SOME REMARKS ON THE STRUCTURE OF ELEMENTARY PARTICLE INTERACTIONS

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1. INTRODUCTION

A FEW years ago, Gell-Mann (1956) suggested a minimal principle for electromagnetic interactions according to which the electromagnetic interactions between elementary particles can only be of the current-field type that occurs in quantum electrodynamics and cannot be purely of the magnetic moment type. In this paper we suggest two specific transformations, as additions to the now well-known mass reversal transformation, invariance under which will guarantee the minimal character of the electromagnetic interactions. Applications of these transformations to strong and weak interactions are considered. These considerations suggest a model of strong interactions and an isotopic spin scheme for leptons which are also discussed.

2. THE MASS GAUGE TRANSFORMATION

To understand the significance of this transformation, we notice that in quantum electrodynamics, the total mass of electrons minus the total mass of positrons is a constant of motion. This implies that the quantum electrodynamical Lagrangian is invariant under the following gauge transformation of the electron field:

$$\psi_e \rightarrow e^{-im_e}\psi_e; \quad \bar{\psi}_e \rightarrow \bar{\psi}_e e^{+im_e} \tag{1}$$

where m_e is the rest mass of the electron. If now we postulate that all the electromagnetic Lagrangians should be invariant under the transformations

$$\psi_A \rightarrow e^{-im_A}\psi_A; \quad \bar{\psi}_A \rightarrow \bar{\psi}_A e^{+im_A} \tag{2}$$

where ψ_A is the field and m_A the rest mass of any fermion, we obtain the minimal principle of Gell-Mann apart from the case where a particle is

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coupled to itself by a magnetic moment type of interaction. The conservation law implied here is that the total mass of fermions minus the total mass of anti-fermions should be a constant of motion in the electromagnetic interactions. In the Feynman picture, the principle may be stated as follows. As we follow a fermion line in a Feynman diagram there is nothing to prevent a coupling that will make a fermion become another fermion of different mass at a vertex, the only conservation law imposed at a vertex being on energy and momentum. However in quantum electrodynamics we notice that as we follow a fermion line, its mass is conserved. If we now insist that this be true in all electromagnetic interactions we obtain the result stated above.

It is obvious that this invariance holds also for the strong pion-baryon interactions in the charge independence approximation where the mass differences within a multiplet are neglected, except for the $\Sigma - \Lambda - \pi$ coupling. In the global symmetry model [Gell-Mann (1957)] where the $\Sigma^0 - \Lambda^0$ mass difference can also be neglected in the absence of K-meson interactions, this symmetry holds for all the π-baryon couplings when the K-meson interactions are switched off.

Turning now to the weak interactions, decays like $\mu^{\pm} \rightarrow e^{\pm} + \nu + \bar{\nu}$ show that the fermion mass conservation is violated in weak decays. To deduce the universal character of the weak interactions from the violation of the fermion mass conservation, we suggest the following argument. The π-baryon interactions, in contrast to the electromagnetic interactions, allow a change of the electric charge of the fermions. These couplings also exhibit a higher degree of symmetry in that the couplings are the same for the different charge states of the same field. Pursuing this analogy, we may expect the weak interactions to involve a universal Fermi coupling (as is experimentally true) since these interactions allow the fermions to change their mass states. In this picture, as far as the weak interactions are concerned, all the fermion fields are different modes of a single fundamental fermion field.

As another application of the mass gauge transformation, we notice that the K-meson interactions connect fermions of different masses. Hence we may similarly expect the K-mesons to be symmetrically coupled to all the baryons. This is the 'cosmic symmetry' model of strong interactions arrived at by Sakurai (1959) from a different point of view. Also, apart from the $\Sigma - \Lambda - \pi$ coupling, the π-baryon couplings connect the same mass states of the fermions, neglecting of course the mass differences within a multiplet. This suggests that perhaps the $\Sigma - \Lambda - \pi$ coupling does not exist.

This has also been suggested by Feynman (1958). This will then ensure parity conservation for all the non-derivative pion interactions from CP invariance and charge independence in the conventional sense [Feinberg and Gursey (1959)]. The K-baryon interactions will also conserve parity because of their 'cosmic symmetry' [Sakurai (1959)].

3. The Mass Reversal Transformation†

Recently when studying the structure of Fermi interactions, Feynman and Gell-Mann (1958) showed that the Dirac equation can be written as a second order equation

$$\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + m\right)\left(\gamma_\nu \frac{\partial}{\partial x_\nu} - m\right)\psi = 0. \tag{3}$$

If ψ is a solution of this new equation, $\psi' = \gamma_5\psi$ is also a solution. So is a linear combination of ψ and $\gamma_5\psi$. This is not true of the first order Dirac equation since γ_5 does not commute with the Dirac Hamiltonian. $\gamma_5\psi$ corresponds to the solution of the equation obtained by replacing m by $-m$ in the Dirac equation and this is equivalent to stating that ψ and $\gamma_5\psi$ are solutions of the Dirac equation of the second order. In considering the interactions of particles, Feynman and Gell-Mann suggested that these solutions of the second order equation should be treated as fundamental, of course imposing conditions depending upon the nature of the interactions. In the case of universal Fermi interactions, the linear combination of solutions used is $\frac{1}{2}(1 \pm \gamma_5)\psi$ which are the eigen states of the chiral operator γ_5 [Sudarshan and Marshak (1958)].

Since ψ and $\gamma_5\psi$ have opposite parities, we may insist that all the interaction. Lagrangians be invariant under the simultaneous reversal of the parities of all the fermion fields by the transformation

$$\psi \to \psi' = \eta\gamma_5\psi \tag{4}$$

with $|\eta|^2 = 1$, since only the relative parities of fermions are physically significant [Wick, Wightman and Wigner (1950)]. This leaves the current-field type electromagnetic Lagrangian $ie\bar{\psi}\gamma_\mu\psi A_\mu$ invariant provided that $A_\mu \to A_\mu' = A_\mu$. If we accept this transformation for A_μ, then the magnetic moment type couplings are not invariant under these transformations and are consequently forbidden.

† This transformation has been discussed in various contents by Sakurai (1958), Okun (1958) and Salam (1959). We merely collect them here as a single connected series of results.

If we now require Yukawa couplings like $ig\bar{\psi}\gamma_5\vec{\tau}\psi\vec{\phi}$ to be invariant under this transformation, we find that $\vec{\phi}$ must transform as

$$\vec{\phi} \rightarrow \vec{\phi}' = -\vec{\phi}. \tag{5}$$

With this transformation for $\vec{\phi}$, it follows that there can be no derivative Yukawa couplings. Invariance under this transformation also forbids couplings between an odd number of spin zero bosons like $\lambda\phi^3$. [*see* also Baym (1960)]. These results are true whether ϕ is scalar or pseudoscalar.

4. THE τ_3 TRANSFORMATION

The minimal principle for the electromagnetic interactions of baryons may be formulated by specifying the form of the corresponding interaction Lagrangians in the isotopic spin space. If ψ_N is the nucleon field, its electromagnetic interaction can be written as $ie/2\,\bar{\psi}_N\gamma_\mu(1+\tau_3)\,\psi_N A_\mu$. Such an interaction is obviously not invariant under rotations about the first or second axis in the isotopic spin space and consequently it is not invariant under a reflection in the plane formed by the first and third axes of the isotopic spin space. Therefore if we define a parity for the fields for such a reflection, this parity is not conserved in electromagnetic interactions. This is analogous to what happens in weak interactions where space parity is not conserved. In the case of weak interactions, the universal $V \pm A$ interaction can be arrived at by arguing that the interaction should be unchanged even if the fields that occur in it are taken over to a state of opposite parity individually, as parity is not conserved. [Sakurai (1958), Sudarshan and Marshak (1958)]. This is achieved by the transformation $\psi \rightarrow \psi' = \pm\gamma_5\psi$. In a similar manner, we can expect the electromagnetic interactions to remain invariant under a transformation which switches the parity of a field under the reflection in the isospin space referred to above to the opposite parity. It is important that this transformation can be applied separately to each of the fields in the interaction. Such a transformation for isospinor fields would be

$$\psi \rightarrow \eta\tau_3\psi \tag{6}$$

with $|\eta|^2 = 1$. Then the electromagnetic coupling of fermions which are isospinors reduce to

$$\lambda\bar{\psi}_A O_i (1 \pm \tau_3)\,\psi_B f_i \tag{7}$$

where O_i and f_i are appropriate γ-matrices and functions of A_μ respectively. The two cases $(1 \pm \tau_3)$ belong to the cases where $\eta = \pm 1$. It is to be noted that couplings between fields of different η are forbidden so that both ψ_A and ψ_B should have the same η. In the d'Espagnat-Prentki scheme [d'Espagnat and Prentki (1956 and 1957)], fields with $\eta = +1$ correspond to isospinors of the first kind and those with $\eta = -1$ to isospinors of the second kind. When $\psi_A \equiv \psi_B$, the transformation has of course to be applied simultaneously to both $\bar{\psi}$ and ψ so that the form is not quite unique in that both $\lambda\bar{\psi}O_i\psi f_i$ and $\lambda\bar{\psi}O_i\tau_3\psi f_i$ are invariant under these transformations. However if we assume that (7) holds unchanged even in this case, we obtain the electromagnetic interaction of an isospinor with itself as $\lambda\bar{\psi}O_i(1 \pm \tau_3)\psi f_i$. For a spinor field which is an isopseudovector, the analogous transformation would be

$$\xi \to \xi' = \eta\theta_3\xi$$

where

$$\theta_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \qquad \xi = \begin{bmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{bmatrix} \tag{8}$$

with $|\eta|^2 = 1$. The coupling $\lambda\bar{\xi}O_i\theta_3\xi f_i$ is evidently invariant under (8). This form is also unique provided we assume the cylindrical symmetry of the electromagnetic interactions about the third axis in the isotopic spin space. This scheme is sufficient to ensure the minimal electromagnetic interaction of the baryons apart from the case where a particle is coupled to itself through a magnetic moment interaction. A coupling between Σ^+ and p or Ξ^- and Σ^- can be forbidden by choosing $\eta = \pm i$ for Σ even if we do not assume any rotational symmetry for electromagnetic interactions about the third axis in the isotopic spin space.

5. A Model of Strong Interactions

Our considerations regarding the mass gauge transformations have suggested a model of strong interactions according to which the K-meson interactions exhibit 'cosmic symmetry' in the sense of Sakurai [Sakurai (1959)] and the $\Sigma - \Lambda - \pi$ coupling is zero. The interaction Lagrangian in this model would be

$$L_{\text{int.}} = L_\pi + L_K$$

$$= iG_1\bar{N}_1\gamma_5\vec{\tau}N_1\vec{\pi} + iG_2\vec{\bar{\Sigma}} \times \vec{\Sigma}\gamma_5\vec{\pi} + iG_3\bar{N}_4\gamma_5\vec{\tau}N_4\vec{\pi}$$

$$+ F\,[(\bar{N}_1N_2)\,K^0 + (\bar{N}_1N_3)\,K^+ + (\bar{N}_4N_2)\,\bar{K}^+$$

$$- (\bar{N}_4N_3)\,\bar{K}^0 + h.c] \tag{9}$$

where, as usual

$$N_1 = \begin{pmatrix} p \\ n \end{pmatrix}, \quad N_2 = \begin{pmatrix} \Sigma^+ \\ Y^0 \end{pmatrix}$$

$$N_3 = \begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix}, \quad N_4 = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix} \tag{10}$$

with

$$Y_0 = \frac{\Lambda^0 - \Sigma^0}{\sqrt{2}}\,; \qquad Z^0 = \frac{\Lambda^0 + \Sigma^0}{\sqrt{2}}\,.$$

We also define

$$N_2{}' = \begin{pmatrix} \Sigma^+ \\ -Z^0 \end{pmatrix}; \quad N_3{}' = \begin{pmatrix} -Y^0 \\ \Sigma^- \end{pmatrix}, \tag{11}$$

which are obtained by replacing Λ^0 by $-\Lambda^0$ in N_2 and N_3.

In writing (9), we have assumed equal parity for all the baryons while the appropriate matrix 1 or $i\gamma_5$ has yet to be inserted in the K-baryon interactions depending upon the parity of the K-meson.

The Lagrangian (9) involves a totality of four different coupling constants and does not lead to any result independent of the perturbation theory in contradiction with experiments. In order to obtain a further reduction in the number of independent coupling constants, we may draw the following analogy between the K- and π-meson interactions and the π- and electromagnetic interactions. The π-meson interactions connect different charge states of a field and are known to be symmetric between the different charge states. The electromagnetic interactions break this symmetry. This breaking is not arbitrary, the couplings introduced for a triplet for instance being such that each member is coupled to itself with a coupling constant equal to eI_z. The electromagnetic interaction of an isotopic triplet thus transforms as the third component of a vector in the isotopic spin space. Since in our model, the K-meson interactions which connect different hypercharge (U) states of the baryons are supposed to be symmetric and the π-meson interactions are supposed to break this symmetry, it is natural introduce the π-meson interactions in analogy with the electromagnetic interactions by setting $G_1 = -G_3$; $G_2 = 0$ since N_1 has $U = 1$, N_4 has $U = -1$ and N_2 and N_3 have $U = 0$. This implies that the π-interactions are cylindrically symmetrical about the third axis of the three-dimensional hypercharge space introduced by Feinberg and Gursey [Feinberg and Gursey (1959)]. The interaction Lagrangian now reduces to

$$
\begin{aligned}
L_{\text{int.}} = iG\, [\bar{N}_1\gamma_5\vec{\tau}N_1 - \bar{N}_4\gamma_5\vec{\tau}N_4]\,\vec{\pi} \\
+ F\, [(\bar{N}_1N_2)\, K^0 + (\bar{N}_1N_3)\, K^+ + (\bar{N}_4N_2)\, \bar{K}^+ \\
- (\bar{N}_4N_3)\, \bar{K}^0 + h.c]. \qquad (12)
\end{aligned}
$$

However this interaction is too highly symmetrical and as Pais has shown [Pais (1958)] leads to a number of results in contradiction with experiment. We may break this symmetry by insisting only on the equality of the absolute values of the K-meson coupling constants and setting for instance

$$f_{\Lambda N_1 K} = f_{\Sigma N_1 K} = f_{N_4 \Lambda K} = -f_{N_4 \Sigma K} = F. \qquad (13)$$

We then obtain, using (11),

$$
\begin{aligned}
L_{\text{int.}} = iG\, [\bar{N}_1\vec{\tau}\gamma_5N_1 - \bar{N}_4\vec{\tau}\gamma_5N_4]\cdot\vec{\pi} \\
+ F\, [(\bar{N}_1N_2)\, K^0 + (\bar{N}_1N_3)\, K^+ + (\bar{N}_4N_2')\, \bar{K}^+ \\
- (\bar{N}_4N_3')\, \bar{K}^0 + h.c). \qquad (14)
\end{aligned}
$$

Considerations analogous to that of Pais [Pais (1958)] show that this removes the $\Sigma^0 - \Lambda^0$ degeneracy. However the invariance of (14) under

$$N_1 \to N_4,\ \ N_4 \to N_1,\ \ \vec{\pi} \to -\vec{\pi},\ \ \Lambda^0 \to -\Lambda^0,\ \ K^+ \to -\bar{K}^0,\ \ K^0 \to \bar{K}^+ \qquad (15)$$

implies that the nucleon and the cascade are still degenerate. Invariance under (15) also implies certain equalities between the amplitudes involving the nucleon and the cascade which at present cannot be verified. It is interesting to note that the difficulty of introducing the mass difference between N_1 and N_4 here is rather analogous to the difficulty of introducing the $\Sigma^+ - \Sigma^-$ mass difference which as pointed out by Gell-Mann [Gell-Mann (1957)] is rather too large to be explained as due to purely electromagnetic interactions.

The work of Pais [Pais (1958)] has already shown that such a small number of independent coupling constants as two is insufficient to reproduce all the known facts. It is however felt desirable to point out the suggestive analogy between the π- and electromagnetic interactions on the one hand and the K- and π-interactions on the other which does provide us with a definite way of introducing the π- and K-interactions. One can of course remove the degeneracy of the nucleon and the cascade by choosing different values for G_1 and G_3 still setting $G_2 = 0$ or by choosing $f_{\Lambda N_1 K} = f_{\Sigma N_1 K} \neq f_{\Lambda N_4 K} = - f_{\Sigma N_4 K}$. Such procedures however are rather arbitrary.

This model of course does not imply that the π-Σ-Σ or the π-Σ-Λ interactions are identically null. We always have virtual processes, where for instance the Σ dissociates into a nucleon and a K, the nucleon interacts with the π, and the resultant nucleon reabsorbs the K to become a Σ or a Λ. These are thus two-step processes and we may perhaps consequently expect the π-Σ and π-Λ interaction cross-sections to be smaller than the π-nucleon cross-sections.

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6. SUMMARY

The existence of symmetries which can lead to minimal electromagnetic interactions have been investigated and three specific invariances to obtain this minimal character have been proposed. Applications of these principles to strong and weak interactions are seen to lead to certain suggestive results. A model of strong interactions arising out of these considerations is also discussed.

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APPENDIX

The analogy between the τ_3- and γ_5- transformations discussed in Section 4 leads to an isotopic spin scheme for leptons in which μ^- and e^- are one of the components of doublets whose other components are missing and ν is a singlet. This scheme is discussed below and it is found that it is able to explain many of the features of leptonic interactions in a qualitative way.

THE ISOTOPIC SPIN SCHEME FOR LEPTONS

In Section 4, we have discussed the fact that the electromagnetic interactions of baryons are invariant under the transformation $\psi \rightarrow \eta\tau_3\psi$ with $|\eta|^2 = 1$. This is analogous to the γ_5- transformation of the neutrino field and eliminates one of the components of the baryon field in its interaction with the electromagnetic field. We now extend the above transformation to μ and e fields and assume that they possess the following invariance similar to the γ_5-invariance of the neutrino field:

$$\begin{aligned} \mu &= - \tau_3 \mu \\ e &= - \tau_3 e \end{aligned} \tag{A1}$$

where the μ and e fields are taken to form isodoublets. It follows that μ and e are essentially one component isotopic spinors of the form

$$\mu = \begin{bmatrix} 0 \\ \mu^- \end{bmatrix}, \quad e = \begin{bmatrix} 0 \\ e^- \end{bmatrix}. \tag{A2}$$

We further assume that the neutrino is an isosinglet.

STRONG INTERACTIONS

It will now be shown that with the isotopic spin assignments given above, it is possible to forbid the strong interactions of leptons. We assume

(*a*) Strong interactions conserve the total isotopic spin. It is immediately clear that μ^- and e^- can have no strong interactions since no isotopic scalar can be formed involving μ or e because of their missing components.

If we further assume

(*b*) Strong interactions conserve parity, ν also can have no strong interactions since it always occurs in the combination $\frac{1}{2}(1 + \gamma_5)\,\nu$.

It follows that leptons can have no strong interactions.

In electromagnetic interactions, the combination $\bar{\mu}^{-}\mu^{-}$ or $\bar{e}^{-}e^{-}$ occurs which transforms like a scalar plus the third component of a vector which is the correct transformation of these interactions for isospinors in the isotopic spin space.

It is interesting to note that only the leptons seem to possess invariances under transformations which eliminate one of their components, either in Lorentz space (γ_5-invariance of the neutrino) or in isotopic spin space ($-\tau_3$-invariance of μ and e) while for baryons, such invariances seem to be a property only of some of their interactions and not of the fields themselves.

WEAK INTERACTIONS

Present evidence in weak interactions seems to rule out the existence of neutral lepton currents [Lee and Yang (1960)]. We therefore assume that the only leptonic currents relevant for weak interactions are $(\bar{\mu}\nu)$ and $(\bar{e}\nu)$. These transform as one of the components of an isospinor. We now assume that weak interactions involving leptons in which baryons are also involved obey the rule $|\Delta\vec{I}| = 1/2$ or $3/2$ while weak interactions involving only leptons obey $|\Delta\vec{I}| = 0$ or 1. Strangeness-changing baryonic currents satisfying $|\Delta S| = 1$ necessarily transform as isospinors corresponding to $I = 1/2$ or $3/2$. Consequently an interaction of the form $J_{\alpha}^{SNC+} J_{\alpha}^{l}$ with $|\Delta S| = 1$ will not obey the selection rule $|\Delta\vec{I}| = 1/2$ or $3/2$ and are consequently forbidden. Thus we get the result that strangeness-changing leptonic decays with $|\Delta S| = 1$ are forbidden. Experimentally, apart from the $K \rightarrow \mu + \nu$ decay, all such decays have low probabilities. The $K \rightarrow \mu + \nu$ decay is hard to explain. Thus apart from this decay mode, we are able to explain at least qualitatively the low rates of such processes.

In this scheme, if $|\Delta S| = 2$ currents are allowed, we may have decays like $\Xi^{-} \rightarrow n + e^{-} + \bar{\nu}$. The $K_1^0 - K_2^0$ mass difference however seems to suggest against the existence of such currents [Okun and Pontecorvo (1958)].