Structure of nuclei in the region $A = 70$

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Abstract. The structure of the selenium nuclei in the region $A = 70$ is studied using our deformed configuration mixing (DCM) shell model based on Hartree–Fock states. An effective interaction given by Kuo and modified by Bhatt is used. An attempt is made to understand the coexistence of shapes in selenium nuclei.

Keywords. Coexistence of shapes; large deformation; configuration mixing shell model; band crossing.

PACS Nos 21·10; 21·60; 23·20; 27·50

1. Introduction

In recent years, new and fascinating technological advances have made it possible for nuclear physicists to produce and study nuclides far from the valley of stability. This has led to exciting discoveries of large deformation, coexistence of shapes, rapid variation of structure with changes in neutron and proton number etc in the region $A = 80$. It has been shown in particular that as the neutron or proton number approaches 38–40, nuclei show strong deformations and rapid variation of structure. These features pose interesting theoretical questions about the basic shell structure and nucleon configurations in these nuclei. In this development, studies of coexistence of different shapes in selenium nuclei would be quite interesting.

Theoretically this region has not been amenable to detailed microscopic calculations. Even with the assumption of an inert core of $^{56}$Ni, the number of active nucleons to be treated is quite large and the configuration space must include at least four active single particle orbits: $p_{3/2}, f_{5/2}, p_{1/2}$ and $g_{9/2}$. Only few such calculations have been carried out and that too only for simple systems and with drastic approximations. Most of the theoretical investigations in this region employ vibrational models, coupling of quasi-particles to vibrations, rotational models, the dynamic deformation model, interacting boson model, etc. It would be very useful to have a detailed microscopic model which can track the rapid changes of nuclear structure over the entire sequence of nuclei and give an insight into the role of nuclear interactions as well as the dynamics of the single particle motions. In §2, a brief outline of the model is given. Section 3 discusses the coexistence of shapes in even selenium model. Some of our recent results of odd-even proton-rich isotopes ($A = 69, 71$) are given in §4 and finally our conclusion in §5.

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2. Theory

For several years, we have been developing and applying a deformed shell model approach based on Hartree–Fock states to nuclei in this region as well as in the $f_{7/2}$ region (Dhar et al. 1975) and also to heavier nuclei, e.g. Hg isotopes (Praharaaj and Khadkikar 1983). It has also been found to be quite successful in describing many properties of nuclei in this transitional region of $pf - g_{9/2}$ orbits (Sahu and Pandya 1984). A brief outline of the model is given below. The calculation involves the following three steps: (i) generation of the lowest HF intrinsic state by solving the HF equation self-consistently and other low-lying particle-hole excited intrinsic states by performing a tagged HF calculation (ii) the evaluation of the energies of the states of definite angular momentum projected from each of the intrinsic states generated above (iii) orthonormalization of the states projected from different intrinsic states and the diagonalization of the Hamiltonian in the basis of these orthonormalized states. The details are given by Dhar et al. (1975) and Ripka (1968).

In our microscopic model, we take $^{56}$Ni to be the closed inert core with $p_{3/2}$, $f_{5/2}$, $p_{1/2}$ and $g_{9/2}$ as the active single particle orbits. The single particle energies for the first three orbits are taken as 0.0 MeV, 0.78 MeV and 1.08 MeV from $^{57}$Ni data. The $g_{9/2}$-orbit is placed at 4.75 MeV. An effective interaction, generated for this space by Kuo (private communication) and partially modified by Bhatt and coworkers (Ahalpara et al. 1985), is used.

3. Shape coexistence in Se isotopes

Hamilton et al. (1974) were the first to observe that something unusual was happening in $^{72}$Se and that the properties of the excited states $2^+_1$, $2^+_2$ and $0^+_2$ did not fit in with the then accepted model of a spherical vibrational nucleus. They were able to identify a band of states up to quite high values of angular momentum and showed that while the lowest $0^+$, $2^+$ states appear to conform to a vibrating spherical nucleus, the states with $J = 4^+$ and above belong to a band based on a strongly deformed intrinsic state with $K = 0^+$. Subsequent studies of $^{74}$Se and $^{70}$Se by Hamilton and his group (Piercey et al. 1979; Ahmed et al. 1981) also revealed similar features.

We first carry out a HF calculation to determine the nature of the intrinsic states for the $^{70}$Se, $^{72}$Se and $^{74}$Se nuclei. Apart from the lowest energy intrinsic states, we consider a few additional $K = 0^+$ intrinsic states (with energies close to the ground state) in which two or four particles are placed in the next excited single-particle orbit and a tagged HF calculation is carried out. States of good angular momenta are then projected out from each of these intrinsic states and mixed through the Hamiltonian in a standard band-mixing calculation (Dhar et al. 1975). The electromagnetic properties are then calculated using the effective charges $e_p = 1.6 e$ and $e_n = 1.0 e$. This is the conventional choice for the effective charges in this region and approximately accounts for the limitation of the configuration space chosen.

3.1 Intrinsic states

We first carry out a standard HF calculation which give the lowest energy intrinsic states for the $^{70}$Se, $^{72}$Se, $^{74}$Se isotopes. Figure 1 shows the single-particle spectra
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\[ \begin{array}{c|c|c|c}
\text{Energy (MeV)} & 70\text{Se} & 72\text{Se} & 74\text{Se} \\
\hline
E_{HF} & -30.23 & -32.34 & -32.34 \\
Q_{HF} & 18.28 & 12.38 & 23.11 \\
\beta & 0.18 & 0.12 & 0.22 \\
\end{array} \]

Figure 1. The spectra of HF single particle orbits for the lowest energy prolate intrinsic states for $^{70}\text{Se}$, $^{72}\text{Se}$ and $^{74}\text{Se}$. The protons are represented by circles and the neutrons by crosses. The numbers next to the levels denote $2k$ values.

corresponding to these intrinsic states. For all the isotopes, the HF calculations give an oblate state as the energetically lowest intrinsic state, a few MeV lower than the lowest prolate state. Although it is not easy to deduce precisely the shape of the nucleus in its ground state, the general systematics and the quadrupole moment of the $2^+$ states indicate that all nuclei in this region are prolate at least in the low energy states. We have found it a quite general feature of our calculations in this region that oblate intrinsic states occur that are nearly degenerate or are even lower in energy than the lowest prolate states (Ahalpara et al 1985b). Obviously, in this region, something is wrong with the oscillator potential and presumably the prolate-spherical-oblate degeneracy is extremely sensitive to some aspects of the single particle level scheme. This feature has been discussed in detail by Galeri et al (1986). In our band-mixing calculations, oblate states do not mix in any significant way with the prolate states and thus do not affect the results for spectroscopy or electromagnetic transitions. We, therefore, consider only the prolate or spherical intrinsic states.

For $^{70}\text{Se}$ as well as $^{72}\text{Se}$, the ground states have small deformations, with nucleons being distributed only in $p - f$ orbits. However, it is possible in both cases to obtain an excited intrinsic state with a much larger deformation by exciting two neutrons from the Fermi state to the $k = 1/2^+$ state. A band-mixing calculation involving states of good $J$ projected from these two intrinsic states correctly reproduces the observed coexistence features of the spectra in both $^{70}\text{Se}$ and $^{72}\text{Se}$ as shown in figure 2a and 2b.

For the case of $^{74}\text{Se}$, we find a spherical state with 40 neutrons completely filling
the $p - f$ shell, another deformed intrinsic state with two neutrons in $k = 1/2^+$ orbit, and yet another even more deformed intrinsic state with four neutrons in $k = 1/2^+$ and $3/2^+$ orbits—all nearly degenerate in energy. On projection and band-mixing, however, we find the ground state band to remain yrast up to $J = 12^+$ (see figure 2c), the spherical $0^+$ state at an excitation of several MeV (experimentally not yet identified). Thus we see no band-crossing in $^{74}$Se.

These results as well as the electromagnetic properties etc. are discussed in detail earlier (Sahu et al 1987). Our purpose here is to emphasize that it is the intruder states $g_{9/2}$ and the rapid lowering in energy of the $k = 1/2^+$ and $3/2^+$ components of this orbit that enable easy access to neutrons in Fermi levels giving more deformed configurations. Thus, we see shape coexistence essentially of a weakly deformed prolate shape and a strongly deformed prolate shape. Thus, the separation in energy of the
\( \theta_{9/2} \) state from the \( p - f \) states is an important parameter determining the presence or absence of shape coexistence. The above model can be further improved in detail by taking into account additional configuration mixing, both for protons and neutrons.

4. Structure of \( ^{69}\text{Se} \) and \( ^{71}\text{Se} \)

During the last year, studies of excited levels in \( ^{69}\text{Se} \) and \( ^{71}\text{Se} \) have created new interest in the shapes of these nuclei. As mentioned earlier, all observations on nuclei in this region have always unequivocally shown prolate shapes. Detailed calculations of deformed single particle potentials using Strutinsky method (Bengtsson et al. 1984) or even simple Hartree-Fock calculations based on microscopic effective interactions (Ahalpara and Pandya 1987) predict extra stability for oblate shapes if \( N, Z = 36, 34 \). In view of this, one may expect \( ^{69}\text{Se} \) or \( ^{71}\text{Se} \) to have oblate shapes. An odd neutron can provide a sensitive probe of the quadrupole field of the even-even core and the sign of the quadrupole moment can be determined from the sign of the mixing ratio

\[
\delta = \frac{\langle J_{\pi}|E2|J_f \rangle}{\langle J_{\pi}|M1|J_f \rangle}
\]

for \( \Delta J = 1 \) transitions between favoured and unfavoured bands and the signature splitting of these bands.

We briefly summarize the results of Wiosna et al. (1988) and Eberth et al. (1984). In both the isotopes, one sees positive parity states as well as the negative parity states. For \( ^{71}\text{Se} \), a rotational band of states \( 3/2^-, 7/2^-, 11/2^-, 15/2^- \) and \( 19/2^- \) is identified, but at the same time, there are low-lying \( 1/2^- \) and \( 5/2^- \) states, as well as
an unidentified-spin-parity state between $3/2^-$ and $7/2^-$ which apparently belongs to this set. For $^{69}$Se only negative parity states, identified, are the nearly degenerate $3/2^-$ and $5/2^-$ states. In the positive parity states, both nuclei show a decoupled band based on $k=9/2^+$. The favoured branch $9/2^+, 13/2^+, 17/2^+$ etc is well identified, as well as the unfavoured branch beginning with $11/2^+$. But there are several additional positive parity states whose structure is not quite clear. The experimental data need a lot of cleaning up. The measurement of the mixing ratio for $11/2^+ \rightarrow 9/2^+$ in both the nuclei has been analysed to establish that the intrinsic quadrupole moment is negative, clearly implying an oblate shape. This measurement is so crucial that it would be worthwhile confirming it and making more measurements of other similar mixing ratios. For the first time, oblate shapes have then been seen in nuclei in this mass range and also a clear coexistence of prolate and oblate shapes.

Let us now consider the results of Hartree–Fock calculations similar to those for even isotopes of Se. We first discuss the negative parity states. The only states seen in $^{69}$Se are the ground state $J=3/2^-$ and a very close $J=5/2^-$ at 39 keV excitation. For $^{71}$Se, the ground state is $5/2^-$ with the $3/2^-$ excited state at 282 keV, plus a $\Delta J=2$ band beginning with $7/2^-$ at 1040 keV (see figure 3). We, therefore, discuss only the results for $^{71}$Se. The prolate HF level scheme of $^{70}$Se (figure 1) shows that the additional neutron would be in the $k=1/2^-$ orbit and one could have other excited configurations in which the additional neutron could be in the $k=3/2^-$ (with a pair in $k=1/2^-$) or in $k=5/2^-$ or even with a pair of neutrons in $k=1/2^+$ orbit with the odd neutron in $k=1/2^-$ or $k=3/2^-$ orbits. This last configuration would give a much more deformed state compared to others and would be at a comparable energy because of the deformation energy.

![Figure 3](image.png)

Figure 3. A comparison between experimental and calculated negative parity levels for $^{71}$Se. The quantities near the arrows represent $B(E2)$ values in Weisskopf unit.
We have projected good angular momentum states from these intrinsic states and carried out a proper band-mixing calculation. Some of these results are shown in figure 3 along with the observed negative parity states. It seems reasonable to identify the \( \Delta I = 2 \) band beginning with \( J = 3/2^- \) as the one arising from primarily the highly deformed intrinsic state \( k = 1/2^- \) with two neutrons in \( g_{9/2} \) state. However, the ground state \( 5/2^- \) arising from the less deformed \( k = 1/2^- \) intrinsic state has all nucleons in \( fp \) states. The comparison of calculated and observed \( B(E2) \) values lends credence to such an identification.

We have also considered the oblate HF solutions (see figure 4). In this case 36 neutrons form a fairly close stable core and the last odd neutron can only be in the \( k = 1/2^- \) orbit. We also show in figure 3 the band of states projected from the \( k = 1/2^- \) intrinsic state. Obviously the agreement with observed states is much worse. We prefer to conclude that the negative parity \( \Delta I = 2 \) band has a prolate shape and is generated from a highly deformed state with two neutrons in \( k = 1/2^+ \) orbit.

We next discuss the positive parity band based on the \( 9/2^+ \) state. It seems natural to consider this as based on an oblate intrinsic state with \( k = 9/2^+ \). Figure 4 shows that to generate such a state, one must break the stable \( N = 36 \) core and lift a neutron from the \( k = 9/2^+ \) orbit to \( k = 1/2^- \) or \( 3/2^- \) orbit. Alternatively, one can raise the odd neutron to the \( k = 7/2^+ \) orbit. We have again carried out a band-mixing calculation for states of good \( J \) projected from these intrinsic states and the results are shown in figure 5. While the ordering of the levels is correctly reproduced, we find that the calculated band of states is much more expanded as compared to the observed one. We also consider the possibility of projecting such a band from prolate intrinsic states, which obviously will have \( k = 1/2^+ \) or \( k = 3/2^+ \). The result of a calculation involving projection and band-mixing of good \( J \) states from these intrinsic

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**Figure 4.** The spectra of HF single particle orbits for the lowest energy oblate intrinsic states for \(^{71}\)Se.
states is also shown in figure 5. One sees in this case also that the \( J = 9/2^+ \) state is lowest in energy. One can identify the favoured and unfavoured branches of this band beginning with \( J = 9/2^+ \) as the bandhead. However, the favoured branch consisting of \( J = 9/2^+ \), \( 13/2^+ \), \( 17/2^+ \) etc is somewhat compressed compared to the observed spacings. However, the unfavoured branch appears to agree reasonably well with the observations. Thus, the calculations do show that it is not absolutely necessary to jump to the conclusion that the \( 9/2^+ \) band must be oblate, just because \( J = 9/2^+ \) level is the lowest in energy. As far as the energy level scheme alone is concerned, the prolate shape cannot be ruled out, at least for \(^{71}\text{Se}\).

For the case of positive parity states of \(^{69}\text{Se}\), we have carried out similar calculations with similar results. Here the calculated oblate band is in reasonable agreement with observations at least for the first few states, but later diverges considerably. On the other hand, for the prolate band, the calculated favoured band is much more compressed than the observed band, and also is well separated from the unfavoured branch, so that the calculated \( J = 11/2^+ \) lies above the \( J = 13/2^+ \) etc. We note that in this case, the measurement of the \( M1/E2 \) branching ratio has also indicated an oblate shape for the nucleus. We propose to do more detailed calculations for both \(^{69}\text{Se}\) and \(^{71}\text{Se}\) in the near future.

5. Conclusion

We have attempted to understand the coexistence of shapes in selenium nuclei from a purely microscopic approach. HF calculations with suitable extensions provide a good insight into the shapes and deformations of nuclei and their characteristic energies. An analysis of the single particle HF spectra shows very clearly the role played by neutron excitation into the \( g_{9/2} \) orbit in producing states of large deformation. The rapid change of nuclear properties with neutron number in Se isotopes is no longer a mystery. Our results show band-crossing for \(^{70}\text{Se}\) and \(^{72}\text{Se}\).
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Figure 6. A comparison between experimental and calculated positive parity levels for $^{69}\text{Se}$.

yrast spectra but not for $^{74}\text{Se}$. The ground band is nearly spherical in both cases, with a strongly deformed band becoming yrast at relatively low angular momentum states. Similar calculations for $^{69}\text{Se}$ and $^{71}\text{Se}$ indicate support for an oblate shape for the positive parity band in $^{69}\text{Se}$, but do not rule out the possibility of a prolate shape for a similar band in $^{71}\text{Se}$. The negative parity band in $^{71}\text{Se}$ definitely appears to be prolate.

Acknowledgement

This research was supported by the Department of Science and Technology, New Delhi.

References

Ahalpara D P, Abzouzi A and Bhatt K H 1985 *Nucl. Phys.* A445 1
Ahalpara D P, Bhatt K H and Sahu R 1985b *J. Phys.* G11 735
Ahalpara D P and Pandya S P 1987 *Proc. fifth Int. Conf. on nuclei far from stability* (AIP Conf. Proc. 164) 278
Eberth J et al 1984 *Int. Symp. on beam spectroscopy* Debrecen, Hungary, p. 23
Galeriu D, Bucurescu D and Ivascu M 1986 *J. Phys.* G12 329
Wiosna M et al 1988 *Phys. Lett.* B200 255