ON DIFFUSER EFFICIENCY IN COMPRESSIBLE FLOW

BY P. NILAKANTAN

Received June 22, 1945
(Communicated by Sir C V. Raman, Kt., F.R.S., N.L.)

INTRODUCTION

THE diffuser is a device whereby the kinetic energy of flow of a high velocity fluid jet is transformed into pressure energy. It has many practical applications in turbines, wind tunnels, pumps and various duct systems.

The conditions of fluid flow in the diffuser are determined by the energy equation and the equation of continuity of fluid dynamics. In the simple case of the flow of an incompressible fluid, or the case of a compressible fluid at very low Mach numbers, these equations are:

$$P_1U_1A_1 + \tfrac{1}{2}\rho U_1^3A_1 = P_2U_2A_2 + \tfrac{1}{2}\rho U_2^3A_2 \tag{1}$$

$$U_1A_1 = U_2A_2, \tag{2}$$

where P_1, U_1, A_1 and P_2, U_2, A_2 denote the pressure, velocity and area of cross-section at the entrance and exit sections of the diffuser, respectively, and ρ is the density of the fluid. The efficiency of the diffuser is defined as the fraction of the difference in the kinetic energy of the jet at the entrance and exit sections that is transformed into pressure energy; that is,

$$\eta = \frac{P_2U_2A_2 - P_1U_1A_1}{\tfrac{1}{2}\rho U_1^3A_1 - \tfrac{1}{2}\rho U_2^3A_2} = \frac{P_2 - P_1}{\tfrac{1}{2}PU_1^2 - \tfrac{1}{2}PU_2^2} = \frac{P_2 - P_1}{q_1 - q_2}. \tag{3}$$

Another definition of diffuser efficiency used in design expresses it as the ratio of the difference of pressure between the entrance and exit sections to the dynamic pressure at the entrance: that is,

$$\eta = \frac{P_2 - P_1}{P_0 - P_1} = \frac{P_2 - P_1}{q_1}, \tag{4}$$

where P_1 and P_2 are the pressures at the entrance and exit respectively and P_0 is the stagnation pressure. This quantity, however, does not give any indication of the energy losses involved. On the other hand, the efficiency as defined by (3) does not give the fraction of the kinetic energy at the entrance that is actually transformed into pressure. Hence both definitions are useful when designing a diffuser in order to obtain maximum pressure recovery with a minimum of energy loss. Formulæ (3) and (4) are directly applicable only under ideal flow conditions. And in view of the fact that under actual conditions the velocity of the jet is not constant across any section of the diffuser, the definition of efficiency according to equation (3) has to be suitably modified in order that it may be calculated from actual experimental data. We shall assume that the static pressure and the density are constant across any section, which is justifiable when the divergence angle of the diffuser is small. In this case, we get the following expression:[1]

$$\eta = \frac{P_2 - P_1}{\frac{1}{2}\rho\bar{U}_1^2\left\{\alpha - \beta\left(\frac{A_1}{A_2}\right)^2\right\}} \tag{5}$$

where,

$$\int_{A_1} \tfrac{1}{2}\rho U^3 dA = \tfrac{1}{2}\rho\bar{U}_1^3 A_1\alpha$$

$$\int_{A_2} \tfrac{1}{2}\rho U^3 dA = \tfrac{1}{2}\rho U_2^3 A_2\beta$$

A_1 and A_2 are the entrance and exit areas of cross-section as defined before and $\bar{U}_1$ and $\bar{U}_2$ are the average velocities respectively, defined by the relations,

$$\bar{U}_1 = \frac{\int_{A_1} U dA}{A_1} \tag{6}$$

$$\bar{U}_2 = \frac{\int_{A_2} U dA}{A_2}.$$

Hence the determination of the efficiency at low speeds involves the measurement of P_1, P_2, U_1 and U_2 and the calculation of $\bar{U}_1$, $\bar{U}_2$, α and β under conditions of steady flow.

At high speeds, when compressibility effects become prominent, the above definitions of the diffuser efficiency are no longer sufficient and we run into thermodynamic complications. The object of the present paper is to present a practical solution of the problem in the case of compressible flow that has been employed by the author in testing scale models of several diffusers for a high speed wind-tunnel.

Theoretical Considerations

Ackeret[2] has defined diffuser efficiency in compressible flow as

$$\eta = \frac{P_2 - P_1}{P_0 - P_1} \tag{7}$$

which is the same as formula (4) in the case of incompressible fluid flow. But η is not equal to $\frac{P_2 - P_1}{q_1}$ since, in compressible flow,

$$(P_0 - P_1) = q_1\left(1 + \frac{1}{4}M_1^2 + \frac{1}{40}M_1^4 \ldots\right)$$

We may therefore write $\eta = \dfrac{P_2 - P_1}{q_1 \left(1 + \frac{1}{4} M_1^2\right)}$, to a first approximation. The efficiency as defined above does not again give any indication of the energy losses. A satisfactory definition in the case of compressible fluid flow has been used by L. Crocco[3] in his important paper on high speed wind-tunnels, taking into consideration the non-isentropic character of the flow. According to the Bernoulli equation for the flow of a non-viscous compressible fluid, we have the following fundamental relation:

$$U dU = -\frac{dP}{\rho}. \tag{8}$$

L. Crocco assumes that the actual equation of flow, taking into account frictional dissipation of heat, is of the form,

$$\eta U dU = -\frac{dP}{\rho} \tag{9}$$

where η is the efficiency factor.

The problem of determining η reduces to that of integrating the above expression. If in the first place, η is assumed to be constant for any flow under specified conditions in the diffuser, then,

$$\eta \frac{(U_1^2 - U_2^2)}{2} = -\int_{P_1}^{P_2} \frac{dP}{\rho}. \tag{10}$$

Due to the frictional evolution of heat during the transformation, the changes taking place are no longer isentropic and in order to integrate $\frac{dP}{\rho}$ it will be necessary to know the actual law governing the variation of P with ρ. This law will be different from the usual adiabatic relation, $\frac{P}{\rho^{\gamma}} = \text{constant}$, although of the same form. It may be noticed that in equation (10), η is the fraction of the kinetic energy which is transformed into pressure energy and hence is the efficiency of the diffuser while $(1 - \eta)$ is the fraction of the energy which is rendered unavailable from the standpoint of pressure recovery.

The definition of the equation of flow expressed by (9), which is characteristic of the diffuser, defines also the adiabatic non-isentropic law of variation of pressure with density, if we regard η as a constant for the diffuser under any specified condition of velocity and pressure distribution. In this connection it must be clearly borne in mind that η is not a constant for the diffuser *under all conditions of flow*. In other words, if we specify any condition of fluid by the Mach number at the entrance of the diffuser, then η will be a function of this Mach number. Corresponding to each Mach number, we shall have a certain velocity distribution across the entrance and exit sections of the diffuser, a corresponding pressure distribution along the length of the diffuser, and a certain value of η (regarded as constant only for the flow from the entrance of the diffuser to its exit) which we shall now express in terms of the other quantities. This can be accomplished with the help of the energy equation,

$$d\left(\frac{U^2}{2}\right) + J di = 0, \tag{11}$$

where di is equal to $C_P\, dT$ and J is the mechanical equivalent of heat.

Hence,
$$J\, di = \frac{1}{\eta}\frac{dP}{\rho} = J\, C_P\, dT = \frac{\gamma}{(\gamma-1)} R dT, \tag{12}$$

where γ is the ratio of specific heats and R is the gas constant defined by the relation $P/\rho = RT$. Equation (12) may then be reduced to the form,

$$\frac{dP}{P} = \frac{\gamma\eta}{(\gamma-1)}\frac{dT}{T} \tag{13}$$

which gives an integration,

$$\log P = \frac{\gamma\eta}{(\gamma-1)} \log T + C \tag{14}$$

i.e., $\dfrac{P}{T^{\left(\frac{\gamma\eta}{\gamma-1}\right)}}$ is constant.

From the equation of state we also get the relation connecting pressure and density as follows:

$$\frac{P_1}{P_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\frac{\gamma\eta}{(\gamma\eta-\gamma+1)}} \tag{15}$$

The above relation enables us to integrate equation (10) and we get,

$$\tfrac{1}{2}(U_1^2 - U_2^2) = \frac{\gamma}{\gamma-1} RT_1 \left\{\left(\frac{P_2}{P_1}\right)^{\frac{(\gamma-1)}{\gamma\eta}} - 1\right\}. \tag{16}$$

The subscripts 1 and 2 refer to the entrance and the exit of the diffuser respectively.

The following special case of equation (16) is interesting. If U_2 approaches zero, P_2 becomes, in the limit, the stagnation pressure at the exit, and we have,

$$\tfrac{1}{2}U_1^2 = \frac{\gamma}{(\gamma-1)} - RT_1 \left\{\left(\frac{P_2}{P_1}\right)^{\frac{(\gamma-1)}{\gamma\eta}} - 1.\right. \tag{17}$$

Dropping the subscript 1 and designating the exit pressure as P_0, the equation becomes,

$$\frac{1}{2}\frac{U^2}{a^2}=\frac{1}{(\gamma-1)}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{(\gamma-1)}{\gamma\eta}}-1\right\} \tag{18}$$

where a is the local velocity of sound: $a=\sqrt{\frac{\gamma P}{\rho}}$.

Hence,

$$\frac{P_0}{\rho}=\left\{1+\frac{(\gamma-1)}{2}\frac{U^2}{a^2}\right\}^{\frac{\gamma\eta}{(\gamma-1)}} \tag{19}$$

The corresponding formula for the isentropic case given by Glauert[4] is,

$$\frac{P_0}{P}=\left\{1+\frac{(\gamma-1)}{2}\frac{U^2}{a^2}\right\}^{\frac{\gamma-1}{\gamma}} \tag{20}$$

Equation (16) is still not in a suitable form for making any computations from experimental data, since the velocity distribution is not uniform across the entrance and exit sections.

If $\bar{U}_1$ and $\bar{U}_2$ are the average velocities, then by the continuity condition,

$$A_1\bar{U}_1\rho_1=A_2\bar{U}_2\rho_2. \tag{21}$$

We shall make the simplifying assumption that P_1, P_2, ρ_1 and ρ_2 are constant across any section of the diffuser. Further, we shall define,

$$\int_{A_1} U^2 dA=\alpha\bar{U}_1^2A_1,$$

$$\int_{A_2} U^2 dA=\beta\bar{U}_2^2A_2,$$

Also, $$\bar{U}_2^2=\left(\frac{A_1}{A_2}\right)^2\left(\frac{\rho_1}{\rho_2}\right)^2\bar{U}_1^2 \text{ from (21).}$$

Then equation (16) becomes,

$$\frac{1}{2}\left\{\alpha\bar{U}_1^2-\beta\left(\frac{A_1}{A_2}\right)^2\left(\frac{\rho_1}{\rho_2}\right)^2\bar{U}_1^2\right\}=\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1}\left\{\left(\frac{P_2}{P_1}\right)^F-1\right\} \tag{22}$$

where $$F=\frac{\gamma-1}{\gamma\eta}.$$

If further, it is assumed that the velocity is constant across the entrance of the diffuser (which is generally the case) so that $\alpha=1$, we have,

$$1-\beta\left(\frac{A_1}{A_2}\right)^2\left(\frac{P_1}{P_2}\right)^{2(1-F)}=\frac{\gamma}{(\gamma-1)}\frac{P_1}{q_1}\left\{\left(\frac{P_2}{P_1}\right)^F-1\right\} \tag{23}$$

since $$\left(\frac{P_1}{P_2}\right)^{1-F}=\frac{\rho_2}{\rho_1}.$$

Hence

$$1+\frac{\gamma}{(\gamma-1)}\frac{P_1}{q_1}=\beta\left(\frac{A_1}{A_2}\right)^2\left(\frac{P_1}{P_2}\right)^{2(1-F)}+\frac{\gamma}{(\gamma-1)}\frac{P_1}{q_1}\left(\frac{P_1}{P_2}\right)^{-F}. \tag{24}$$

We have to solve the above equation for F.

Putting $\frac{\gamma}{(\gamma-1)}\frac{P_1}{q_1}=R$, and $\left(\frac{P_1}{P_2}\right)^{-F}=S$

we have,

$$1+R=\beta\left(\frac{A_1}{A_2}\right)^2\left(\frac{P_1}{P_2}\right)^2 S^2+RS, \tag{25}$$

which is a quadratic in S and gives as a solution

$$S=\frac{-R+\sqrt{R^2+4\beta(1+R)\left(\frac{A_1}{A_2}\right)^2\left(\frac{P_1}{P_2}\right)^2}}{2\beta\left(\frac{A_1}{A_2}\right)^2\left(\frac{P_1}{P_2}\right)^2}$$

taking only the positive value of S.

Then, $$\log S=-F\log\left(\frac{P_1}{P_2}\right)$$

i.e., $$F=\frac{\log S}{\log\left(\frac{P_2}{P_1}\right)} \tag{26}$$

$$\therefore \quad \eta=\frac{(\gamma-1)}{\gamma}\frac{\log\left(\frac{P_2}{P_1}\right)}{\log S}. \tag{27}$$

Outline of Experimental and Computational Procedure

The quantities that have to be determined experimentally are P_1, P_2, q_1 and β. P_1, P_2 and q_1 may be easily determined by means of a suitable Pitot-static tube, whereas the determination of β will involve the evaluation of $\int_{A_2} U^2 dA/\bar{U}_2^2 A_2$, as defined earlier. This involves a velocity survey of the exit section for the integration of $U^2 dA$. The value of U at various points on the exit section may be determined by pitot traverses across the section, reasonably sufficient to cover the whole area and to take into account all variations of velocity. $\bar{U}_2$ is given by the relation $\frac{\int_{A_2} U dA}{A_2}$ and may also be evaluated from the velocity survey.

The steps in the calculation of η from the experimental data obtained are as follows:

(1) Calculate $R=\frac{\gamma}{\gamma-1}\frac{P_1}{q_1}$

(2) Evaluate $\bar{U}_2 = \dfrac{\int_{A_2} U dA}{A_2}$

and $\beta = \dfrac{\int_{A_2} U^2 dA}{\bar{U}_2{}^2 A_2}$ from the velocity survey at the exit section.

(3) Calculate $S = \dfrac{-R + \sqrt{R^2 + 4\beta(1+R)\left(\dfrac{A_1}{A_2}\right)^2\left(\dfrac{P_1}{P_2}\right)^2}}{2\beta\left(\dfrac{A_1}{A_2}\right)\left(\dfrac{P_1}{P_2}\right)^2}\lambda$, and

finally find $\qquad \eta = \dfrac{(\gamma-1)}{\gamma}\dfrac{\log\left(\dfrac{P_2}{P_1}\right)}{\log S}.$

Discussion

No attempt will be made here to discuss the results of earlier experimental work on diffuser efficiency at comparatively low speeds. Regarding these, reference may be made to the article by G. N. Patterson, on modern diffuser design[1] and also to the article on turbulent flow by H. Bateman[5] in which he discusses flow in a conical channel. Published data on the efficiency of high speed diffusers are unfortunately meagre since the subject has come into prominence only recently. The formulæ for efficiency presented in this paper are expected to be useful in future experimental work.

It may be remarked that theoretical formulæ for estimating losses in any new design of a diffuser are of little practical value. Empirical formulæ based on previous experimental work may be of limited use. It has been established that at low speeds a conical diffuser of about 7 degrees divergence angle is about the most satisfactory one. High speed jets, however, exhibit a tendency to separation for divergence angles even as low as 5 degrees. The character of the divergence of the cross-section from the entrance to the exit also appears to be a critical factor. Straight walls are more satisfactory than curved walls and, contrary to conventional ideas based on experiments at low speeds, greater efficiency is achieved by designing for a rapid expansion at the entrance followed by a slower expansion rate near the exit than *vice versa*. In any case the design of a high speed diffuser to meet specified conditions remains at present, a problem that can be solved satisfactorily only after considerable experimentation. Accumulation of a vast amount of data will no doubt provide theoretical bases for future design.

Summary

An expression for diffuser efficiency in compressible flow has been developed. Experimental procedures for the determination of the efficiency are outlined and the steps for calculating it from the experimental data are indicated.

References

1. Parkinson .. "Modern Diffuser Design," *Aircraft Engineering*, 1938, **10**, 267.
2. Ackeret .. "High Speed Wind Tunnels," *N. A. C. A. Tech. Memo.*, 1936, No. 808.
3. Croco .. *Atti dell' Associazione Italiana di Aerotecnica*, 1939-40, **19**.
4. Glauert .. *Aerofoil and Airscrew Theory*, Cambridge University Press, 1926.
5. Bateman .. *Bulletin of the National Research Council*, Washington, D.C., 1931, No. 84, 469.