Mass Determination on Steeply Dipping Tracks in Emulsion Block Detectors.

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In the study of particle tracks in Emulsion Block Detectors one can frequently observe appreciable length of trajectory even for those tracks which traverse individual emulsion sheets with a large dip angle. The customary methods for deducing the mass of such particles from multiple Coulomb scattering and range or from multiple Coulomb scattering and grain density give very unreliable results. The failure of obtaining accurate mass values with these methods is due to the small distortions which are introduced into the emulsions during processing. We will show in this paper that the effect of such distortions can be completely eliminated if instead of deducing the Coulomb scattering from second co-ordinate differences one makes use of third or higher order differences. In a small sample of proton tracks with dip angles between 32° and 52° good mass determinations have been made.

The Relation Between Second, Third and Fourth Order Co-ordinate Differences.

If P_k designates points located on the track and separated from each other by the cell-length t_k and if y_k denotes the distance to the point P_k from a line nearly parallel to the track then the second difference D_2 is customarily defined by:

$$D_2^{(k)} = y_k - 2y_{k+1} + y_{k+2}\,.$$

Similarly we define the third difference D_3 by:

$$D_3^{(k)} = y_k - 3y_{k+1} + 3y_{k+2} - y_{k+3}$$

and in general the difference D_n by:

$$D_n^{(k)} = \sum_{\lambda=0}^{n} a_\lambda y_{k+\lambda}$$

with

$$a_\lambda = \frac{n!\,(-1)^\lambda}{\lambda!\,(n-\lambda)!}.$$

One can then easily show on the basis, for instance, of the distribution function given for D_2 by SCOTT [1], that the mean absolute values of the second, third and fourth differences are related as follows:

$$\langle|D_2|\rangle = \Delta$$
$$\langle|D_3|\rangle = \sqrt{\tfrac{3}{2}}\,\Delta$$
$$\langle|D_4|\rangle = 2\Delta$$

and that in the approximation in which the distribution function of D_2 is taken to be a Gaussian, the distribution functions for the higher order differences will also be Gaussian.

It is easily seen that while D_2 measures the deviation of the track from a straight line, D_3 measures its deviation from a circle of arbitrary radius, and D_4 measures its deviation from a curve whose curvature varies linearly. The Δ-values obtained from measurements of D_3 will, therefore, be independant of a uniform curvature and those of D_4 independent of a uniformly varying curvature irrespective of whether this curvature is introduced by distortion of the emulsion or by waviness of the microscope stage.

The customary curvature correction which is made by adding to each of the experimentally determined second difference values (D_2) a constant, such that their algebraic sum vanishes, is a satisfactory approximation for long tracks, but quite inapplicable to steep tracks where the number of available cells per emulsion sheet is very small. It represents an over correction since even for an undistorted track the algebraic sum of second differences should not vanish except in the limit of a very large number of readings.

The Effect of Distortion on Scattering Measurements.

The most common distortion in emulsions consists of a displacement S in the plane of the emulsion (x, y plane) whose direction and magnitude remains constant over large areas of the plate, but increases quadratically with depth (Z direction), having the value zero at the glass surface and the maximum value S_0 at the air surface of the processed emulsion. One can easily show that under these conditions the contribution of the distortion to the measured second differences (δ_2) is given by:

$$\delta_2 = \frac{2S_0}{N^2} \sin \alpha \,,$$

where N is the total number of cell-lengths available for measurements in the particular emulsion sheet and α is the angle which the projection of the track in the x, y plane makes with the distortion vector $\boldsymbol{S}$. Thus even if the distortion is moderate ($S_0 \leqslant 20\ \mu$) in a 600 μ emulsion a steep track for which only four or five cell-lengths are available can give spurious second differences of the order of 1 μ, which is equal to or more than the expected scattering per cell-length. On the other hand the contribution of this type of distortion to the third difference will be smaller by a factor of the order of $(1/N)(S_0/l_0)^2$ where l_0 is the projected track length in the emulsion sheet. Assuming the same emulsion thickness and distortion as before, for a track which traverses the emulsion with a dip angle of 45°, this factor is of the order of 1/3000.

We expect, therefore, that for steep tracks in moderately distorted emulsions the scattering measured by third and higher order differences will give identical mass values but will differ substantially from those obtained from second order differences.

The Effect of Noise.

Before making a comparison with experiment, it is necessary to investigate how the emulsion noise, and noise due to reading errors must be taken into account when third or higher order differences are used for mass determination. (As mentioned before the contribution of stage noise will be negligible in third order differences) (*).

Experimentally the noise contribution can be easily determined by measuring second, third and higher order differences on the flat track of a very energetic particle. It is also possible to calculate the noise distribution on the basis of some model and determine the expected mean absolute values of ε_2, ε_3, etc.. It turns out that the theoretical distribution function is not very sensitive to the details of the model of noise which is used. We made have calculations for instance, on the basis of the following assumptions.

(*) Noise in the focussing motion (z-motion) of the microscope can be neglected for measurements on flat tracks, but will affect scattering measurements on steeply dipping tracks and may have to be taken into account.

The probability of making a grain developable is proportional to the length of particle trajectory which lies inside the grain. The grains have spherical shape, are of uniform size and uniformly distributed in the emulsion. A reading represents the average y-co-ordinate of the centres of the small number of grains (for instance, 5 grains). Readings can only be made in discrete steps which (in appropriate units) are represented by the possible values: 0, ± 1, ± 2, ± 3.

With such a model the predicted ratios $\varepsilon_2 : \varepsilon_3 : \varepsilon_4 : \varepsilon_5$ are in excellent agreement with the experimental values and furthermore, the distribution function of the noise resembles a normal distribution sufficiently closely to justify the commonly used relation between the « measured » scattering values D_n and the « true » scattering values Δ_n:

$$\Delta_n = \sqrt{D_n^2 - \varepsilon_n^2}\,.$$

Experimental Results.

By measuring flat tracks due to very energetic particles for which scattering may be considered to be negligible compared to noise, we obtain the following mean absolute values for differences in various orders:

$$\varepsilon_2 = 0.156\ \mu, \qquad \varepsilon_3 = 0.285\ \mu, \qquad \varepsilon_4 = 0.526\ \mu, \qquad \varepsilon_5 = 0.997\ \mu\,.$$

These noise values were used in Table I to derive, from the « measured » differences D_n, the corresponding « true » differences Δ_n.

TABLE I.

Particles	Scattering scheme [2]	Second Differ.		Third Difference			Fourth Difference		
		No. of cells	Δ_2 (μ)	No. of cells	Δ_3 (μ)	$\sqrt{\frac{2}{3}}\Delta_3$ (μ)	No. of cells	Δ_4 (μ)	$\Delta_4/2$ (μ)
10 flat π-mesons	π (1.6)	807	1.605±.042	746	2.030	1.657	686	3.388	1.694
12 flat Protons	p (1.6)	779	1.600±.045	741	2.061	1.680	712	3.395	1.698
6 flat protons	π (1.6)	890	0.712±.019	869	0.932	0.760	840	1.522	0.761
11 dipping protons (5 with $45° < \theta < 52°$) (6 with $32° < \theta < 45°$)	π (1.6)	764	0.924±.026	635	0.925	0.756	519	1.488	0.744

In order to correct for the effect of large angle single scattering individual values of D_n which exceed four times the average of the remaining values have been eliminated. The first line in Table I represents results of making measurements on the tracks of ten flat π-mesons, using the method of varying cell-length [2] (π (1.6) scheme). These cell-lengths are so chosen that, irrespective of range, the mean absolute value of the second difference for π-meson is expected to be 1.6 μ. The second and third lines represent the result of measurements on flat proton tracks, scattered according to the p(1.6) and π(1.6) schemes, respectively. In each case about 800 readings have been obtained. The Table shows that the values obtained for Δ_2, $\sqrt{\frac{2}{3}}\Delta_3$ and $\frac{1}{2}\Delta_4$ are identical within experimental errors and give the correct mass values for the particles whose tracks have been measured. There is, however, an indication that slightly better agreement between the results from second, third and fourth order differences can be obtained if the theoretically derived conversion factors ($\sqrt{\frac{2}{3}}$ and $\frac{1}{2}$) are decreased by about 6%. The fourth line of Table I represents the results of measurements on protons which traverse the emulsion stack with dip angles between 32° and 52°. Here we find that as expected the results obtained from third and fourth differences are identical with each other and with values obtained for flat tracks. The values derived from second differences on the other hand are quite different for steep and for flat proton tracks.

Since the mass of a particle varies as $\Delta^{-2.276}$ [2] the application of second difference measurements to steep tracks gives for the dipping tracks a mass value which is too low by a factor $(.924/.712)^{2.276} = 1.8$ but the application of third and fourth difference measurements gives factors of 0.98 and 0.90 respectively.

We have purposely omitted to indicate the standard deviation in the results obtained from third and fourth order differences. Individual measurements of Δ_3 and Δ_4 are not strictly independent of each other and the number of tracks we have measured so far is as yet insufficient to derive an empirical estimate of the error from the internal consistency of the data.

REFERENCES

[1] W. T. Scott: *Phys. Rev.*, 76, 212 (1949).

[2] S. Biswas, E. C. George and B. Peters: *Proc. Ind. Acad. Sci.*, **38**, 418 (1953).