AN ELECTRICAL AIRPLANE C.G. POSITION INDICATOR*

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Introduction

Although several types of Airplane C.G. Determinators are known which work on the mechanical principle of the lever just as the common balance, their manipulation is rather cumbersome and the devices themselves are not very handy. In the present paper an electrical circuit is described which is capable of indicating the centre of gravity position of an airplane for any arbitrary manner of loading after a few adjustments requiring very little skill. The design of the circuit for the case of a typical airplane is explained with the help of a numerical example. This should serve the purpose of further clarifying the theoretical considerations.

Basic Principle

The position of the centre of gravity of an airplane in the horizontal plane and at the normal attitude corresponding to the level flight condition at cruising speed, is determined by the relation,

$$x = \frac{\sum_{i=0}^{i=n} w_i \, x_i}{\sum_{i=0}^{i=n} w_i} = \frac{\sum_{i=0}^{i=n} m_i}{\sum_{i=0}^{i=n} w_i}, \qquad (1)$$

where w_i is the weight of load item i, and x_i is the corresponding moment arm measured from any convenient centre of gravity datum point, the total number of items being n. A simple electrical analogy of the division indicated in relation (1) may be envisaged by considering a voltage that is proportional to the algebraic sum of the moments, applied at the ends of a resistance that is proportional to the sum of the weights: the resulting current in the resistance will be proportional to the distance of the centre of gravity of the airplane from the datum point. This is the basic principle of the centre of gravity indicator.

Theory of the Electrical Circuit

The arrangement of an electrical circuit in order that the conditions mentioned above may be realized in practice is shown in Fig. 1.

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* Patents applied for.

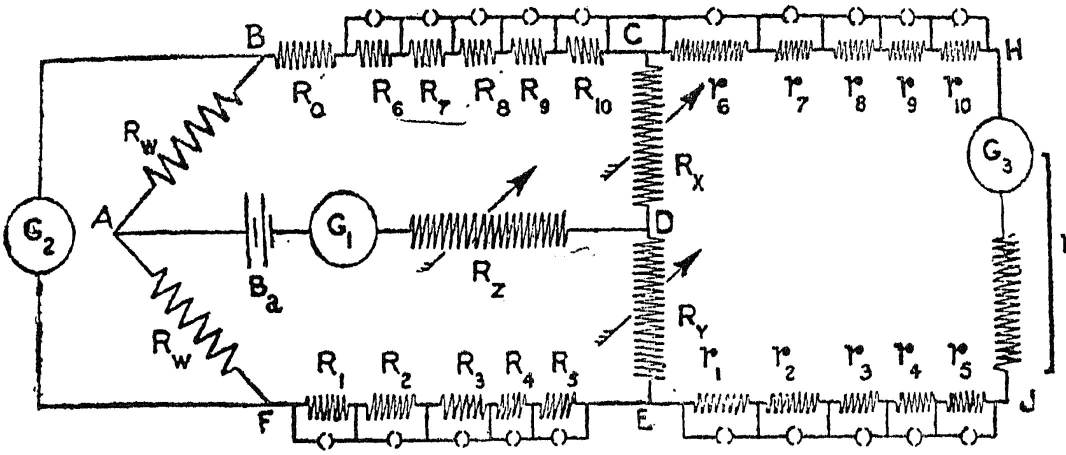

FIG. 1

The resistances, R_0, R_1, R_2,.... R_{10} shown in the figure, are made proportional to the moments m_0, m_1, m_2,.... m_{10} of the airplane weight empty and the altogether 10 items of the disposable load respectively (n is arbitrarily assumed to be 10 in this case). Similarly, the resistances r_0, r_1, ...r_{10} are made proportional to the corresponding weights w_0, w_1,.... w_{10} respectively. All the positive moments (clockwise) are included in the arm ABC while all the negative moments (anti-clockwise) are in the arm AFE of the electrical circuit. The R_w's are two equal resistances. G_1 and G_3 are current meters while G_2 is a sensitive galvanometer. G_2 has its zero reading in the middle of the scale.

The rheostats R_x and R_y may be so adjusted that G_2 reads zero: then the currents in the arms ABC and AFE are equal. The rheostat R_z could then be adjusted such that the current registered by G_1 has a specific value say c_1. c_1 is an instrument constant and its significance will become evident later. The currents in the arms ABC and AFE of the circuit will have the same value, say c_2, which again will be equal to $\frac{1}{2}c_1$. This is due to the fact that the two resistances R_w are equal and G_2 reads zero. The currents in the arms CD, DE. and CHJE will be equal to $c_2 + c_3$, $c_2 - c_3$, and c_3 respectively. Applying the well-known law of electrical networks, the following relations are obtained:—

$$c_2R_A - c_2R_B - c_3r = 0 \tag{2}$$

$$(c_2 + c_3)\,R_X - (c_2 - c_3)\,R_Y + c_3r = 0 \tag{3}$$

$$c_2\,(R_w + R_A) + (c_2 + c_3)\,R_X = c_2\,(R_w + R_B) + (c_2 - c_3)\,R_Y$$
$$= E_{Ba} - (R_{G1} + R_Z)\,c_1 = E \text{ say} \tag{4}$$

where,

$$R_A = R_0 + R_6 + R_7 + R_8 + R_9 + R_{10},$$
$$R_B = R_1 + R_2 + R_3 + R_4 + R_5,$$
$$r\;\; = r_0 + r_1 + r_2 \ldots\ldots + r_{10};$$

R_{G_1} is the resistance of G_1 and

r_0 includes the resistance of G_3.

From equations (2), (3) and (4), the following relations are easily obtained, namely,

$$c_3 = \frac{(R_A - R_B)\, c_2}{r} \tag{5}$$

$$R_X = \left\{\frac{E}{c_2} - (R_w + R_A)\right\} \frac{r}{r + (R_A - R_B)} \tag{6}$$

$$R_Y = \left\{\frac{E}{c_2} - (R_w + R_B)\right\} \frac{r}{r - (R_A - R_B)} \tag{7}$$

Although the value of c_1 and therefore of c_2 also may be arbitrarily decided upon initially in order to suit the most advantageous design of the instrument, the absolute values of R_X and R_Y are not uniquely determined; their values will depend upon the values of E and R_w. The relative values of R_X and R_Y are, however, fixed by the values of R_A, R_B and r, according to the relation,

$$\frac{R_A - R_B}{r} = \frac{R_Y - R_X}{R_X + R_Y + r} \tag{8}$$

The operations performed may now be considered in the light of the above equations. The essential object of the manipulations has been to make c_2 equal to $\frac{1}{2}c_1$. The relation (5) then directly gives the answer to the problem. For, $(R_A - R_B)$ corresponds to the algebraic sum of the moments and r corresponds to the total weight of the airplane, c_2 being an instrument constant. Hence, c_3 is always proportional to $\frac{(R_A - R_B)}{r}$.

Assuming the resistances to be so chosen that 1 ohm of the resistances R_i is equivalent to a inch-pounds and that 1 ohm of the r_i's is equivalent to b pounds, we have then from equation (5),

$$\frac{c_3}{c_2} \cdot \frac{a}{b} = \frac{(R_A - R_B)\, a}{r \cdot b} \text{ inches.} \tag{9}$$

In other words, if c_3, read in milli-amperes, is multiplied by the factor $\frac{a}{b} \cdot \frac{1}{c_2}$ in which c_2 is also in milli-amperes, we get directly the C.G. position in inches from the datum point. For practical purposes G_3 can be calibrated in inches of C.G. position fore and aft. Since c_1 is an instrument constant, its value read on G_1 may also be indicated by an index mark on the dial of the instru-
t.

SOME PRACTICAL CONSIDERATIONS

It is not the purpose here to go into minute details of design of the circuit for any airplane. But attention may be drawn to some points of practical interest by considering the case of a typical airplane of 6,000 lbs. gross weight. The weight and moment data of the airplane are given in Table I below.

TABLE I

Item	Weight W_i (lbs.)	Moment arm X_i (inches)	Moment m_i (inch—lbs.)
Airplane weight empty ..	4000	0	0
Pilot ..	170	−35	−5950
1st Passenger ..	170	− 5	− 850
2nd ,, ..	170	− 5	− 850
3rd ,, ..	170	25	4250
4th ,, ..	170	25	4250
5th ,, ..	170	55	9350
6th ,, ..	170	55	9350
Fuel (full) ..	450	10	4500
Cargo (full) ..	360	90	32400
Total ..	6000		56450

The datum point in the horizontal plane for the measurement of moment arms has been chosen for convenience as that corresponding to the centre of gravity position of the airplane weight empty. In fact, it may be preferable to choose the datum point given by the manufacturer himself and the centre of gravity limits specified in the *Airplane Manual* with reference to this datum point may be indicated on the dial of G_3. This however, is only a design detail.

TABLE II

Item No. i	Item Description	Resistances r_i (ohms)	Resistances R_i (ohms)
0	Airplane weight empty ..	400	0
1	Pilot ..	17	5·95
2	1st Passenger ..	17	·85
3	2nd ,, ..	17	·85
4	3rd ,, ..	17	4·25
5	4th ,, ..	17	4·25
6	5th ,, ..	17	9·35
7	6th ,, ..	17	9·35
8	Fuel ..	*0–45	*0–4 ·5
9	Cargo ..	*0–36	*0–32·4

* Adjustable in fractions ¼, ½ and ¾ of full value.

The electrical circuit for the airplane under consideration may be designed such that 1 ohm of the R_i's is equivalent to 1000 inch-pounds and 1 ohm of the r_i's to 10 pounds weight. The actual values of r_i and R_i for the various items are then as given in Table II.

The value of c_1 is fixed at 20 milli-amperes. Hence c_2 will be 10 milli-amperes. The multiplying factor is 10 when c_3 is read in milli-amperes, in order to get the C.G. position in inches. For the battery a two-volt accumulator cell may be used.

Considering the case of the fully loaded airplane, we have

$$R_A = 64 \cdot 1\ \Omega$$

$$R_B = 7 \cdot 65\ \Omega, \text{ and}$$

$$r = 600\ \Omega.$$

Hence $c_3 = \cdot 941\ m.a.$ This evidently corresponds to a C.G. position of 9·41 inches aft of the datum point. If the R_w's are each equal to 10 Ω, then it may be easily shown that, corresponding to a value of $(R_{G_1} + R_Z) = 50\ \Omega$,

$$R_X = 23 \cdot 7\ \Omega, \text{ and}$$

$$R_Y = 90 \cdot 8\ \Omega,$$

from equations (4), (6) and (7).

It may also be shown that if R_X is initially kept at some arbitrary value, say 20 Ω, the appropriate values of R_Y and $(R_{G_1} + R_Z)$ are 86·48 Ω and 52 Ω respectively.

The above considerations show that the manipulation of the instrument is a very simple matter and often only R_Y and R_Z need be adjusted.

By means of the short-circuiting plugs provided, any arbitrary manner of loading of the airplane may be reproduced on the circuit (see Fig. 1), and the C.G. position in inches read directly on the dial of G_3 after two or three adjustments.

Summary

The theory of the electrical circuit of an Airplane C.G. Position Indicator has been developed. The practical application of the circuit to the case of a typical airplane has been demonstrated with the help of a numerical example.