and $\delta$ of the present theory. In this connection, it will be useful to note the relations:

$$\frac{e^2}{6m^2(Z_1) \sin^2 \chi \cos^2 \chi} = \delta_1 G / \sqrt{2}; \quad \frac{e^2}{6m^2(Z_2) \sin^2 \chi} = \delta_2 G / \sqrt{2};$$

(9)

where we define:

$$\delta_1 = 1 - \frac{\delta}{6 + 4\delta}, \quad \delta_2 = 1 + \frac{1}{2} \delta.$$  

(10)

3. Model A

In this model we take, for leptons, $(e_L, \nu_e, E^0_L)$ and $(\mu_L, \nu_\mu, M^0_L)$ to belong to the $2^s$ representation of $SU_3(W)$ with $G_0 = -\frac{1}{3}$, where subscript $L$ means left-handed
components \((e_\mu = \frac{1}{2}(1 + \gamma_5)e, \text{ etc.})\) and \(E^0\) and \(M^0\) are neutral heavy leptons of electronic and muonic type, respectively.

For quarks we assume six flavours: \(u, d, s, c, t\) and \(g\). The charges of \(t\) and \(g\) are \(+\frac{2}{3}\), just as of \(u\) and \(c\), while \(d\) and \(s\) carry charge \(-\frac{1}{3}\). From these we form two \(3^*\)-representations of \(SU_3(W)\): \((d'_L, u_L, t'_L)\) and \((s'_L, c_L, e'_L)\) with \(G_0 = +\frac{2}{3}\). The primes on \(d'\) and \(s'\) represent the standard Cabibbo rotated combinations of \(d\) and \(s\). Similarly, \(t'\) and \(g'\) represent some suitable mixtures of \(t\) and \(g\).

The right-handed \((R-\) components of all the fermions will be taken to be singlets of \(SU_3(W)\) with \(G_0\) values equal to the respective electric charges.

Additional quarks and leptons may also be suitably included, but they are not relevant for the phenomenology discussed here. The leptons \(\nu, \tau\) along with another heavy \(\tau\)-type lepton \(T\) may thus be included. Anomaly cancellation using this device can be arranged along the lines already spelled out in detail in Pandit (1976).

We now discuss the standard weak current phenomena of current experimental interest. For this we employ the relevant \(Z_1\) and \(Z_2\) mediated effective interactions using the results listed in §2. We note, for this purpose, that \(\mathcal{G}_1 = (-\frac{3}{3}, +\frac{1}{3}, +\frac{1}{3})\) and \(\mathcal{G}_2 = (0, -1, +1)\) for the \(3^*\)-triplets of the \(L\)-components. The \(R\)-components being all singlets of \(SU_3(W)\) have \(\mathcal{G}_1 = \mathcal{G}_2 = 0\).

(i) The \(\nu\)-nucleon neutral current processes are described (using parton picture) by the effective interaction Lagrangian in the notation of Abbott and Barnett (1978):

\[
\begin{align*}
\mathcal{L}_{\text{eff}}(N) & = - \frac{G}{\sqrt{2}} \gamma^\mu (1 + \gamma_5) \nu \left[ u_L \bar{u} \gamma^\mu (1 + \gamma_5) u + u_R \bar{u} \gamma^\mu (1 - \gamma_5) u \right] \\
& + d_L \bar{d} \gamma^\mu (1 + \gamma_5) d + d_R \bar{d} \gamma^\mu (1 - \gamma_5) d,
\end{align*}
\]

where \(u_L, u_R, d_L, d_R\) are coupling parameters (not to be confused with spinor symbols used above) given in this model as follows:

\[
\begin{align*}
\frac{1}{8} \left( \delta_1 + 3\delta_2 \right) - \frac{1}{8} \delta_1 \sin^2 \chi, & \quad d_L = \delta_1 (\frac{1}{3} + \frac{1}{3} \sin^2 \chi), \\
\frac{1}{8} \delta_1 \sin^2 \chi, & \quad d_R = \delta_1 \sin^2 \chi.
\end{align*}
\]

(ii) For the asymmetry parameter characterising the inelastic scattering of longitudinally polarised electrons off unpolarised deuterons, we have, following the considerations and notations of Cahn and Gilman (1978),

\[
A_{\text{eD}} = - \frac{G q^2}{2 \sqrt{2} \pi a} \left( \frac{4}{5} - 2 \sin^2 \chi \right) \left[ \frac{1}{5 - 2 \sin^2 \chi} \right] \delta_1. \quad (13)
\]

The values of \(u_L, u_R, d_L, d_R\) determined by Abbott and Barnett (1978), and the SLAC measurement (Prescott et al 1978) of \(A_{\text{eD}}\) may now be used to determine the parameters \(\delta\) and \(\sin^2 \chi\) for our model. Values of \(\delta\) between \(-0.9\) and \(-1\) and \(\sin^2 \chi \simeq 0.25\) give very good agreements. Thus, merely as an example, for \(\delta = -0.95\), and \(\sin^2 \chi = 0.25\), we obtain

\[
\begin{align*}
u_L & \simeq +0.38, \\
d_L & \simeq -0.42, \\
U_R & \simeq -0.12, \\
d_R & \simeq +0.06,
\end{align*}
\]

\[\text{for } \nu_L \simeq 0.38, \quad d_L \simeq -0.42, \quad U_R \simeq -0.12, \quad d_R \simeq 0.06, \quad (14)\]
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to be compared with the fits of Abbott and Barnett (1978): $u_L = 0.33 \pm 0.07$, $d_L = -0.40 \pm 0.07$, $u_R = -0.18 \pm 0.06$ and $d_R = 0.0 \pm 0.11$. For the same values of the parameters and $\gamma = 0.21$, we find

$$A_{AB} \approx -8.8 \times 10^{-5} \text{GeV}^2,$$

in very good agreement with the value $(-9.5 \pm 1.6) \times 10^{-5} \text{GeV}^2$ measured at SLAC [Prescott et al 1978].

(iii) For $(\nu-e)$-scattering we have the effective interaction

$$L_{\text{eff}} = -\frac{G}{\sqrt{2}} \left[ \bar{\nu}_\mu \gamma_\lambda (1 + \gamma_5) \nu_\mu \{ \bar{e} \gamma_\lambda (C_{\nu} + C_A \gamma_5) e \} ight. \\
\left. + \bar{\nu}_e \gamma_\lambda (1 + \gamma_5) \nu_e \{ \bar{e} \gamma_\lambda (C'_{\nu} + C'_A \gamma_5) e \} \right],$$

where

$$C_{\nu} = (-\frac{1}{3} + \sin^2 \chi) \delta_3, \ C_A = -\frac{1}{3} \delta_3, \ C'_{\nu} = C_{\nu} + 1, \ C'_A = C_A + 1.$$  

Again, as an example, with $\sin^2 \chi \approx 0.25$ and $\delta \approx -0.95$, we obtain

$$\sigma(\nu \mu e) \approx 0.17 \times 10^{-41} \text{ cm}^2 (E_{\nu}/\text{GeV}), \ \sigma(\bar{\nu} \mu e) \approx 0.11 \times 10^{-41} \text{ cm}^2 (E_{\nu}/\text{GeV}).$$

The value of $\sigma(\nu \mu e)$ agrees well with the latest result, $(0.18 \pm 0.08) \times 10^{-41} \text{ cm}^2 E_{\nu}/\text{GeV}$, from BNL-Columbia (Cnops et al 1978. See also Faisser et al 1978).

(iv) For the atomic physics parity violation in heavy atoms arising from the neutral axial vector current of the electron coupled to the neutral vector currents of the $u$ and $d$ quarks, we have the characteristic parameter

$$Q_W(Z, N) = \delta_3 [-4 (\sin^2 \chi) Z - 2N].$$

For atomic bismuth $(Z=83, N=126)$ we find for $\delta \approx -0.95$ and $\sin^2 \chi \approx 0.25$:

$$Q_W(\text{Bi}) \approx -480.$$  

Here the earlier results from Oxford (Baird et al 1977) and Seattle (Lewis et al 1977) do not agree with the latest result from Novosibirsk (Barkov and Zolotorev 1978). We must thus reserve judgment for future. For comparison, in the standard $W-S$ model with $\sin^2 \theta_W \approx 1/4$, $Q_W(\text{Bi}) \approx 130$.

4. Model B

In this model the choice of leptons and their representations is identical with that in model A. The difference comes in the quark sector. Again we assume six quarks $u, d, s, c, b$ and $h$, where $b$ and $h$ are now different heavy quarks both carrying electric charge $-\frac{2}{3}$. From these we form two $3$-representations of $SU_3(W)$: $(u_L, d_L, b_L)$
and \((e_L, s'_L, h'_L)\), both having \(G_0 = 0\). The meanings of the primes and subscript \(L\) are as before. The \(R\)-components of all the fermions are again taken to be \(SU_3(W)\) singlets with \(G_0\) equal to their respective electric charges. Again additions of further heavy quarks and leptons, and anomaly cancellation, will follow the lines of Pandit (1976).

The neutral current phenomena may be discussed again as in the last section. We only have to note that (in distinction with model A) for the quark triplets we must now take

\[
\mathcal{G}_1 = (\pm \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}), \quad \mathcal{G}_2 = (0, +1, -1).
\]

Then using the notations introduced in §3, we note below the relevant results.

(i) Concerning \(\nu\)-nucleon neutral current processes we now have:

\[
u_L = \delta_1 (\frac{1}{3} - \frac{1}{3} \sin^2 \chi), \quad d_L = -\frac{1}{6} (\delta_1 - 3\delta_2) + \frac{1}{6} \delta_4 \sin^2 \chi,
\]

\[
u_R = -\frac{1}{3} \delta_1 \sin^2 \chi, \quad d_R = \frac{1}{6} \delta_4 \sin^2 \chi.
\]

(21)

(ii) For the asymmetry parameter for longitudinally polarised \(e\)-deuteron inelastic scattering we have

\[
A_{eD} = -\frac{Gq^2}{2\sqrt{2}\pi a} \left[ (1 - 2 \sin^2 \chi) \delta_1 + \frac{1 - (1 - y)^2}{1 + (1 - y)^2} (1 - 3 \sin^2 \chi) \delta_4 \right].
\]

(22)

Again we can obtain good agreement with the fits of Abbott and Barnett (1978) for \(\nu\)-\(N\) data and with the measured value (Prescott et al 1978) of \(A_{eD}\), this time with \(\delta\) ranging from \(\sim -1\) to \(-0.75\) and \(\sin^2 \chi \simeq 0.3\). Merely as an example, we take \(\delta \simeq -0.8\) and \(\sin^2 \chi \simeq 0.3\) and obtain:

\[
u_L \simeq 0.30, \quad d_L \simeq 0.45, \quad \nu_R \simeq -0.13, \quad d_R \simeq 0.07;
\]

(23)

and, with \(y = 0.21\),

\[
A_{eD} \simeq -9.9 \times 10^{-5} \left(\frac{q^2}{\text{GeV}^2}\right),
\]

(24)

in excellent agreement with Abbott and Barnett (1978) and Prescott et al (1978), respectively.

(iii) For \((\nu-e)\)-scattering, the coupling parameters are formally the same as for model A (since the lepton representations are the same in both the models) and are given by equation (17). The difference would arise from the different values of \(\sin^2 \chi\) and \(\delta\) needed in the two models for processes (i) and (ii). For \(\delta \simeq -0.8\) and \(\sin^2 \chi \simeq 0.3\), we have in this model:

\[
\sigma(\nu_{\mu}e) \simeq 0.12 \times 10^{-41} \text{ cm}^2 \left(\frac{E_p}{\text{GeV}}\right); \quad \sigma(\bar{\nu}_{\mu}e) \simeq 0.10 \times 10^{-41} \text{ cm}^2 \left(\frac{E_p}{\text{GeV}}\right).
\]

(25)

These are to be compared with the values of equation (18) for model A. The value of \(\sigma(\nu_{\mu}e)\) is again consistent with the experimental result of Cnops et al (1978).
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(iv) For the atomic physics parity-violation in heavy atoms we now have

$$Q_{W}(Z, N) = 2Z \left(1 - 2 \sin^2 \chi\right) \delta_x,$$

independent of $N$. Taking, for illustration, $\delta \simeq -0.8$ and $\sin^2 \chi \simeq 0.3$ we obtain

$$Q_{W}(\text{Bi}) \simeq 86.$$

(27)

Thus the two models differ drastically in predicting this parameter. However, in view of the already noted unsettled experimental situation, we have to reserve judgment for future.

In view of the results displayed above, we feel that the models presented here deserve further attention and study.

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