NUMERICAL CALCULATIONS ON THE NEW APPROACH TO THE CASCADE THEORY—II

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ABSTRACT

Numerical results on the mean numbers of electrons produced in small thicknesses in a shower initiated by a single electron are presented on the basis of the new approach to the cascade theory.

IN a recent contribution to these *Proceedings*, Ramakrishnan and one of us (S. K. S.) (1956) have suggested a new approach to the cascade theory. In a subsequent contribution (1957, hereinafter referred to as Paper I), we have presented numerical results on the mean numbers of electrons produced for large thicknesses. We here deal with the mean numbers for small thicknesses.

The mean number of electrons produced between 0 and t with the energy of each electron being $> E_c$ at the point of its production can be written as

$$\epsilon\{N(y;t)\}$$

$$= \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} \frac{B_s C_s}{\mu_s - \lambda_s} \cdot \frac{e^{y(s-1)}}{s-1}\left[\left(\frac{1}{\lambda_s} - \frac{1}{\mu_s}\right) - \left(\frac{e^{-\lambda_s t}}{\lambda_s} - \frac{e^{-\mu_s t}}{\mu_s}\right)\right] ds \quad (1)$$

where $y = \log E_0/E_c$ and A_s, B_s, C_s, λ_s and μ_s are defined in Paper I. For small thicknesses, we cannot neglect the term containing $e^{-\mu_s t}/\mu_s$ as has been done in Paper I. But the term

$$\frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} e^{y(s-1)} \frac{B_s C_s}{\mu_s - \lambda_s} \frac{e^{-\mu_s t}}{\mu_s (s-1)} ds$$

does not have any saddle point and hence cannot be evaluated in the same manner as in Paper I. This can be done if we adopt the device of re-grouping the terms in (1) as

$$\epsilon\{N(y;t)\} = \epsilon\{N_\lambda(y;t)\} - \epsilon\{N_\mu(y;t)\} \quad (2)$$

where

$$\epsilon\{N_\lambda(y;t)\} = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} \frac{B_s C_s}{\mu_s - \lambda_s} \cdot \frac{e^{y(s-1)}}{\lambda_s(s-1)}\, ds$$

$$- \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} \frac{B_s C_s}{\mu_s - \lambda_s} \cdot \frac{e^{-\lambda_s t + y(s-1)}}{\lambda_s(s-1)}\, ds \qquad (3)$$

$$\epsilon\{N_\mu(y;t)\} = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} \frac{B_s C_s}{\mu_s - \lambda_s} \frac{e^{y(s-1)}}{s-1}\left(\frac{1-e^{-\mu_s t}}{\mu_s}\right) ds \qquad (4)$$

$\epsilon\{N_\lambda(y;t)\}$ can be evaluated in the same manner as has been done in Paper I.

To evaluate $\epsilon\{N_\mu(y;t)\}$ we adopt the following procedure. Since we are interested only in small thicknesses, the term $(1-e^{-\mu_s t})/\mu_s$ can be expanded and we need retain the first three or four terms of the expansion. Thus to a very good approximation we can write

$$\epsilon\{N_\mu(y;t)\} = \sum_{m=0}^{4} \frac{I_m\, t^{m+1}}{(m+1)!} \qquad (5)$$

where

$$I_m = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{\chi_m(s)}\, ds \qquad (6)$$

$$\chi_m(s) = \log\frac{B_s C_s}{\mu_s - \lambda_s} - \log(s-1) + m\log\mu_s + y(s-1) \qquad (7)$$

As s tends to $+1$, it is easy to see that $\chi_m(s)$ tends to $+\infty$ since C_s tends to $+\infty$ as s tends to $+1$. As s tends to $+\infty$, through real values it can be proved, as has been done in Paper I that $\chi_m(s)$ tends to $+\infty$ and that I_m has a saddle point s_0. We obtain the formula

$$I_m = \frac{e^{\chi_m(s_0)}}{\sqrt{2\chi_m''(s_0)}} \qquad (8)$$

where s_0 is the saddle point given by

$$\left(\frac{d\chi_m(s)}{ds}\right)_{s=s_0} = 0. \qquad (9)$$

The mean numbers corresponding to $t = 0 \cdot 5$ and $t = 0 \cdot 8$ have been calculated for various values of y. In this case, the saddle points fall between $s = 1$ and $s = 2$. *In this interval the basic functions cannot be obtained by sub-tabulation from any of the standard cascade tables due to the fact that* C_S, λ_S *and* μ_S *become infinite at* $s = 1$. Hence it is considered worthwhile to tabulate these functions from their definitions and these are given in Table I. Table II gives the mean number of electrons produced

TABLE I

S	A_s	B_s	C_s	λ_s	μ_s
1·1	·152041	1·400998	12·821039	3·786712	4·712431
1·12	·180151	1·375115	10·575366	3·348058	4·301886
1·14	·207592	1·350215	8·975642	3·002095	3·983364
1·16	·234398	1·326244	7·779412	2·719330	3·727407
1·18	·260595	1·303151	6·851997	2·482030	3·516303
1·2	·286213	1·280890	6·112601	2·278786	3·338677
1·22	·311253	1·259418	5·509812	2·101890	3·186821
1·24	·335756	1·238694	5·009375	1·945888	3·055322
1·26	·359742	1·218679	4·587573	1·806813	2·940233
1·28	·383234	1·199349	4·227471	1·681704	2·838617
1·3	·406253	1·180640	3·916663	1·568256	2·748187
1·32	·428816	1·162552	3·645837	1·464717	2·667211
1·34	·450943	1·145050	3·407887	1·369668	2·594289
1·36	·472649	1·128091	3·197285	1·281956	2·528284
1·38	·493951	1·111666	3·009666	1·200662	2·468291
1·4	·514864	1·095745	2·841548	1·125010	2·413552
1·42	·535401	1·080305	2·690106	1·054357	2·363437
1·44	·555577	1·065325	2·553038	·988160	2·317415
1·46	·575404	1·050785	2·428441	·925958	2·275040
1·48	·594894	1·036665	2·314730	·867356	2·235928
1·5	·614060	1·022949	2·210578	·812012	2·199750
1·52	·632911	1·009618	2·114858	·759632	2·166221

TABLE I (*contd.*)

S	A_s	B_s	C_s	λ_s	μ_s
1·54	·651458	·996656	2·026617	·709956	2·135092
1·56	·669712	·984050	1·945035	·662759	2·106149
1·58	·687681	·971783	1·869409	·617841	2·079200
1·6	·705375	·9598433	1·799129	·575026	2·054079
1·62	·722802	·948217	1·733663	·534155	2·030635
1·64	·739970	·936892	1·672550	·495088	2·008736
1·66	·756888	·925857	1·615383	·457698	1·988264
1·68	·773562	·915101	1·561801	·421874	1·969113
1·7	·790001	·904612	1·511491	·387511	1·951189
1·72	·806210	·894383	1·464169	·354517	1·934405
1·74	·822196	·884401	1·419587	·322809	1·918683
1·76	·837966	·874659	1·377520	·292309	1·903953
1·78	·853526	·865148	1·337768	·262948	1·890152
1·8	·868881	·855860	1·300152	·234663	1·877222
1·82	·884037	·846788	1·264509	·207393	1·865108
1·84	·899000	·837922	1·230692	·181084	1·853762
1·86	·913773	·829258	1·198570	·155689	1·843140
1·88	·928364	·820786	1·168023	·131159	1·833201
1·9	·942775	·812502	1·138941	·107455	1·823907
1·92	·957012	·804399	1·111223	·084534	1·815224
1·94	·971079	·796470	1·084780	·062360	1·807117
1·96	·984979	·788711	1·059528	·040901	1·799558
1·98	·998719	·781115	1·035392	·020124	1·792521
2	1·012300	·773678	1·012300	0	1·785978

for $t = 0{\cdot}5$ and $t = 0{\cdot}8$. For comparison we have also tabulated the mean number of electrons that exist at t given by standard cascade theory.

TABLE II

Mean numbers for small thicknesses

	y	$\epsilon\{N_\lambda(y;t)\}$	$\epsilon\{N_\mu(y;t)\}$	$\epsilon\{N(y;t)\}$ [$\epsilon\{n(y;t)\}$* is given in brackets]
$t=0\cdot5$	8	20·6	·532	20·1† (2·62)
	7	10·2	·503	9·70†(2·33)
	6	5·1	·445	4·66 (2·08)
	5	2·73	·459	2·27 (1·78)
$t=0\cdot8$	8	35·7	·670	35·0† (4·93)
	7	16·7	·617	16·1† (4·13)
	6	8·65	·430	8·22 (3·40)
	5	4·59	·556	4·03 (2·74)

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Note.—In a private communication, Dr. Fay at Gottingen has pointed out that our values for $\epsilon\{N(y;t)\}$ are too large in the case of $y = 7$ and 8 (indicated by † in Table II) as compared with his experimental data. We are inclined to agree with him and admit that the technique we have adopted of splitting the terms in equation (3) is not suitable for large values of y and small t. In (3), $\epsilon\{N_\lambda(y;t)\}$ is given by the difference of two terms and since each of these terms is large and their difference is small, we cannot assert that the computed differences are accurate. We admit that our method is unsuitable in such cases though it yields very good results for all other cases. The results for large y and small t are presented to invite an improvement to our technique.

REFERENCES

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* $\epsilon\{n(y;t)\}$ denotes the mean number of electrons that exist at t.

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