

K EMISSION-LINE WIDTHS AND THE SOLAR CHROMOSPHERE

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Abstract. Closely spaced microphotometer tracings parallel to the dispersion of one excellent frame of a K-line time sequence have been utilized for a study of the nature of the K_{2V} , K_{2R} intensities in the case of the solar chromosphere. The frequency of occurrence of the categories of intensity ratio $I_{K_{2V}}/I_{K_{2R}}$ are as follows: $I_{K_{2V}} > I_{K_{2R}} = 45.3$ per cent; $I_{K_{2V}} = I_{K_{2R}} = 4.7$ per cent; $I_{K_{2V}} < I_{K_{2R}} = 25$ per cent; $I_{K_{2R}} = 0 = 22.3$ per cent; $I_{K_{2V}} = 0 = 0.7$ per cent. Two types of absorbing components are postulated to explain the pattern of observed K_{2V} , K_{2R} intensity ratios. One component with minor Doppler displacements acting on the normal K_{232} profile, where $K_{2V} > K_{2R}$, produces the cases $K_{2V} \gg K_{2R}$, $K_{2V} = K_{2R}$, $K_{2V} < K_{2R}$. The other component arises from 'dark condensations' which are of size 3500 kms as seen in K_{2R} . They have principally large down flowing velocities in the range 5–8 km/sec and are seen on K_3 spectroheliograms with sizes of about 5000 kms, within the coarse network of emission. These 'dark condensations' give rise to the situation $K_{2R} = 0$.

K_2 -line widths are measured for all tracings where K_{2V} , K_{2R} are measurable simultaneously. The distribution curve of these widths is extremely sharp. The K_2 emission source is identified with the bright fine mottles visible on the surface. Evidence for this interpretation comes from the study of auto-correlation functions of K_2 intensity variations and the spacing between the bright fine mottles from both spectrograms and spectroheliograms. The life time of the fine mottling is 200 sec.

The supergranular boundaries which constitute the coarse network come in two intensity classes. A low intensity network has the fine mottles as its principal contributor to the K emission. When the network is bright, the enhancement is caused by increased K emission due to the accumulation of magnetic fields at the supergranule boundary. The K_2 widths of the low intensity supergranular boundary agree with the value found for the bright mottles. Those for the brighter network are lower than this value, similar to the K_2 widths as seen in the active regions.

It is concluded that bright fine mottling is responsible for the relation, found by Wilson and Bappu, between K emission line widths and absolute magnitudes of the stars.

The paper discusses the solar cycle equivalents that stellar chromospheres can demonstrate and indicates a possible line of approach for successful detection of cyclic activity in stellar chromospheres.

The last decade has witnessed much interest in the chromospheres of stars, especially on problems regarding the physical parameters that characterize them. There is also the added charm of speculating on the detection of cyclic effects in stellar chromospheres, similar to that which we are aware of in the solar case. The discovery that the widths of the K emission reversals are related to the absolute magnitudes of the stars (Wilson and Bappu, 1957; Bappu, 1954; Wilson, 1954) and that the intensities have an age dependence (Wilson, 1963), together with the high resolution spectroscopic efforts of Tousey and his collaborators (Tousey, 1967) in the vacuum ultraviolet, have been the principal stimulants to progress in the field. The photoelectric measurements of Liller (1968) and Wilson (1968) of the structure and intensities of stellar K-line reversals, as well as the secular changes of these aspects, hold forth the promise of an early detection of solar cycle equivalent phenomena in other stellar chromospheres.

Over the decades since Hale, Deslandres and Young first observed the bright reversals on the solar surface, there has been much effort directed towards the observations of these characteristics in the solar chromosphere. These cover spatial variations over active and quiet regions of the Sun, and in recent years in particular, the interpretation of the double reversal, especially in the context of the absolute magnitude – K-line width relation. An excellent review on the solar H and K-lines has recently been prepared by Linsky and Avrett (1970). Many aspects of this remarkable relation are still not clear and undoubtedly will need much observation and theoretical speculation for a final interpretation.

Our purpose in this study is to investigate the detailed K-emission line behaviour in a typical quiet region of the Sun and examine the possibility of obtaining an identification on the solar surface of those characteristics that enable the Sun to be one of those stars that obeys the K-line width-absolute magnitude relation.

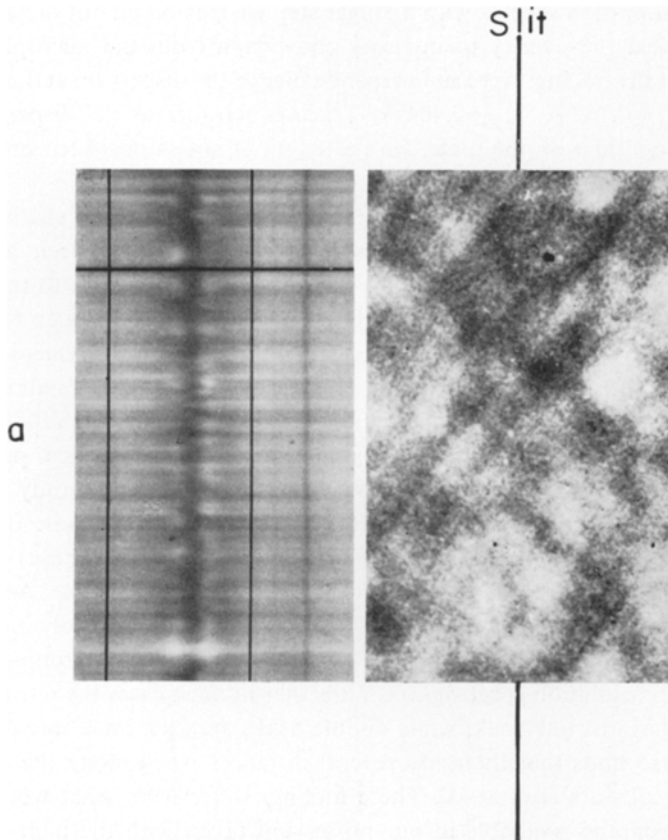


Fig. 1a. The K-line spectrogram of 12 June 1964, Scale:5.5"/mm. The 2 vertical dark lines are shadows of wires stretched in the focal plane. The horizontal dark line is caused by a wire stretched across the entrance slit of the spectrograph.

Fig. 1b. K_{232} spectroheliogram taken within 4 min of the spectrogram and enlarged to the same scale. The supergranules intersected by the slit can be identified with the help of the spectroheliogram.

We have used for this purpose one frame of exceptional quality exposed on the K-line. This frame forms part of a time sequence obtained with the spectrograph slit on a quiet region near the centre of the solar disc. There were no plages in the near vicinity. The time sequence was obtained at the Kodaikanal solar tower with an image scale of $5.5''/\text{mm}$ in the telescope focal plane and with the 18 m Littrow spectrograph that gives a dispersion of $9.4 \text{ mm}/\text{\AA}$ in the sixth order. Overlapping orders were eliminated with wide band glass colour filters. Careful centering of the image on a guide plate together with a knowledge of the orientation of the slit allow an exact identification of the location of the spectrograph slit on a K_{232} spectroheliogram. This spectroheliogram was taken within 4 min of the K spectrum that we have utilized in this study.

Figure 1 is an enlargement of the K-line alongside a high contrast copy of the spectroheliogram that shows the supergranular network intersected by the spectrograph slit. A 45 mm length of slit permitted coverage of nearly 180000 km of the solar surface. Calibration spectra with a Hilger step wedge and an out of focus image of the Sun enabled the density to intensity conversion from the microphotometer tracings. Some of the tracings were run perpendicular to the dispersion at the following settings: K_3 , $K_3 \pm 0.162 \text{ \AA}$, $K_3 \pm 0.404 \text{ \AA}$. Tracings parallel to the dispersion were run at intervals of 160μ on the plate, with a length of slit as projected on the plate equal to 160μ .

Of 159 tracings obtained along the dispersion in a sequence with spatial interval on the plate of 160μ or 640 km on the solar surface, 148 have been reduced to intensities. These have been normalized in wavelength with reference to the line Fe I 3932.639 \AA and in intensity at a point in the K_{1V} profile at 3933.13 \AA . Such a procedure enables intercomparison of intensity values at any point of the K emission profile amongst the different scans. Normalization of the wavelength provides all scans with a zero point in wavelength that enables evaluation of the Doppler shifts of the K_3 core relative to the Fe line, for different positions along the slit. Typical profiles can be seen in Figure 2. Of the 148 cases that we have included in this study we see no trace of K_{2R} ($I_{K_{2R}}=0$) on 33 scans. We have $I_{K_{2V}} > I_{K_{2R}}$ for 67 scans. By far this situation is the most common, since it happens in 45.2 per cent of the cases examined. The situation $I_{K_{2V}} = I_{K_{2R}}$ prevails in 7 cases or 4.7 per cent of the whole. And we find that $I_{K_{2V}} < I_{K_{2R}}$ in 37 cases or 25 per cent of the number examined. Only on one scan can we say with certainty that $I_{K_{2V}} = I_{K_{2R}} = 0$. Pasachoff (1969, 1970) from an analysis of Kitt Peak high resolution spectrograms finds that in most cases the normal profile of the emission has just one peak, while double peaks are seen on 10 per cent of the occasions. He also finds that there are several instances when along the lengths of the slit no K_2 peaks are seen at all. These findings differ from what we encounter on several spectrograms available in our possession taken with the high resolution spectrographs both at Kodaikanal and at Kitt Peak. Pasachoff's findings differ also from those of Jensen and Orrall (1963).

We next adopt the wavelength of K as given in the Rowland Atlas and determine the positive or negative μ shifts of the deepest part of K_3 from the rectified tracings.

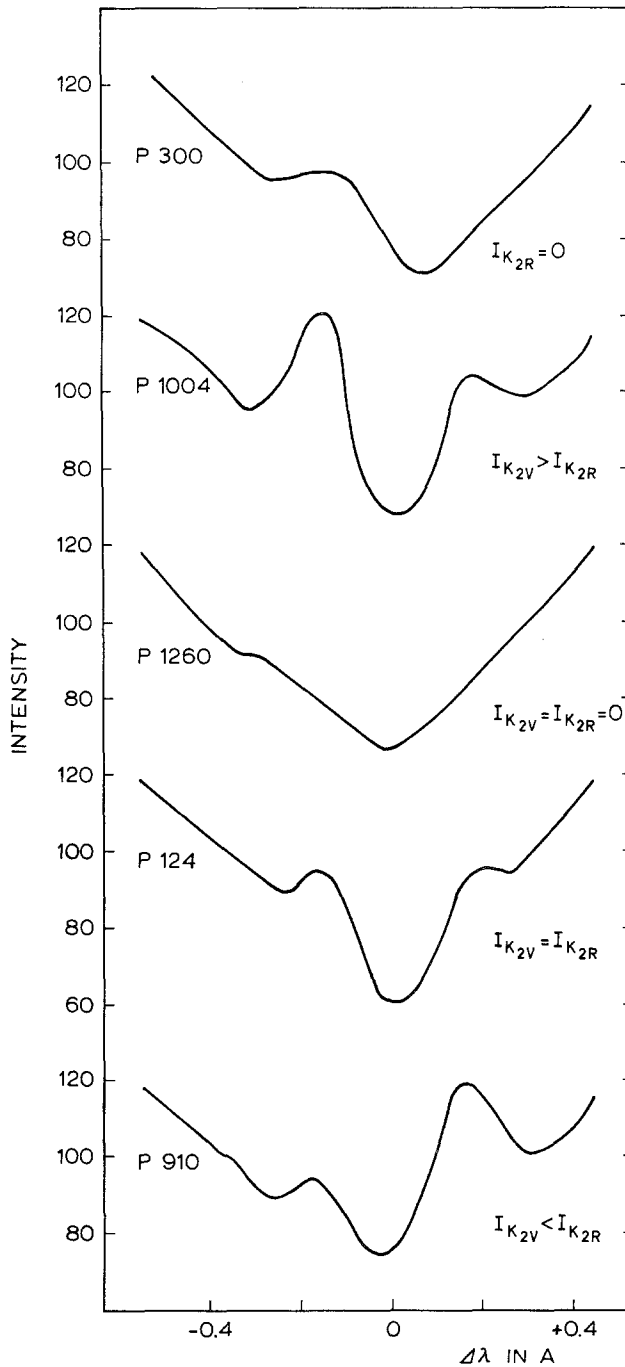


Fig. 2. Samples of profiles at 5 points along the slit showing different ratios of $I_{K_{2V}}$ and $I_{K_{2R}}$. The numbers P 300 etc. give the profile designation along the slit, reckoned from the wire shadow parallel to the dispersion.

We categorize these as negative, 0 and 2 types of positive shifts, one being large and the other inclusive of the medium and small shifts. Figure 3 depicts histograms that indicate the number in each of the four classes of wavelength shifts, for each of the intensity categories. These reveal a relationship between the Doppler shifts of K_3 and

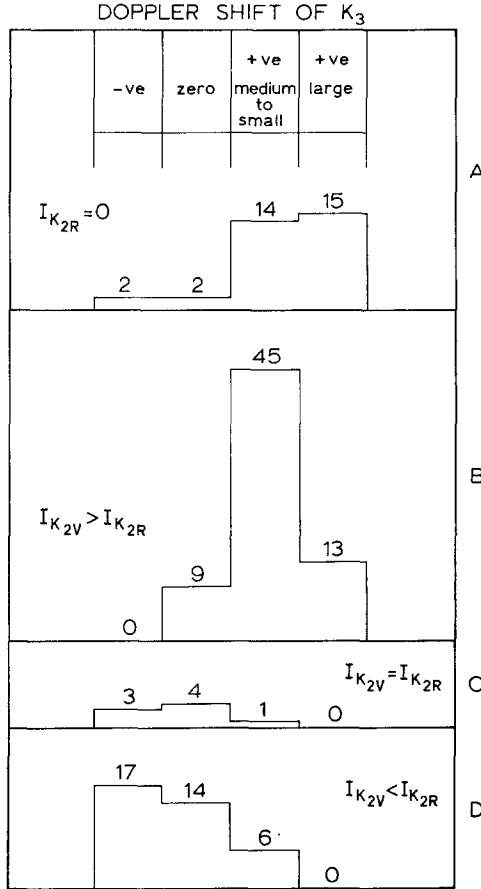


Fig. 3. Relation between Doppler shifts of K_3 and the 4 intensity categories of $I_{K_{2R}}$ and $I_{K_{2V}}$. Number in the diagram represent the number of instances in each category. Note that when $I_{K_{2R}} = 0$ the Doppler shifts of K_3 are predominantly redward while for $I_{K_{2V}} < I_{K_{2R}}$ the K_3 shifts are primarily negative.

the intensity ratio of $I_{K_{2V}}, I_{K_{2R}}$. Consider *B* and *C* alone, and we find that the common characteristic depicted is an excess of positive shifts of K_3 when $I_{K_{2V}} > I_{K_{2R}}$. The reverse trend is seen of numerous negative shifts or 0 shifts when $I_{K_{2V}} < I_{K_{2R}}$. When equality of $I_{K_{2V}}$ and $I_{K_{2R}}$ exists, as in histogram *C* which includes the single case $I_{K_{2V}} = I_{K_{2R}} = 0$, not only do we have an increase in the number of undeviated K_3 positions, but also we find in general negative shifts more often than small positive

shifts. The sample we have, is limited, but it is significant that when we indeed have such an equality we do not have a single instance of a large positive shift. The intensity ratio thus seems to depend on the Doppler characteristic of a veil that decreases the intensity of either the violet or red chromospheric reversal.

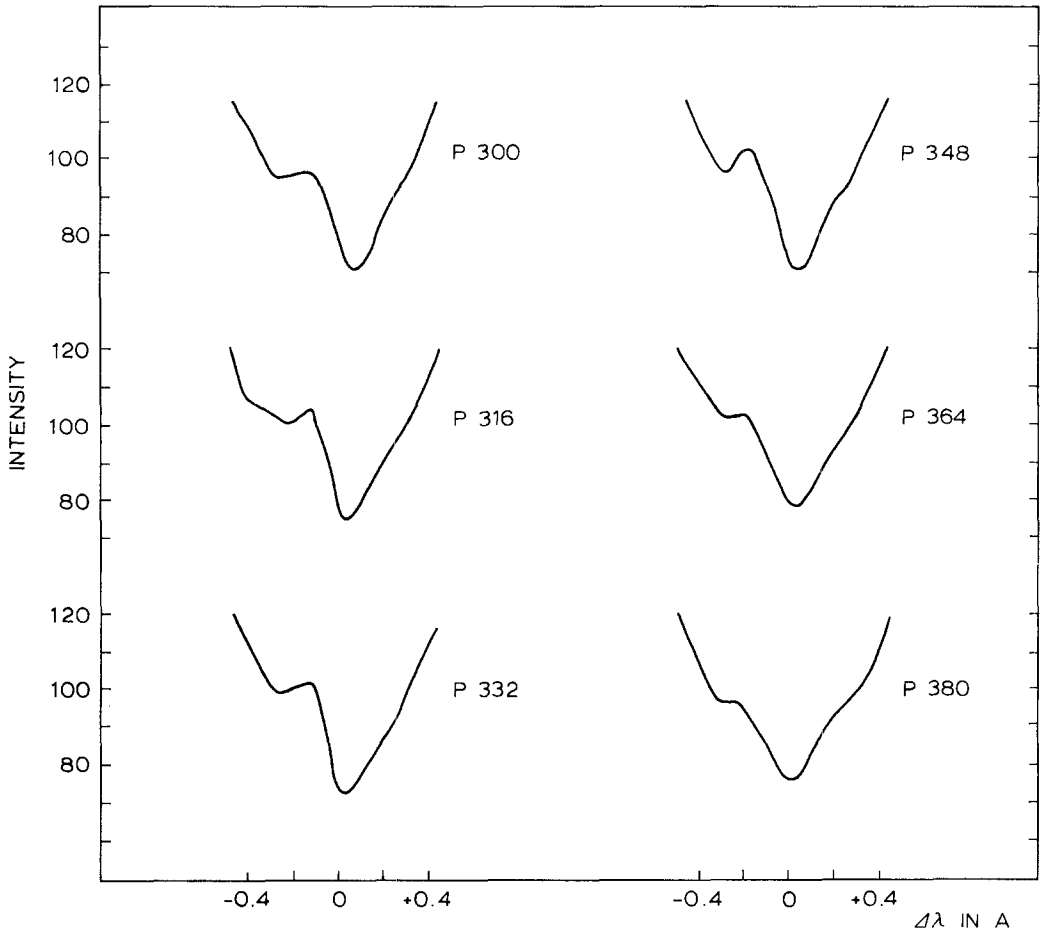


Fig. 4. Sets of profiles from consecutive positions 640 km apart, along the slit, for which $I_{K_{2R}} = 0$.

As indicated earlier we have 33 scans on which no K_{2R} can be seen. If we locate these scan positions on a K_3 intensity tracing obtained perpendicular to the dispersion, we notice that 32 of the scans fall in positions where a low intensity value of K_3 prevails. If these positions of $I_{K_{2R}} = 0$ are located on the print of the spectrum, a striking feature is the detection of areas of increased absorption in K_3 coincident with the $I_{K_{2R}} = 0$ positions. Many of these features are found on consecutive scans (Figure 4) that cover about 800 to 960 μ or 3200 to 3840 km on the solar surface at the

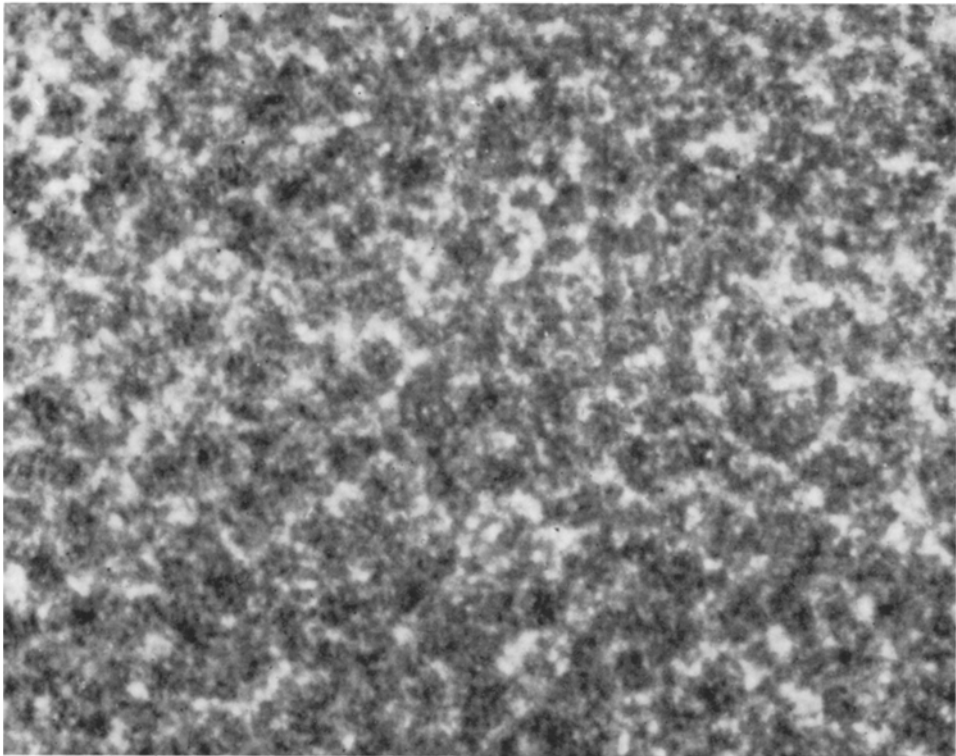
position of K_{2R} . These areas are larger in K_3 . It appears, therefore, that enhanced displaced absorption minimizes the K_{2R} intensity and reduces it to 0. Proof of such a conjecture follows from an examination of the values of I_{K_3} for large positive Doppler displacements of K_3 . The values of I_{K_3} are quite normal for 14 out of the 15 points that have a large positive displacement. The values of I_{K_3} decrease as one examines the 14 cases where positive shifts are small. An examination of the spectrum indicates that most of these 'dark condensations' are predominantly down flowing with extended wings towards the red. We believe that the $I_{K_{2R}}=0$ situation prevails wherever the 'dark condensations' exist that seem to rain into the chromosphere with velocities of about 5 to 10 km/sec.

In brief, we consider the manifestation of two distinct agencies of absorption that are at play in producing the subtle variations in $I_{K_{2V}}/I_{K_{2R}}$ ratio, when we examine the solar K-line chromospheric emission under high resolution. A slight absorption prevails in the position of K_3 which causes the normal $I_{K_{2V}} > I_{K_{2R}}$ to assume the form $I_{K_{2V}} = I_{K_{2R}}$ or $I_{K_{2V}} < I_{K_{2R}}$ when both K_{2V} and K_{2R} exist with measurable intensity. There is the additional absorption with large positive displacement which we call the 'dark condensations' which rain down into the chromosphere with cross-sections of 3000 to 4000 km and which totally obliterate the K_{2R} emission feature thus giving rise to the situation $I_{K_{2R}}=0$, and $I_{K_{2V}}$ bright and conspicuous. In histogram A we also notice that we have two cases of negative K_3 velocity and two of zero K_3 velocity, even when $I_{K_{2R}}=0$. We interpret this as indicating that the K_{2R} disappearance is due to the positive displaced 'dark condensation'. However, the negative and zero displacements of the K_3 minimum imply that if the 'dark condensation' had not existed and thereby affected the intensity of K_{2R} we would have had the situation of $I_{K_{2V}} < I_{K_{2R}}$ by virtue of the negative shift. We, therefore, see here the resultant effect of both sources of absorption considered above. We find additional justification for such a conjecture from the fact that, indeed for these cases, the intensities of K_{2V} are lower than normal.

The 'dark condensations' that we have described should be seen clearly in their two dimensional characteristic on K_3 spectroheliograms obtained under good conditions of seeing. We find this to be so, as can be seen in the reproduction of one such spectroheliogram in Figure 5, where we find many of these dark condensations of sizes in the 4000 to 6000 km range as seen in K_3 present within the coarse network boundaries.

The intensity tracings perpendicular to the spectrograph dispersion have been utilized for determining the rms values of intensity fluctuations. These are as in Table I.

The largest variations in intensity are in K_3 . A contribution of a significant nature to these variations arises from the presence of the 'dark condensations'. If one allows for this contribution, then one will find that K_{2V} and K_{2R} show the largest rms fluctuations. Hence, maximum detail of emission features on a spectroheliogram will be obtainable when the slit setting is of K_{2V} or K_{2R} . Expanded line profiles as described first by Leighton (1959) and demonstrated by Title (1966) in his collection of spectroheliograms bear out this point of view.



————— 32000 Km

Fig. 5. Part of a high quality K_{232} spectroheliogram obtained at Kodaikanal. The image shows the bright points or fine mottles, the coarse mottles, the network and the dark condensations.

TABLE I

	Designation	rms per cent
3933.684	K_3	15.3
3933.522	K_{2V}	14.5
3933.846	K_{2R}	14.5
3933.280	K_{1V}	12.2
3934.088	K_{1R}	8.3

We now proceed to an evaluation of the K-line widths. The emission line widths as measured for the stars by Wilson and Bappu (1957) are micrometer settings that can be carried out without difficulty by virtue of the low dispersion as compared to solar spectra. We have, therefore, measured two parameters: (1) a K_2 width which measures the peak to peak separation of K_2 from the intensity tracings (2) a K_1 width

which signifies the width of the entire emission feature. Figure 6 describes pictorially our definitions. By virtue of our specification, the widths we have measured are for features wherein K_{2V} and K_{2R} are noticeable and measurable. This brings our sample to 52 cases where such measures can be made. The sample includes the cases $I_{K_{2V}} > I_{K_{2R}}$; $I_{K_{2V}} = I_{K_{2R}}$ and $I_{K_{2V}} < I_{K_{2R}}$. A histogram, where numbers are plotted for

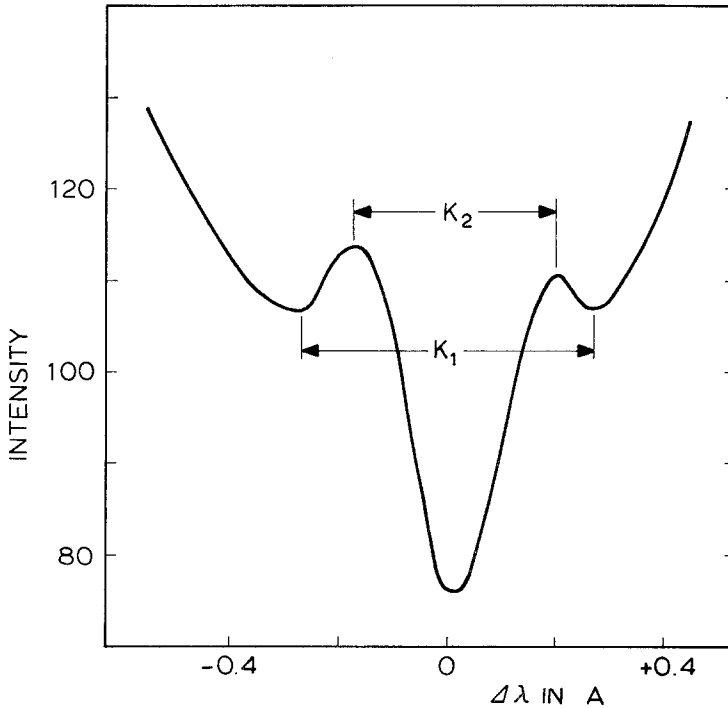


Fig. 6. K-line emission profile showing definitions of K_2 and K_1 -line widths.

different K_2 and K_1 widths, brings out the striking feature of the narrow range in values about a mean velocity, as implied by the width. Figure 7 shows the extreme sharpness that a distribution curve of K_2 widths have on the solar surface. The K_2 widths as defined yield a mean value of 26.2 km/sec. The K_1 widths yield a value of 41.3 km/sec. A mean emission line width defined as the width at half intensity, midway between the K_1 , K_2 points on both violet and red side yields 33.75 km/sec. This result agrees well with the value of 34 km/sec obtained by Wilson and Bappu (1957) from a McMath-Hulbert trailed spectrum of the solar image.

The spectrum seen in Figure 1 has a localized enhancement of K_{2V} , K_{2R} which can be identified on the spectroheliogram as arising from the boundary of the coarse network. We wish to point out at this stage that there appears to be a slight inconsistency of nomenclature in the literature. Linsky and Avrett (1970) consider the coarse network to be coarse mottles. This is incorrect, since the coarse network depicts the

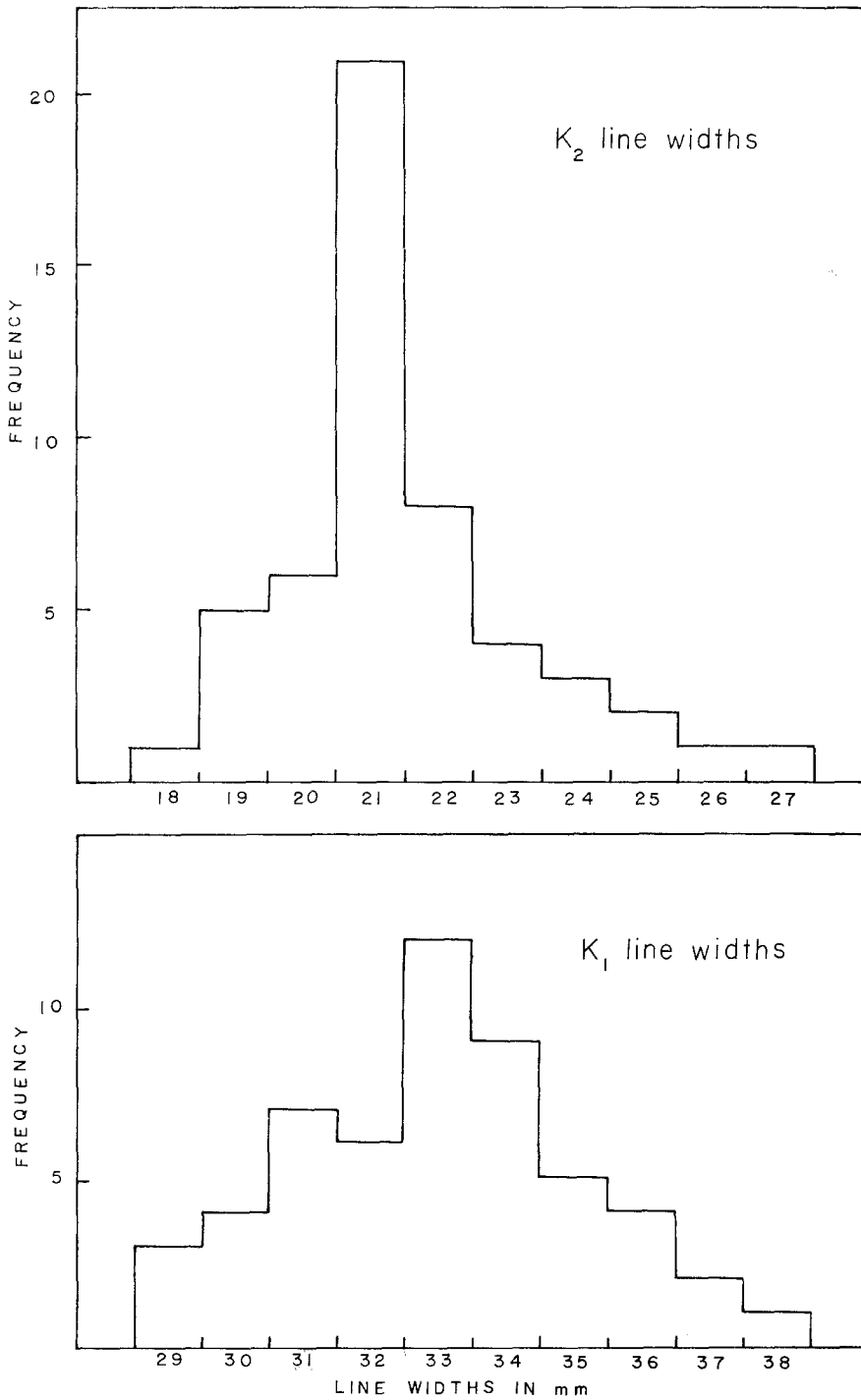


Fig. 7. Distribution of K₂ and K₁-line widths measured from 52 profiles having measurable K_{2V} and K_{2R}. Abscissa gives line widths measured in mm on the tracings (1 mm = 1,268 km/sec).

supergranulation and is a cell of size 30000 km, while a coarse mottle is a localized aggregate of fine mottles and has a size of about 7000 km (De Jager, 1959). The single localized enhancement which we identify as the supergranular boundary is the brightest feature seen along the entire slit. The mean K_2 emission width for this feature is 20.3 km/sec, a value very much less than the 26.2 km/sec found elsewhere along the slit. Other boundaries of supergranules intersected by the spectrograph slit are of quite low intensity and we find no appreciable decrease in their individual K_2 widths when compared to the overall mean value.

We have seen that when $I_{K_{2V}}$ and $I_{K_{2R}}$ are of appreciable intensity as to allow determination of reliable K_1 , K_2 line widths, the mean value fits in well with the relation of Wilson and Bappu (1957). Bright points of K_2 emission are thus the principal contributors to the line width-absolute magnitude relation. The intensity scans perpendicular to the dispersion show that many of the bright points are very narrow in size and are typically of the order of 1-2 sec of arc. Hence, these bright points that enable the Sun to follow the line width-absolute magnitude relation are the fine mottles (De Jager, 1959) seen on the solar surface.

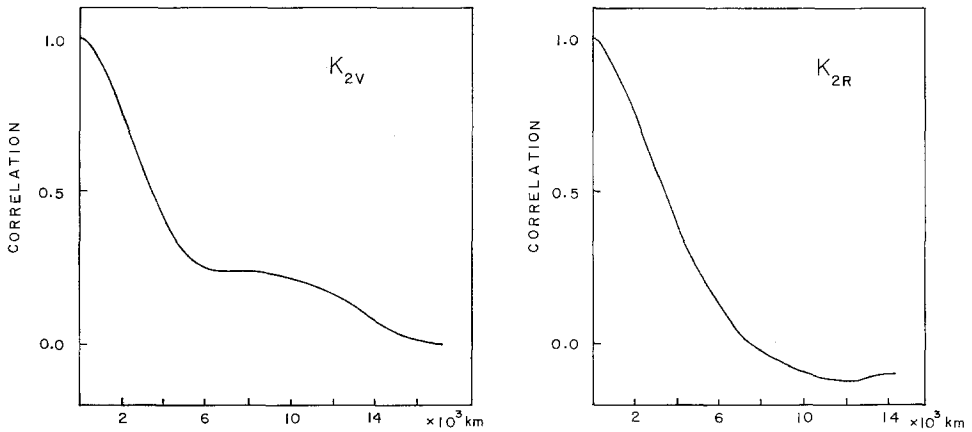


Fig. 8. Auto-correlation curves of intensity fluctuations along slit of K_{2V} and K_{2R} . The full-width at half-maximum gives the average spacing between the bright points,

The spacings of these bright points along the spectrograph slit will no doubt serve to stabilize our identification better. We have used the intensity scans perpendicular to the dispersion and evaluated intensities at intervals of 300 km along the slit length. These values permit the derivation by a computer of an auto-correlation function the full width at half maximum of which as can be judged from Figure 8 is 6800 km. We interpret this value as indicative of the spacing between the bright points as derived from a one dimensional scan along the slit length.

It would be of interest to make this identification secure by a similar study of bright points on a two dimensional scale using a high quality K_{232} spectroheliogram.

Following Bahng and Schwarzschild (1961), we project an enlarged image of the spectroheliogram negative on its own print, enlarged to the same size and measure the amount of scattered light photoelectrically off the print, when it is displaced over distances smaller than the spacing of the bright points. This gives an auto-correlation function, the full width at half maximum of which yields the average spacing between the bright points. A value of 7000 km is typical of the spacing and agrees well with that derived by auto-correlation methods from the spectrum. This confirms that the bright points, that yield the K-line widths which correlate with absolute magnitude, are in fact the fine mottles.

Our picture of the calcium coarse network, which can be seen also from Figure 5, is that it is built up of fine mottles and aggregates of fine mottles called the coarse mottles (De Jager, 1959). In addition, the recent findings of Leighton and his collaborators imply a piling up of magnetic field at the boundary of the supergranule leading to a magnetohydrodynamic origin of extra calcium emission along the network. It is well known from the work of Elske Smith (1960) and others subsequently, that the K_2 widths as defined in Figure 6 decrease over plage areas where magnetic fields are appreciable or high. We have ascribed above, two contributors to the network emission. One is the contribution of the fine mottles to the intensity. The other is the emission that is associated with the presence of the magnetic field at the boundary. As the latter increases in value appreciably, it tends to lower the final K_2 width which is a weighted mean of the two. The single enhancement at the supergranular boundary at the bottom of the spectrum in Figure 1 represents a similar case and explains the lower value of K_2 width that we have obtained.

The fine mottles in most high quality spectroheliograms are seen distributed not only along the network but also within it together with the 'dark condensations' we have discussed earlier. What then is the contributor to the emission from the space between the conspicuous and bright fine mottles that we see in a two dimensional picture? Since our intensity scans reveal K_{2V} or K_{2R} emission to prevail on a scale of a second of arc, we may have an unresolved background of fine mottles providing the emission intensity. The fact that the emission line widths are independent of intensity of the emission implies the validity of our assumption of a background of fine mottles of low intensity, that are unresolved under the combined conditions of seeing and instrumental resolution.

The time sequence in K permits the determination of the life times of these fine mottles. Since the solar image was guided to better than a second of arc it is possible to follow individually the life history of each bright feature. Only those bright points were included that were born during the time sequence. The life times can be judged from Figure 9 for the 45 cases that have been studied. A mean life time around 200 sec is typical. We have no spectroheliogram sequence that can permit the use of a procedure similar to that employed on the time sequence of spectrograms. We have, however, taken two consecutive spectroheliograms under good seeing conditions and with a time interval between them of 134 sec to indicate indirectly by number counts the order of the life time. Of 205 fine mottles studied on the first plate 47 per cent were

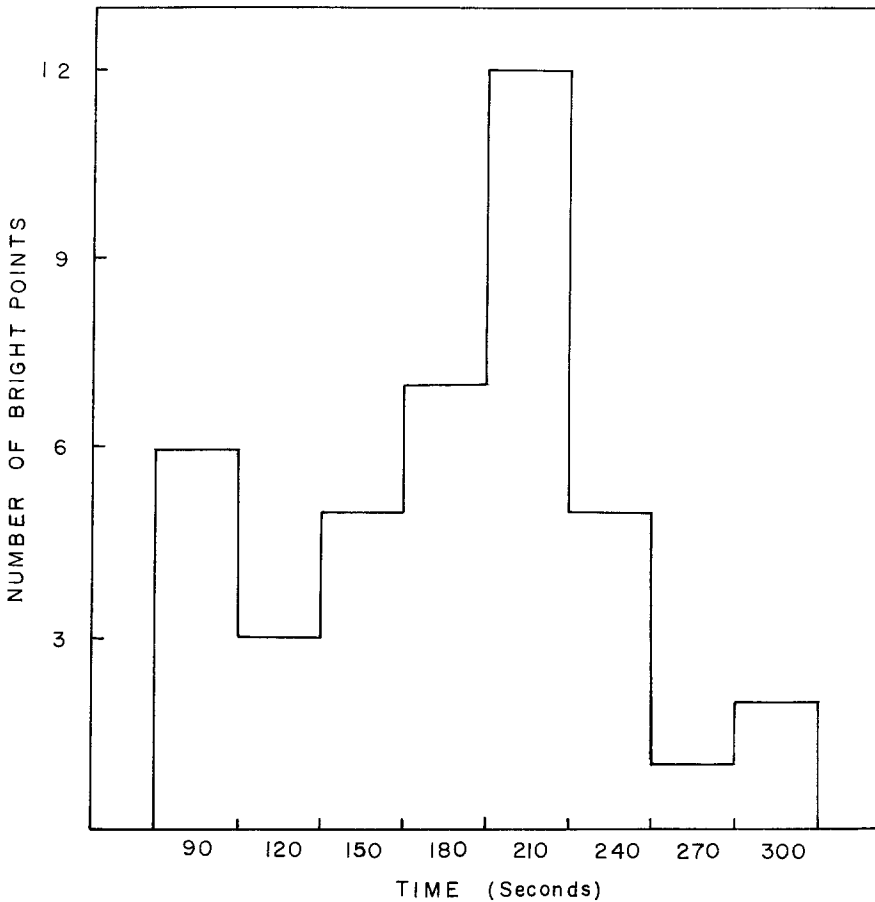


Fig. 9. Distribution of life times of bright points.

completely lost on the second exposure, 37 per cent were reduced by 60 per cent of their original intensity. Nine per cent showed no change in the time interval of 134 sec. This indicates the validity of our estimate, of a 200 sec lifetime as typical for the fine mottles.

A final inference from the K-line behaviour on the Sun will be a useful guide to the detection and measurement of star cycles. The appearance and disappearance of extensive plages with phase in the sunspot cycle makes it obvious that the Sun, if examined in a 0.7 Å band centred on the K-line, would prove to be variable in light. Assuming that the average plage intensities are 2.4 times the dark portions of the mottled background (Dodson and Hedeman, 1954), the K-line variation of the Sun will be about 7 per cent. This estimate is much lower than Sheeley's (1967) value of 40 per cent. However, the light variation will be dependent on the rotation of the star and the rate of birth and decay of plages. We plot in Figure 10 for the years 1960 and 1962 the calcium plage areas on the visible hemisphere of the Sun as measured from

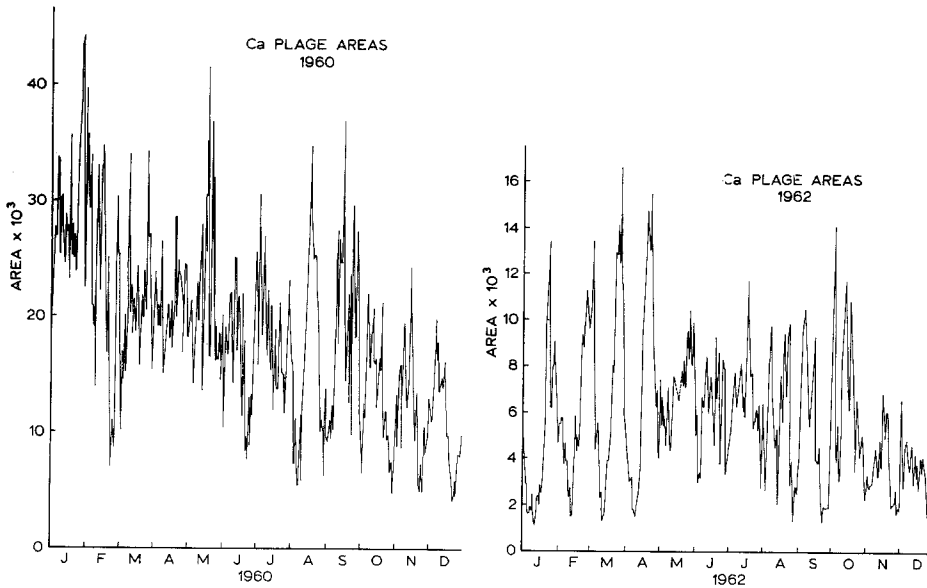


Fig. 10. Plot of Ca^+ plage areas (mean daily areas) for the years 1960 and 1962. The area is in millionths of visible hemisphere of the Sun.

Kodaikanal K_{232} spectroheliograms (Kuriyan, 1967). The changes are large, specially near maximum, when within a rotation period striking variations can be seen. However, the gross amplitude of variation from sunspot maximum to minimum is noticeable and quite measurable. If continuous observations at well chosen times are made, specially when only a single active region is seen on the visible disc, as in the case of the early months of 1962, the information can be used for a measurement of the star's rotational velocity with some accuracy.

It should be remembered that star cycle effects similar to that of the solar cycle necessarily imply the presence of active regions on the visible portion of the star and consequent enhancement of K-line emission associated with the magnetic activity. Such K emission would have K_2 widths that would be narrower than the contribution originating from the fine mottle background. Since the integrated spectrum of the star would represent the weighted mean of the fine mottle contributions as well as those originating from the active regions on the stellar surface, the K emission width of the star as defined in the Wilson-Bappu terminology will tend to be higher than what is consistent with the known absolute magnitude of the star. Viewed from another point of view, one might say that such stars would deviate appreciably from the normal relation of line width and absolute magnitude. It would seem, therefore, that those stars that provide K-line widths larger than what is consistent with their M_V inferred otherwise may prove more suitable for detection of star cycles similar to the solar cycle, than would be the case if only enhanced intensity of K emission were the criterion of selection.

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