Direct Measurement of the Top Quark Mass

The DØ Collaboration*
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(March 10, 1997)

Abstract

We measure the top quark mass $m_t$ using $t\bar{t}$ pairs produced in the DØ detector by $\sqrt{s} = 1.8$ TeV $p\bar{p}$ collisions in a 125 pb$^{-1}$ exposure at the Fermilab Tevatron. We make a two constraint fit to $m_t$ in $t\bar{t} \rightarrow bW^+\bar{b}W^-$ final states with one $W$ decaying to $q\bar{q}$ and the other to $e\nu$ or $\mu\nu$. Events are binned in fit mass versus a measure of probability for events to be signal rather than background. Likelihood fits to the data yield $m_t = 173.3 \pm 5.6$ (stat) $\pm 6.2$ (syst) GeV/$c^2$.

(DØ Collaboration)
1 Universidad de los Andes, Bogotá, Colombia
2 University of Arizona, Tucson, Arizona 85721
3 Boston University, Boston, Massachusetts 02215
4 Brookhaven National Laboratory, Upton, New York 11973
5 Brown University, Providence, Rhode Island 02912
6 Universidad de Buenos Aires, Buenos Aires, Argentina
7 University of California, Davis, California 95616
8 University of California, Irvine, California 92697
9 University of California, Riverside, California 92521
10 LAFEX, Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, Brazil
11 CINVESTAV, Mexico City, Mexico
12 Columbia University, New York, New York 10027
13 Delhi University, Delhi, India 110007
14 Fermi National Accelerator Laboratory, Batavia, Illinois 60510
15 Florida State University, Tallahassee, Florida 32306
16 University of Hawaii, Honolulu, Hawaii 96822
17 University of Illinois at Chicago, Chicago, Illinois 60607
18 Indiana University, Bloomington, Indiana 47405
19 Iowa State University, Ames, Iowa 50011
20 Korea University, Seoul, Korea
21 Kyungsung University, Pusan, Korea
22 Lawrence Berkeley National Laboratory and University of California, Berkeley, California 94720
23 University of Maryland, College Park, Maryland 20742
24 University of Michigan, Ann Arbor, Michigan 48109
25 Michigan State University, East Lansing, Michigan 48824
26 Moscow State University, Moscow, Russia
27 University of Nebraska, Lincoln, Nebraska 68588
28 New York University, New York, New York 10003
29 Northeastern University, Boston, Massachusetts 02115
30 Northern Illinois University, DeKalb, Illinois 60115
31 Northwestern University, Evanston, Illinois 60208
32 University of Notre Dame, Notre Dame, Indiana 46556
33 University of Oklahoma, Norman, Oklahoma 73019
34 University of Panjab, Chandigarh 16-00-14, India
35 Institute for High Energy Physics, 142-284 Protvino, Russia
36 Purdue University, West Lafayette, Indiana 47907
37 Rice University, Houston, Texas 77005
38 Universidade Estadual do Rio de Janeiro, Brazil
39 University of Rochester, Rochester, New York 14627
40 CEA, DAPNIA/Service de Physique des Particules, CE-SACLAY, Gif-sur-Yvette, France
41 Seoul National University, Seoul, Korea
42 State University of New York, Stony Brook, New York 11794
43 Tata Institute of Fundamental Research, Colaba, Mumbai 400005, India
44 University of Texas, Arlington, Texas 76019
45 Texas A&M University, College Station, Texas 77843
The top quark has a large mass $m_t$ that can be determined to greater fractional precision than is possible for the lighter quarks, which decay after they form hadrons. Since $m_t$ is large, it controls the strength of quark-loop corrections to tree-level relations among electroweak parameters. If these parameters and $m_t$ are measured precisely, the Standard Model Higgs boson mass can be constrained.

Direct measurements of $m_t$ have been published as part of the initial observations [1] of $t\bar{t}$ production in $\sqrt{s} = 1.8$ TeV $p\bar{p}$ collisions. At present, the best accuracy in $m_t$ is achieved for lepton + jets ($\ell$+jets) final states in which one $W$ boson (from $t \to bW$) decays to $e\nu$ or $\mu\nu$ and the other $W$ decays to a $q\bar{q}$ pair that forms jets. We report a measurement of $m_t$ in the $\ell$+jets channel using the $\approx 125$ pb$^{-1}$ exposure of the DØ detector during the 1992–96 Fermilab Tevatron runs. Since Ref. [1] appeared, our data sample has doubled, and for a fixed sample size our error on $m_t$ has halved.

The DØ detector and our basic methods for triggering, reconstructing events, and identifying particles are described elsewhere [2]. Recent advances include enhanced triggering and reconstruction efficiency for $\mu$+jets events, due in part to better use of calorimeter data. As a signature of $W \to \ell\nu$, we require missing energy transverse to the beam ($E_{T}^{\ell}$) $> 20$ GeV, and one isolated $e$ or $\mu$ ($\ell$) with $E_{T}^{\ell} > 20$ GeV and pseudorapidity $|\eta_{\ell}| < 2$ or $|\eta_{\mu}| < 1.7$. We also demand $E_{T}^{\text{cal}} > 25$ (20) GeV for $e$+jets ($\mu$+jets) events, where $E_{T}^{\text{cal}}$ is $E_{T}$ measured only in the calorimeter. As signatures of the $q\bar{q}$ from $W$ decay and the $b$ and $\bar{b}$ from $t$ and $\bar{t}$ decay, we require $\geq 4$ jets reconstructed with cones of half-angle $\Delta R \approx (\Delta\phi^2 + \Delta\eta^2)^{1/2} = 0.5$, having $E_{T} > 15$ GeV and $|\eta| < 2$.

Within $\Delta R = 0.5$ of a jet axis, additional muons ($\mu$ tags) satisfying $p_{T}^{\mu} > 4$ GeV/c and $|\eta_{\mu}| < 1.7$ arise mainly from $b$ and $c$ quark semileptonic decay. These occur in $\approx 20\%$ of $t\bar{t}$ events but only $\approx 2\%$ of background events [2]. In untagged events, to suppress background we require $E_{T}^{F} (\equiv |E_{T}^{L}| + |E_{T}^{R}|) > 60$ GeV and $|\eta_{W}| < 2$ for the $W \to \ell\nu$. The latter cut, exhibited in Fig. 1(a), reduces the difference in $\eta_{W}$ distributions between data and Monte Carlo (MC) simulated background. We use the HERWIG MC [3] to simulate top signal, and the VECBOS MC [4] (with HERWIG fragmentation of partons into jets) to simulate (but not to normalize) the dominant $W$+multijet background. The $\approx 20\%$ of background events from non-$W$ sources are modeled by multijet data that barely fail the lepton identification criteria.

To each event passing the above cuts, we make a two constraint (2C) kinematic fit [3] to the $t\bar{t} \to \ell$+jets hypothesis by minimizing a $\chi^2 = (\mathbf{v} - \mathbf{v}^{*})^{T}G(\mathbf{v} - \mathbf{v}^{*})$, where $\mathbf{v}$ ($\mathbf{v}^{*}$) is the vector of measured (fit) variables and $G^{-1}$ is its error matrix. Both reconstructed $W$ masses are constrained to equal the $W$ pole mass, and the same fit mass $m_{\text{fit}}$ is assigned to both the $t$ and $\bar{t}$ quarks. If the event contains $>4$ accepted jets, only the four jets with highest $E_{T}$ are used. In $\approx 50\%$ of MC top events, these jets correspond to the $b$, $\bar{b}$, $q$, and $\bar{q}$. With (without) a $\mu$ tag in the event, there are 6 (12) possible fit assignments of these jets to the quarks, each having two solutions to the $\nu$ longitudinal momentum $p_{T}^{\nu}$. We use $m_{\text{fit}}$ only from the permutation with lowest $\chi^2$, the correct choice for $\approx 20\%$ of MC top events. Because of the ambiguities, $m_{\text{fit}}$ is not the same as $m_{t}$, though they are strongly correlated. Our best estimate of $m_{t}$ is obtained from the best match between MC samples and the data.

From the 90-event distribution shown in Fig. 1(b) we select 77 events with a 2C fit satisfying $\chi^2 < 10$. Of these, 5 are $\mu$ tagged and $\approx 65\%$ are background. Further separation of signal and background events is based on four kinematic variables $x \equiv \{x_1, x_2, x_3, x_4\}$
chosen to have small correlation with $m_{\text{fit}}$. On average, all are larger for MC top events than for background events, selected to have the same $\langle m_{\text{fit}} \rangle$ as the top events [6]. The simpler variables are $x_1 \equiv E_T$ and $x_2 \equiv A$, where aplanarity $A$ is \( \frac{2}{3} \times \) the least eigenvalue of the normalized laboratory momentum tensor of the jets and the $W$ boson. The third variable $x_3 \equiv H_{T2}/H_z$ measures the event’s centrality, where $H_z$ is the sum of $|p_z|$ of $\ell$, $\nu$, and the

FIG. 1. Events per bin vs. event selection variables defined in the text, plotted for (a–b, g–h) top quark mass analysis samples, and (c–f) $W + 3$ jet control samples. Histograms are data, filled circles are expected top + background mixture, and open triangles are expected background only. Solid arrows in (a–b) show cuts applied to all events; the open arrow in (g) illustrates the LB cut. The nonuniform bin widths in (g–h) are chosen to yield uniform bin populations.
jets, and $H_{T2}$ is the sum of all jet $|E_T|$ except the highest. Finally, $x_4 \equiv \Delta R_{jj}^{\min} E_T^{\min}/E_T^{\sum}$ measures the extent to which jets are clustered together, where $\Delta R_{jj}^{\min}$ is the minimum $\Delta R$ of the six pairs of four jets, and $E_T^{\min}$ is the smaller jet $E_T$ from the minimum $\Delta R$ pair. As shown for the background dominated $W+3$ jet sample in Fig. 1(c–f), $x_1$–$x_4$ are reasonably well modeled by MC; this is true also for the $W+2$ jet and top mass samples (not shown).

We bin events in a two-dimensional array with abscissa $m_{\text{fit}}$ and ordinate $D(x)$, where $D$ is a multivariate discriminant. To show that our results are robust, we use two methods for which the definition of $D$, the granularity with which it is binned, and the additional requirements are different. In our “low bias” (LB) method, we first parametrize $L_i(x_i) \equiv s_i(x_i)/b_i(x_i)$, where $s_i$ and $b_i$ are the top signal and background densities in each variable, integrating over the others. We form the log likelihood $\ln L \equiv \sum_i \omega_i \ln L_i$, where the weights $\omega_i$ are adjusted slightly away from unity to nullify the average correlation (“bias”) of $L$ with $m_{\text{fit}}$, and for each event we set $D_{\text{LB}} = L/(1 + L)$. Finally, we divide the ordinate coarsely into signal- and background-rich bins according to whether the LB cut is passed. This cut is satisfied if a $\mu$ tag exists; otherwise it is not satisfied if $D_{\text{LB}} < 0.43$ (Fig. 1(g)) or if $H_{T2} < 90$ GeV.

Our neural network (NN) method is sensitive to the correlations among the $x_i$ as well as to their individual densities. We use a three layer feed-forward NN with 4 input nodes fed by

![Figure 2](image-url)

FIG. 2. Events per bin ($\propto$ areas of boxes) vs. $D_{\text{NN}}$ (ordinate) and $m_{\text{fit}}$ (abscissa) for (a) expected 172 GeV/$c^2$ top signal, (b) expected background, and (c) data. $D_{\text{NN}}$ is binned as in Fig. 1(h).
x, 5 hidden nodes, and 1 output node, trained on samples of top signal (background) with density $s(x)/(b(x))$. For a given event, the network output $D_{\text{NN}}$ approximates the ratio $s(x)/(s(x)+b(x))$. We divide the ordinate finely into ten bins in $D_{\text{NN}}$, independent of $H_T^2$ or $\mu$ tagging. Figure 4(g–h) shows that $D_{\text{LB}}$ and $D_{\text{NN}}$ are distributed as predicted and provide comparable discrimination, as we expect when the $\omega_i$ are close to unity and the $L_i$ are not strongly correlated. Figure 2 exhibits the arrays for the NN method. Little correlation between $D_{\text{NN}}$ and $m_{\text{fit}}$ is evident in the expected signal or background distributions, which are distinct; the data clearly reveal contributions from both sources. Figure 3 shows the distributions of $m_{\text{fit}}$ for data (a) passing and (b) failing the LB cut.

![Figure 3](image-url)

**FIG. 3.** (a–b) Events per bin vs. $m_{\text{fit}}$ for events (a) passing or (b) failing the LB cut. Histograms are data, filled circles are the predicted mixture of top and background, and open triangles are predicted background only. The circles and triangles are the average of the LB and NN fit predictions, which differ by <10%. (c) Log of arbitrarily normalized likelihood $L$ vs. true top quark mass $m_t$ for the LB (filled triangles) and NN (open squares) fits, with errors due to finite top MC statistics. The curves are quadratic fits to the lowest point and its 8 nearest neighbors. In MC studies, 7% (27%) of simulated experiments yield a smaller LB (NN) maximum likelihood.
Table I. Results of fits to data and MC events. Fits to data yield values and errors $\sigma_{\text{stat}}$ for $m_t$, $n_s$, and $n_b$ (described in the text). Systematic errors are combined in quadrature. The resulting $m_t$ and its statistical error $\sigma_m$ are the combined LB and NN values. Fits to MC use ensembles of 10,000 simulated experiments composed of top + background, with $m_t$, $\langle n_s \rangle$, and $\langle n_b \rangle$ as listed. They yield a mean result $\langle m_t \rangle$, a mean statistical error $\langle \sigma_m \rangle$, and a range $\pm \delta m$ within which 68% of the results fall. Using the LB (NN) method, 6% (25%) of the simulated experiments produce a $\sigma_m$ which is smaller than we obtain. For an “accurate subset” of the MC ensembles with mean $\sigma_m/m_t$ that matches our value, $\delta m$ is smaller.

<table>
<thead>
<tr>
<th>Fits to data</th>
<th>---LB fit---</th>
<th>---NN fit---</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity fit</td>
<td>value</td>
<td>$\sigma_{\text{stat}}$</td>
</tr>
<tr>
<td>$m_t$(GeV/c$^2$)</td>
<td>174.0 ± 5.6</td>
<td>171.3 ± 6.0</td>
</tr>
<tr>
<td>$n_s$</td>
<td>23.8 +8.3 −7.8</td>
<td>28.8 +8.4 −9.1</td>
</tr>
<tr>
<td>$n_b$</td>
<td>53.2+10.7 −9.3</td>
<td>48.2+11.4 −8.7</td>
</tr>
</tbody>
</table>

Systematic error on $m_t$: energy scale ± 4.0, generator ± 4.1, other ± 2.2

Resulting $m_t$(GeV/c$^2$): $173.3 \pm 5.6$ (stat) ± 6.2 (syst)

<table>
<thead>
<tr>
<th>Fits to MC type</th>
<th>full ensemble</th>
<th>accurate subset</th>
</tr>
</thead>
<tbody>
<tr>
<td>type</td>
<td>---input---</td>
<td>---output---</td>
</tr>
<tr>
<td>(top + background) of fit</td>
<td>$m_t$</td>
<td>$\langle n_s \rangle$</td>
</tr>
<tr>
<td>LB</td>
<td>175</td>
<td>24</td>
</tr>
<tr>
<td>NN</td>
<td>172</td>
<td>29</td>
</tr>
<tr>
<td>LB</td>
<td>175</td>
<td>24</td>
</tr>
<tr>
<td>NN</td>
<td>172</td>
<td>29</td>
</tr>
</tbody>
</table>

To each $m_t$ for which we have generated MC, we assign a likelihood $L$ which assumes that all samples obey Poisson statistics. Bayesian integration\[8\] over possible true signal and background populations in each bin yields

$$L(m_t, n_s, n_b) = \prod_{i=1}^{M} \sum_{j=0}^{n_i} \left( \begin{array}{c} n_{si} + j \\ j \end{array} \right) \left( \begin{array}{c} n_{bi} + k \\ k \end{array} \right) p_s^j (1 + p_s)^{-n_{si} - j - 1} p_b^k (1 + p_b)^{-n_{bi} - k - 1},$$

where $n_s$ ($n_b$) is the expected number of signal (background) events in the data; $n_i$, $n_{si}$, and $n_{bi}$ are the actual number of data, MC signal, and MC background events in bin $i$; $k \equiv n_i - j$; $p_{s,b} \equiv n_{s,b}/(M + \sum_i n_{si,bi})$; and $M = 40$ (200) bins for the LB (NN) methods. Maximizing $L$ for each $m_t$ gives the best estimates $n_s^*(m_t)$ and $n_b^*(m_t)$ for $n_s$ and $n_b$. Figure 3(c) displays ln $L(m_t, n_s^*(m_t), n_b^*(m_t))$ vs. $m_t$, where the curves determine the best fit $m_t$ and its statistical error $\sigma_m$.

Table I presents the fit results, which are consistent with Ref. [1] and with recent reports [3]. The LB and NN results $m_t^{\text{LB}}$ and $m_t^{\text{NN}}$ are mutually consistent; in 21% of MC experiments they are further apart. Nevertheless we include half of $m_t^{\text{LB}} - m_t^{\text{NN}}$ in the systematic error. To obtain our result, shown in Table I, we combine $m_t^{\text{LB}}$ and $m_t^{\text{NN}}$ allowing
for their (88 ± 4)% correlation (determined by MC experiments). Figures 3(a–b) show that this result represents the data well. From the MC experiments summarized in Table I we measure the interval ±δm within which 68% of the MC estimates fall. For the full ensemble, δm is larger than σm from our data. However, for “accurate subsets” of the ensemble for which the average σm/mt is the same as we observe, δm is close to σm [10].

A principal systematic error in mt arises from uncertainty in the jet energy scale, which is calibrated in three steps. In step 1, applied before events are selected, the summed energy Ejet of particles emitted within the jet cone is related to the measured energy Em by 

\[ E_{\text{jet}} = \frac{(E_m - O)}{R(1 - S)} \]

Here the calorimeter response R is calibrated using Z → ee decays and ET balance in γ+jet events, the fractional shower leakage S out of the jet cone is set by test beam data, and the energy offset O due to noise and the underlying event is determined using events with multiple interactions. Steps 2 and 3 are applied only to jet energies used to find mfit. In step 2, top MC is used to correct Ejet to the parton energy in both data and MC. This sharpens the resolution in mfit. Step 3 is a final adjustment based on more detailed study of γ+jet events in data and MC, particularly focused on the dependence of the ET balance upon η of the jet. We assign a jet-scale error of ±(2.5% + 0.5 GeV) based on the internal consistency of step 3, on variations of the γ+jet cuts and the model for the underlying event, and on an independent check of the ET balance in Z+jet events. This leads to an error on mt of ±4.0 GeV/c².

We estimate the uncertainties in modeling of QCD by substituting the ISAJET MC generator for HERWIG, independently for top MC and for VECBOS fragmentation, and by changing the VECBOS QCD scale from jet ⟨pT⟩² to M². The resulting systematic error due to the generator is ±4.1 GeV/c². Other effects including noise, multiple p̅p interactions, and differences in fits to ln L contribute ±2.2 GeV/c². All systematic errors (Table I) sum in quadrature to ±6.2 GeV/c². Therefore our direct measurement of the top quark mass is mt = 173.3 ± 5.6 (stat) ± 6.2 (syst) GeV/c².

We thank the staffs at Fermilab and the collaborating institutions for their contributions to this work, and the Department of Energy and National Science Foundation (U.S.A.), Commissariat à L’Energie Atomique (France), State Committee for Science and Technology and Ministry for Atomic Energy (Russia), CNPq (Brazil), Departments of Atomic Energy and Science and Education (India), Colciencias (Colombia), CONACyT (Mexico), Ministry of Education and KOSEF (Korea), CONICET and UBACyT (Argentina), and the A.P. Sloan Foundation for support.
REFERENCES

∗ Visitor from IHEP, Beijing, China.
† Visitor from Universidad San Francisco de Quito, Quito, Ecuador.

[10] We have varied our analysis procedures (e.g. the binning of $D$) in ways which have little systematic effect on MC results. From data we observe little change in $m_t$, together with variations in $\sigma_{m_t}$ which are of the same order as those of $\delta m_t$ in Table I. We interpret the variations in $\sigma_{m_t}$ as stochastic effects to which the MC studies in Table I are relevant.