Proc. Indian Acad. Sci., Vol. 83 A, No. 5, 1976, pp. 188–201

Flow and heat transfer for a power-law electrically conducting fluid flowing between parallel plates under transverse magnetic field with viscous dissipation

K. M. SUNDARAM* AND G. NATH

Department of Applied Mathematics, Indian Institute of Science, Bangalore 560012

MS received 6 November 1975

ABSTRACT

The flow and heat transfer problem with viscous dissipation for electrically conducting non-Newtonian fluids with power-law model in the thermal entrance region of two parallel plates with magnetic field under constant heat flux and constant wall temperature conditions has been studied. The governing equations have been solved numerically using quasilinearization technique and implicit finite-difference scheme. It has been found that the effect of viscous dissipation on heat transfer is quite significant for heating and cooling conditions at the wall.

NOMENCLATURE

a, half of the distance between the plates;

b_x, induced magnetic field;

$\bar{b}_x$, dimensionless induced magnetic field;

B_0, applied magnetic field;

Br, Brinkman number;

C_v, specific heat at constant volume;

E_y, electric field along y-direction;

h, enthalpy;

Hm, Hartmann number defined by equation (3 b);

y, current in y-direction;

J_y, dimensionless current in y-direction;

* Department of Chemical Engineering.

Presently at Laboratorium Voor Petrochemisc e Technick, Krijgalaan 271, 9000 Ghent, Belgium.

K, dimensionless parameter defined by equation (3 *a*);

k_1, thermal conductivity;

n, index of the power-law model;

Nu, Nusselt number defined by equation (15);

p_x, pressure gradient along axial direction given by (3 *c*);

P_x, dimensionless pressure gradient in axial direction defined by equation (3 *b*);

q_w, wall heat flux;

Re, Reynold's number defined by equation (3 *b*);

Rm, magnetic Reynold's number;

T, temperature;

T_b, bulk temperature;

u, axial velocity;

$\bar{u}$, average axial velocity defined by equation (3 *a*);

U, dimensionless axial velocity;

$\bar{U}$, dimensionless axial velocity defined by equation (11);

z, vertical distance;

Z, dimensionless vertical distance;

θ, dimensionless temperature;

θ_b, dimensionless bulk temperature;

μ, viscosity;

μ_1, magnetic permeability;

ξ, dimensionless axial distance;

ρ, density;

σ, electrical conductivity;

τ, shear stress.

Superscripts

$'$, prime denotes differentiation with respect to Z.

Subscripts

c, critical value;

o, inlet condition;

w, wall condition.

1. INTRODUCTION

THE study of convective heat transfer in a magnetohydrodynamic channel has received considerable attention during the past several years. The interest has been motivated by three principal applications: the magnetohydrodynamic generator, the magnetohydrodynamic pump and the electromagnetic flow meter. The laminar flow of a uniform conducting incompressible fluid between two parallel plates under a uniform transverse magnetic field was studied by Hartmann and Lazarus (known as Hartmann's flow) and was summarized by Sutton and Sherman.[1] The corresponding heat transfer problem in the thermal entrance region of two parallel plates with constant heat flux and constant wall temperature conditions with and without viscous dissipation and Hall effect has been investigated by several authors.[2-10]

The importance of flow and heat transfer problem for the power-law non-Newtonian electrically conducting fluid in practical applications has been discussed by Sarpkaya[11] and Moore.[12] However, Sarpkaya obtained an approximate solution of the flow problem.

In the present paper, both the flow and heat transfer problem under constant heat flux and constant wall temperature conditions in the thermal entrance region of two parallel plates for a power-law non-Newtonian electrically conducting fluid in the presence of a uniform transverse magnetic field has been studied neglecting the Hall effect, but taking into account the effect of viscous dissipation. It is has been shown that the viscous dissipation has a considerable effect on the heat transfer.

2. VELOCITY, CURRENT, AND INDUCED MAGNETIC FIELD

2.1 GOVERNING EQUATIONS

We consider the steady flow of an incompressible electrically conducting power-law fluid $(\tau \alpha \mid du/dz \mid^{n-1} (du/dz))$ along the length of the channel with a uniform and constant magnetic field B_0 in the transverse direction. The fluid properties like density, viscosity, thermal and electrical conductivity, etc. are assumed to be independent of temperature. There are no external (body) forces acting on the fluid. The end effect for the velocity profiles is neglected and secondary flows are not considered. The Hall effect is not taken into account. The electrodes are assumed to be made of materials of infinite electrical conductivity. The insulators have finite conductivity and their electrical resistances are considered as parallel to the channel. There is no slip at the wall. Heat conduction in the axial direction is negligible in comparison with the heat transport in same direction by overall fluid motion. Under these assumptions, the governing equation for the velocity in dimensionless form can be expressed as[1]

$$(U'^{n})' - Hm^2\, U - Re\, P_x + KHm^2 = 0 \tag{1}$$

with boundary conditions

$$U'(0) = 0, \quad U(1) = 0 \tag{2}$$

where

$$Z = z/a, \quad U = u/\bar{u}, \quad \bar{u} = \frac{1}{2a}\int_{-a}^{a} u dz, \quad K = E_y/\bar{u}B_0 \tag{3 a}$$

$$Hm^2 = \sigma B_0{}^2\, a^{n+1}/\bar{u}^{n-1}\,\mu, \quad Re = a^n\, \bar{u}^{2-n}\, \rho/\mu, \quad P_x = p_x/\rho\bar{u}^2 \tag{3 b}$$

$$p_x = \sigma E_y B_0 - \sigma u B_0{}^2 + d\,[\mu\,(du/dz)^n]/dz. \tag{3 c}$$

The expression for current in dimensionless form is given by[1]

$$J_y(Z) = K - U(Z) \tag{4}$$

where

$$J_y = j_y/\sigma B_0 \bar{u} \tag{5}$$

It may be noted that eq. (4) corresponds to short-circuit or open-circuit condition according as $K = 0$ or 1. The dimensionless induced magnetic field can be calculated from

$$\bar{b}_x' = K - U(Z) \tag{6}$$

with

$$b_x(0) = 0 \tag{7}$$

where $\bar{b}_x = b_x/B_0\, Rm, \quad Rm = \mu_1 \bar{u}\sigma a.$

2.2 Results and Discussion

Equation (1) is a nonlinear equation and it has been solved numerically using quasilinearization technique.[13] The quasilinear version of eq. (1) can be expressed in the form

$$\begin{aligned} U''_{i+1} &+ (n-1)\,[Hm^2\,(U_i - K) + Re\, P_x]\,(U_i')^{-n}\, U'_{i+1}/n \\ &- (Hm^2/nU_i'^{\,n-1})\, U_{i+1} = (Re\, P_x - K\, Hm^2)\,(U_i')^{1-n} \\ &+ [(n-1)\, Hm^2\, U_i\,(U_i')^{1-n}]/n. \end{aligned} \tag{8}$$

The boundary conditions (2) can be written as

$$U'_{i+1}(0) = U_{i+1}(1) = 0. \tag{9}$$

Knowing the ith iteration, we can obtain U_{i+1}. The initial profile is assumed to be

$$U_0(Z) = 1 \cdot 5\,(1 - Z^2), \quad U_0'(Z) = -3Z \tag{10}$$

which satisfies the boundary conditions exactly. Using these results, eq. (8) under conditions (9) has been solved numerically using Runge-Kutta-Gill subroutine with step size of 0·05. Since we want to compare our velocity profiles for Newtonian fluids ($n = 1$) with those of the analytical results given in ref. (1), we use the result $\int_{-1}^{1} UdZ = 2$ (which is evident from the definition of $\bar{u}$) in defining the velocity in the form

$$\bar{U} = 2U / \int_{-1}^{1} UdZ \tag{11}$$

where U is obtained from (8) and $\int_{1}^{1} UdZ$ is obtained using Simpson's (1/3) rule. $\bar{U}$ and the corresponding analytical results of ref. (1) are shown in figure 1 and there is very close agreement between them. It can be seen from the figure that the velocity profiles become flatter and the velocity

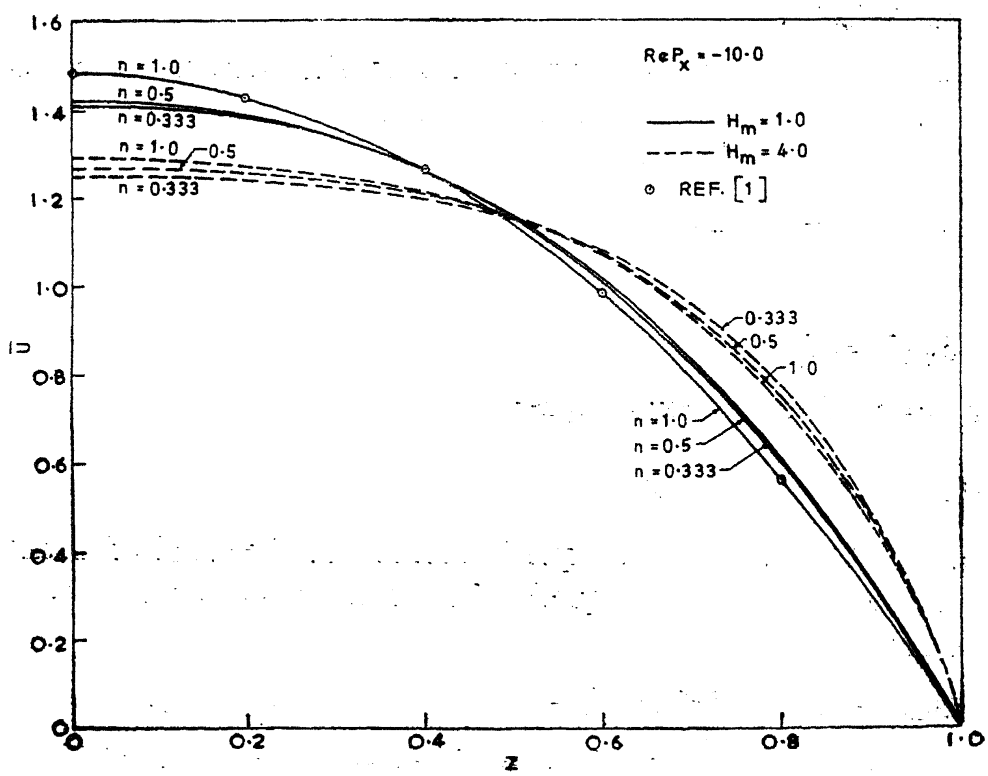

Figure 1. Velocity profiles.

gradient near the channel wall becomes steeper when either Hartmann number Hm increases or the power-law index n decreases.

The induced magnetic field $\bar{b}_x$ is obtained by solving numerically eq. (6) under condition (7). The distribution of $\bar{b}_x$ with Z for $n = 1$, 0·333 and $Hm = 1 \cdot 0$ is depicted in figure 2. It is evident from the figure or from eqs (6) and (7) that $\bar{b}_x(1) = K - 1$. It can be seen that $\bar{b}_x$ is negative when

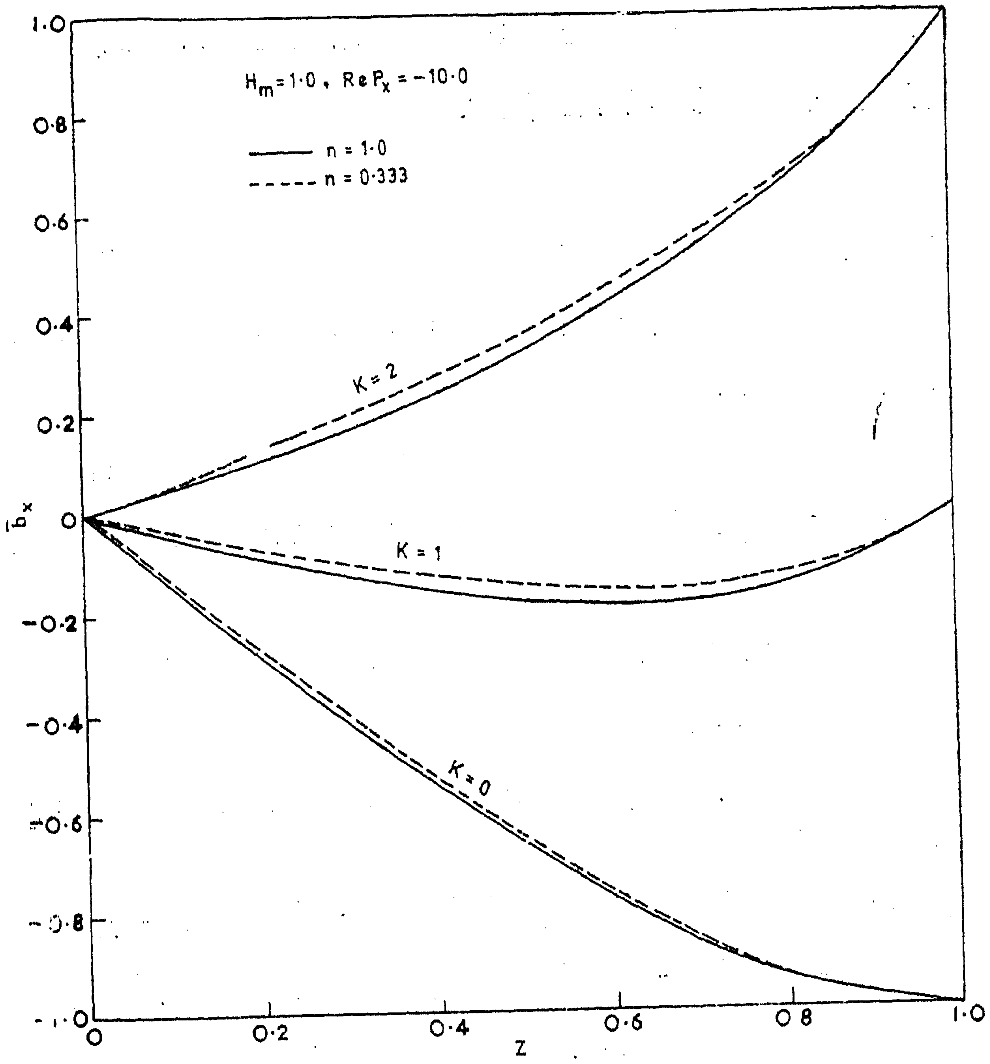

Figure 2. Induced magnetic field profiles.

$K = 0$ and 1 and it is positive for $K = 2$. It has been observed that the effect of K on $\bar{b}_x$ is more pronounced as compared to that of Hm or n.

3. Temperature and Heat Transfer

3.1 Governing equations

Under the assumptions mentioned earlier, the energy equation in dimensionless form can be expressed as[1]

$$\bar{U}\partial\theta/\partial\xi = \partial^2 \theta/\partial Z^2 + Br\,[(d\bar{U}/dZ)^{n+1} + Hm^2 J_y{}^2]. \tag{12}$$

The boundary conditions are

$$\theta = 0 \text{ at } \xi = 0 \text{ for } Z \geqslant 0 \text{ (for constant heat flux)}$$

$$\theta = 1 \text{ at } \xi = 0 \text{ for } Z \geqslant 0 \text{ (for constant wall temperature)}$$

$$\theta = 0 \text{ at } Z = 1 \text{ for } \xi \geqslant 0 \text{ (for constant wall temperature)}$$

$$\partial\theta/\partial Z = 1 \text{ at } Z = 1 \text{ for } \xi \geqslant 0 \text{ (for constant heat flux)}$$

$$\partial\theta/\partial Z = 0 \text{ at } Z = 0 \text{ for } \xi \geqslant 0 \quad (13)$$

where

$$\theta = (T - T_0)/(q_w a/k_1) \quad \text{(for constant heat flux)}$$

$$\theta = (T - T_w)/(T_0 - T_w) \quad \text{(for constant wall temperature)}$$

$$\xi = x/a\, Re\, Pr$$

$$Br = Pr\, \bar{u}^{n+1}\, k_1/C_v a^n\, q_w \quad \text{(for constant heat flux)}$$

$$Br = Pr\, \bar{u}^{n+1}/C_v\, (T_0 - T_w)\, a^{n-1} \quad \text{(for constant wall temperature)} \quad (14)$$

The Nusselt number for constant heat flux case is given by

$$Nu = 2ha/k_1 = 4q_w a/k_1 (T_w - T_b) = 4/(\theta_w - \theta_b). \quad (15)$$

Similarly, the Nusselt number for the case of constant wall temperature can be expressed as[14]

$$Nu = 2\,[(\partial\theta/\partial Z)_{Z=1}]/\theta_b = [-11\theta(\xi, 1) + 18\theta(\xi, 1 - \triangle Z) - 9\theta(\xi, 1 - 2\triangle Z) + 2\theta(\xi, 1 - 3\triangle Z)]/(3\triangle Z\, \theta_b) \quad (16)$$

where the bulk temperature θ_b is expressed in the form

$$\theta_b = \int_0^1 \theta \bar{U} dZ / \int_0^1 \bar{U} dZ \quad (17)$$

and

$$h = 2q_w/(T_w - T_b). \quad (18)$$

3.2 RESULTS AND DISCUSSION

Equation (12) under conditions (13) has been solved numerically for various values of the parameters using Crank-Nicholson implicit finite-difference scheme and the resulting simultaneous equations have been solved by Thomas algorithm. To check the accuracy of the numerical method, calculations were made for the case $Br = 0$ taking $n = 1$, $\bar{U} = 1 \cdot 5\, (1 - Z^2)$. The limiting Nusselt number (for large ξ) was found to be 3·771, whereas the value obtained by Vlachopoulos and Keung[14] is 3·767.

(*a*) Constant heat flux case

The effect of Hartmann number *Hm* and the power-law index *n* on the variation of the wall temperature θ_w with axial distance ξ has been shown in figure 3. It is evident from the figure that for all *n*, θ_w increases as *Hm* or ξ increases. For a given *Hm*, θ_w decreases as *n* decreases. It

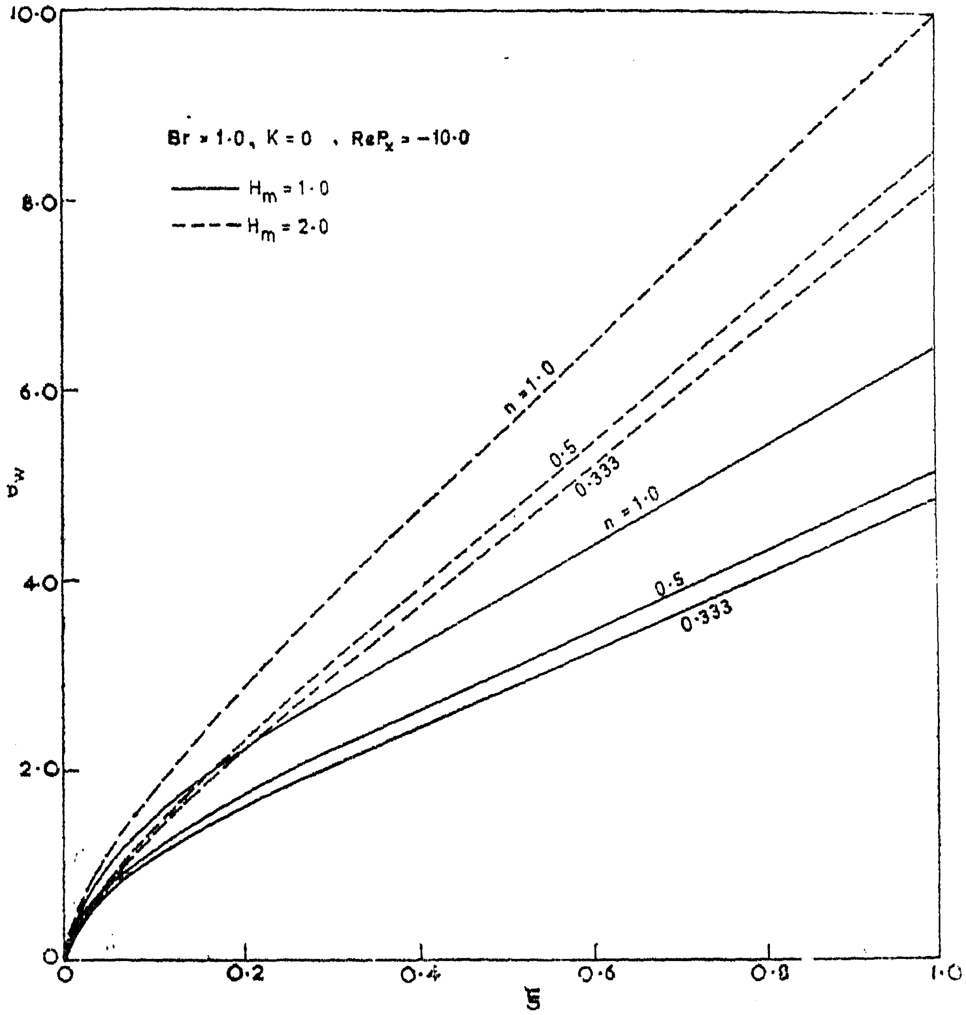

Figure 3. Wall temperature *versus* axial distance (constant heat flux).

may be noted that in this analysis both viscous heating and Joule heating have been included. Therefore, when the channel operates in the short-circuit condition ($K=0$), the current flow is large and since the Joule heating depends on the square of the current, the increase in Hartmann number *Hm* amounts to a large increase in temperature θ_w. Similar effect is observed for $K=2$, although the increase is less compared to $K=0$. For open-circuit ($K=1$), the current flow is small and viscous dissipation dominates the temperature distribution. In this case, θ_w is very small compared to $K=0$ or 2 (θ_w for $K=0$ and 2 is not shown graphically for the sake of brevity).

The Brinkman number *Br* is positive ($q_w > 0$) or negative ($q_w < 0$) according as there is heating or cooling situations on the wall. The effect of Brinkman number Br on wall temprature θ_w and bulk temperature θ_b has been shown in figure 4. It is observed that θ_w and θ_b are positive when $Br \geqslant 0$ and negative when $Br < 0$ and they increase or decrease with ξ according as $Br \geqslant 0$ or $Br < 0$. The negative value of θ_w implies that the

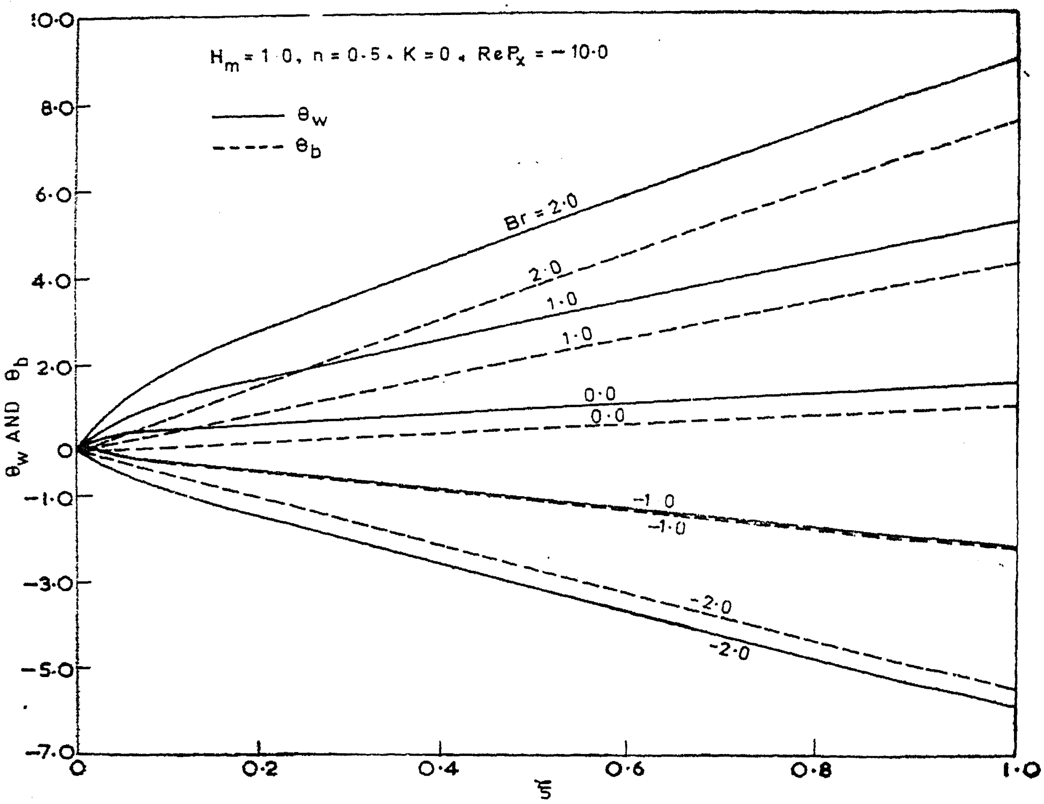

Figure 4. Wall and bulk temperatures *versus* axial distance (constant heat flux).

fluid temperature T_w is greater than the uniform entrance temperature T_0. It may be noted that the distribution of θ_b with ξ can be represented by a straight line in the form $\theta_b = (1 + m\,Br)\,\xi$ where m is a function of n, K and Hm.

However, θ_w, for small ξ, can be represented by a curve, but for large ξ, it can be represented by a straight line.

Figure 5 depicts the effect of K on θ_w, θ_b, and Nu. It is observed that the values of θ_w and θ_b are much less for $K = 1$ as compared to the corresponding values for $K = 0$ or 2, but the values of Nu are less for $K = 2$ as compared to those for $K = 0$ or 2. θ_b for $K = 0$ nearly coincides with that of $K = 2$.

The effect of Br on local Nusslet number Nu is shown in figure 6. Nu for all values of Br reaches an asymptotic value after certain ξ. The results reveal that there exists a critical Brinkman number Br_c ($Br_c < 0$) which depends on Hm and n beyond which Nu becomes negative, and Br_c decreases as n decreases or Hm increases. From the definition of Brinkman number

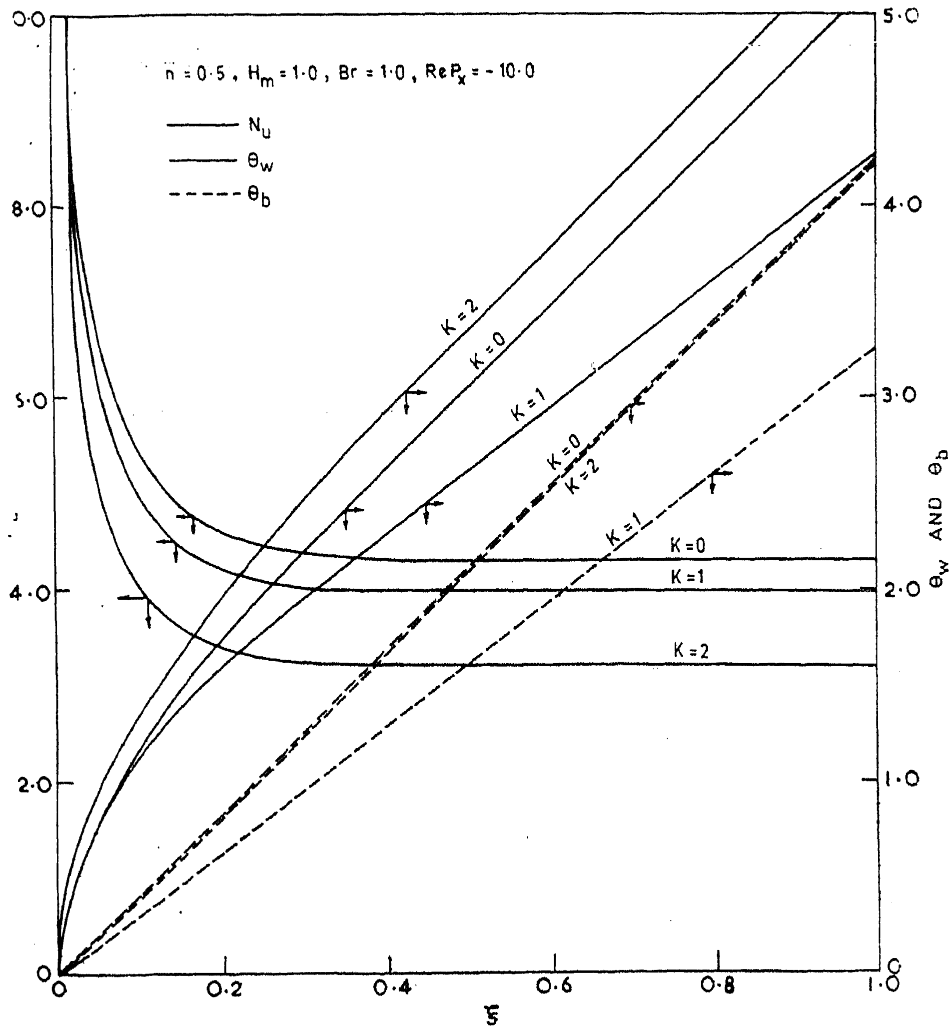

Figure 5. Nusselt number, wall and bulk temperatures *versus* axial distance (constant heat flux).

we know that if $Br < 0$, $q_w < 0$. Similarly, from the definition of Nu [see eq. (15)], we find that $Nu > 0$ for $Br_c < Br < 0$ when $T_b > T_w$. On the other hand, $Nu < 0$ for $Br < Br_c < 0$ when $T_b < T_w$. The reason for this behaviour is that the viscous dissipation yields higher temperature within thin layers along the wall only, because in these regions the gradient of the velocity is high enough to produce appreciable effects. The occurrence of negative values in Nusselt number is due to its definition since viscous dissipation may make $T_b < T_w$ for $q_w < 0$. It may be noted that similar effects have been observed by Ou and Cheng,[15] and Sundaram and Nath[16] for Newtonian and non-Newtonian fluids without magnetic field in a pipe and in parallel plates.

(*b*) Constant wall temperature case

The local temperature distribution θ at various axial distances for some representative values of Hm are shown in figure 7. It is seen that θ increases as Hm or ξ increases. The effects of n, Br, and K on θ_b and Nu are depicted in figures 8–10. Their behaviour is qualitatively similar to those for constant heat flux conditions.

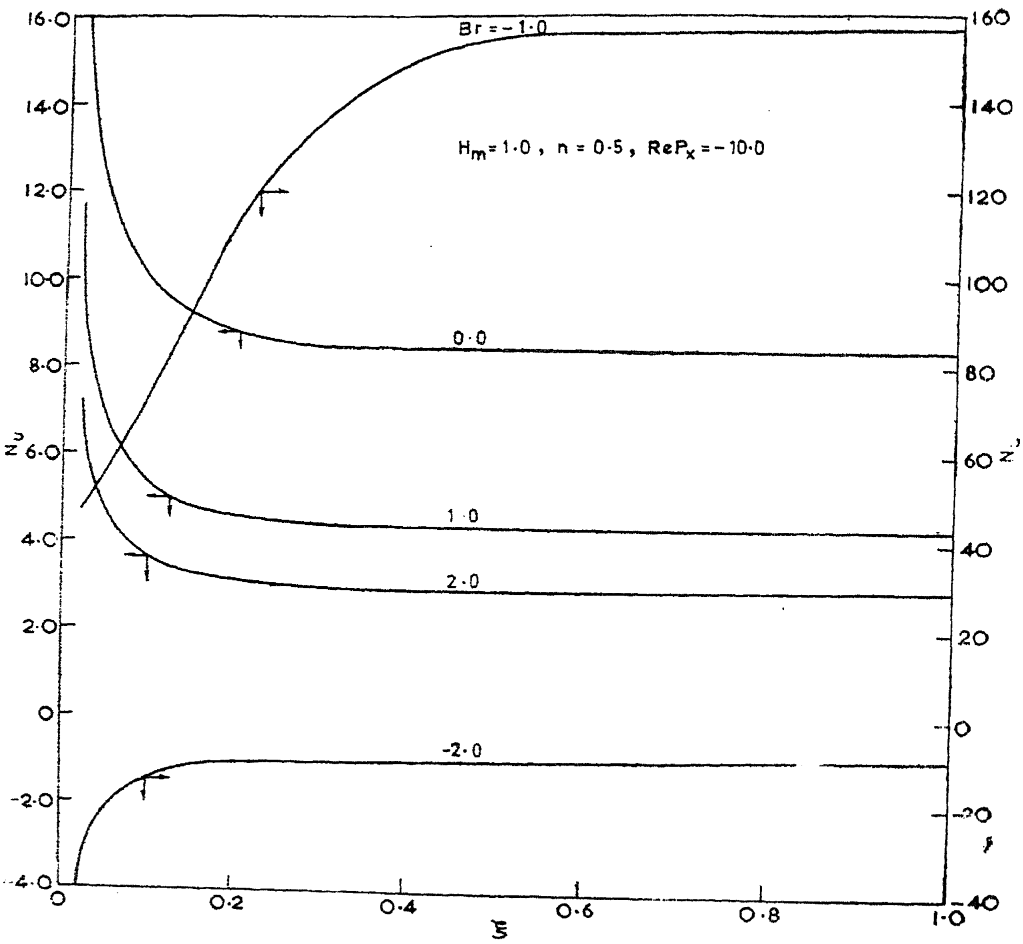

Figure 6. Nusselt number *versus* axial distance (constant heat flux).

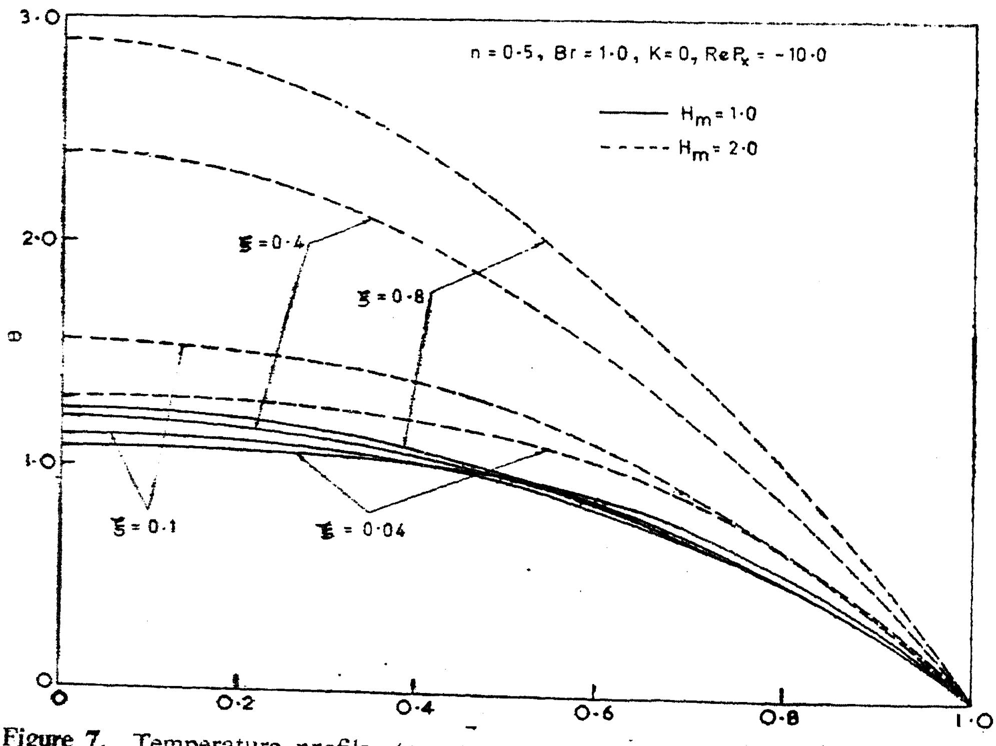

Figure 7. Temperature profiles (constant wall temperature).

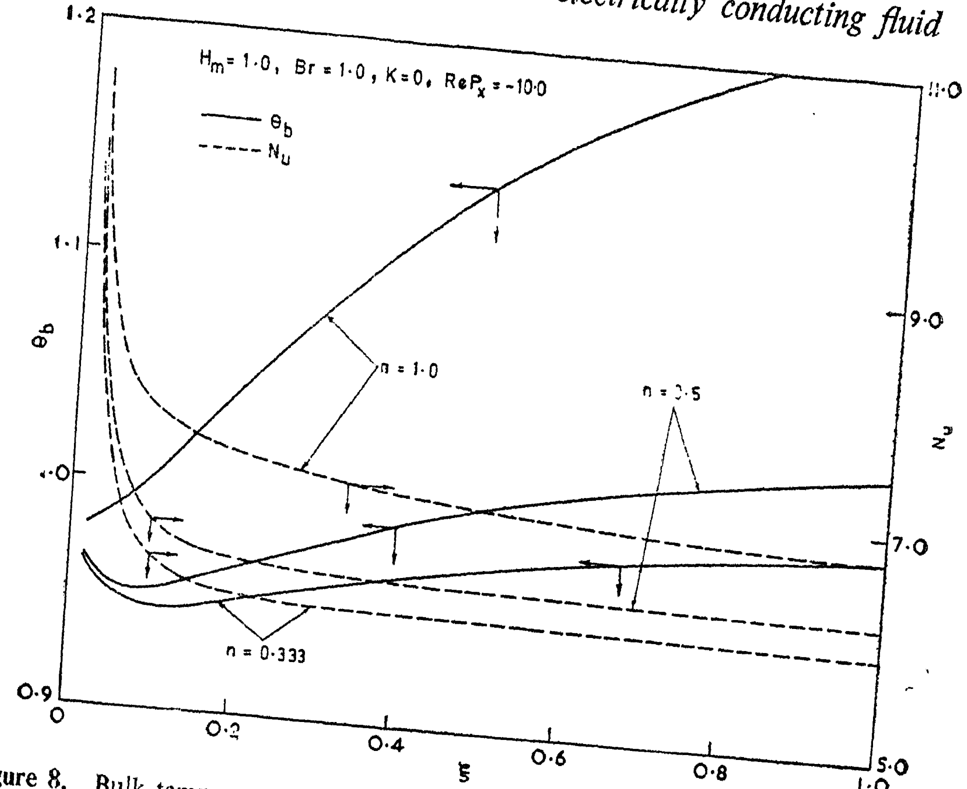

Figure 8. Bulk temperature and Nusselt number *versus* axial distance (constant wall temperature.

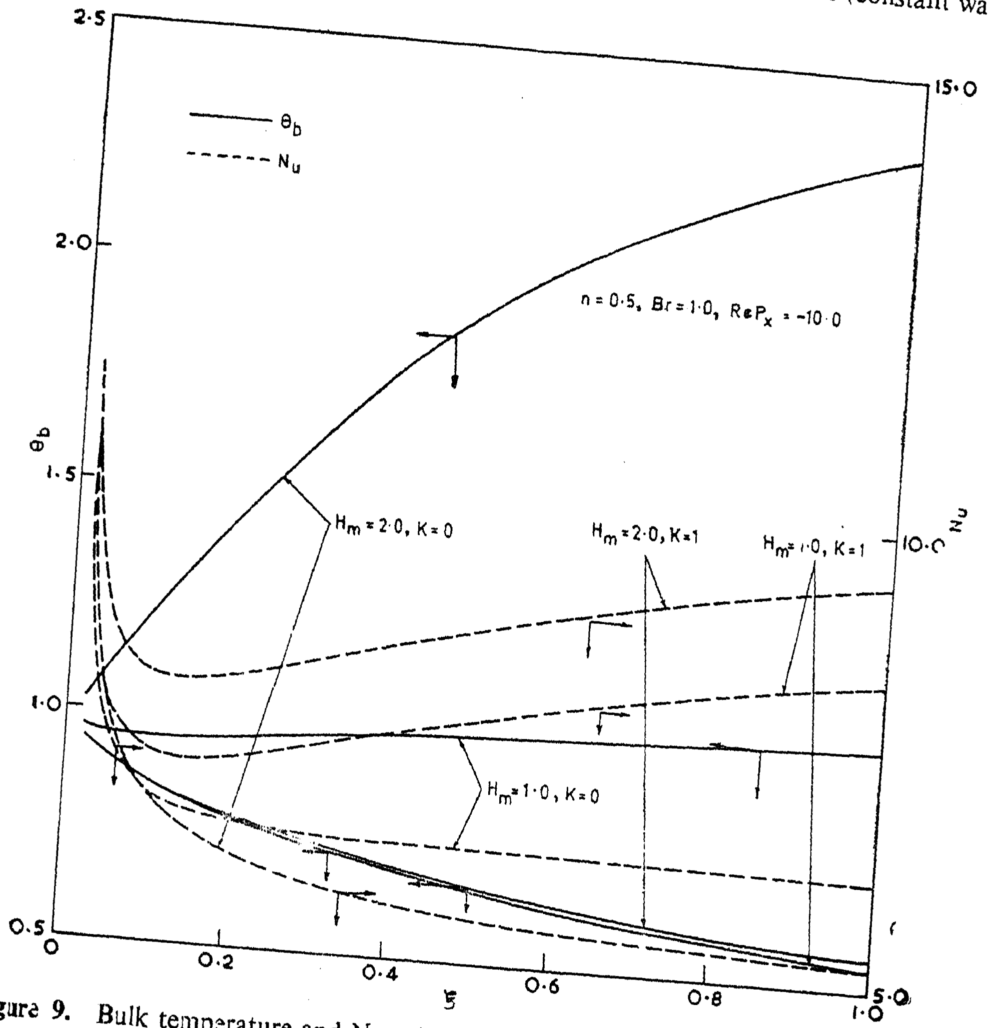

Figure 9. Bulk temperature and Nusselt number *versus* axial distance (constant wall temperature)

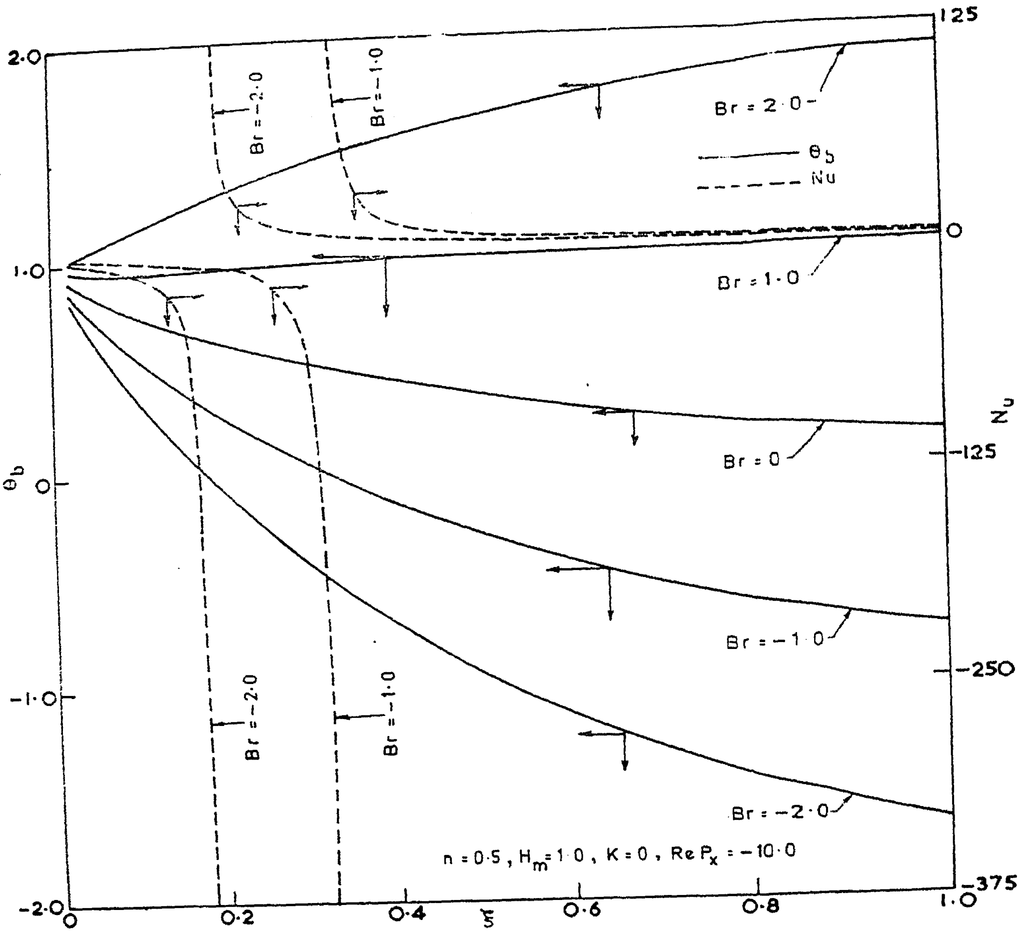

Figure 10. Bulk temperature and Nusselt number *versus* axial distance (constant wall temperature).

4. CONCLUSIONS

The results show that the heat transfer is strongly dependent upon the viscous dissipation (Brinkman number). In the case of cooling at wall, there exists a critical value of Brinkman number Br_c (which depends on the Hartmann number Hm and index of the power-law mode! n) below which ($Br < Br_c < 0$) the bulk temperature is always less than the wall temperature and Nusselt number is negative.

REFERENCES

1. Sutton, G. W. and Sherman, A., *Engineering Magnetohydrodynamics*, McGraw-Hill **341** (1965).
2. Seigel, R., *Trans. ASME, J. Appl. Mech.* **25** 415 (1958).
3. Nigam, S. D. and Singh, S. N., *Quart. J. Mech. Appl. Math.* **13** 85 (1960).
4. Alpher, R. A., *Int. J. Heat Mass Transfer* 3 108 (1961).
5. Perlmutter, M. and Siegel, R., Heat transfer to an electrically conducting fluid flowing in a channel with a transverse magnetic field, NASA Tech. Note D–875 (1961).

6. Michiyoshi, I. and Matsumoto, R., *Int. J. Heat Mass Transfer* 7 101 (1964).

7. Hwang, C. L., Kniper, P. J. and Fan, L. T., *Int. J. Heat Mass Transfer* 9 773 (1966).

8. Hsia, E. S., *Trans. ASME, J. Appl. Mech.* 33 665 (1971).

9. Sato, H., *J. Phys. Soc.* Japan 16 1423 (1961).

10. Tani, I., *J. Aerospace Sci.* **29** 297 (1962).

11. Sarpkaya, T., *AIChE J.* 7 324 (1961).

12. Moore, F. K., *Theory of Laminar Flows*, Vol. 4, Princeton Univ. Press, 691 (1964).

13. Bellman, R. E. and Kalaba, R. E., *Quasilinearization and Non-linear Boundary Value Problems*, Elsevier (1965).

14. Vlachopoulos, J. and Keung, C. K. J., *AIChE. J.* 18 1272 (1972).

15. Ou, J W. and Cheng, K. C., *Appl. Sci. Res.* **28** 289 (1973).

16. Sundaram, K. M. and Nath, G., *Proc. Indian Acad. Sci.* **83** A 50 (1976).