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Shape-excited states of ^{28}Si

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Abstract. The Hartree-Fock (HF) minima for the nucleus ^{28}Si were obtained for the prolate, oblate and spherical shapes using the interaction obtained by Preedom and Wildenthal. The interaction gives rise to large energy separation between the prolate and the oblate shapes. The spherical solution is just 2 MeV above the lowest HF (oblate) minimum. The spectrum projected from the oblate HF state is in good agreement with the experimental spectrum. The transition probabilities for the different energy levels also agree reasonably well. The configuration mixing calculations performed on the basis of states projected from the three shapes indicate that there is no significant mixing of different projected states. The second 0_2^+ state, thus obtained, corresponds to the third 0_3^+ state in the experimental spectrum and stems dominantly from the spherical HF state. It is seen that the structure of the energy levels of ^{28}Si, especially the second 0_2^+ level is very sensitive to the two body interaction. The results are compared with those obtained using the renormalised interaction of Kuo.

Keywords. Nuclear spectroscopy; shape excitations; ^{28}Si.

1. Introduction

The problem of understanding the observed structure of ^{28}Si has been a challenge to nuclear theorists. The reason for the interest in ^{28}Si is clear. The nucleus is in the centre of the d-s shell where transition from rotational motion to vibrational motion is expected to occur. Also it is one of the most complex (numerically) shell model calculations in this region. Experimentally, the ^{28}Si nucleus exhibits a fairly clear rotational band based on the 0^+ ground state. However, apart from the states of this band, the structure of the other excited states is far from clear. Till recently it was hardly possible to carry out even reasonably reliable shell model calculations for this nucleus. Hence most of the theoretical results were based on Hartree-Fock calculations of varying degrees of sophistication (Castle and Parikh 1970, Das Gupta and Harvey 1967). Some shell model calculations in truncated configuration spaces consisting of a few lowest SU_3 representations were also attempted (Bernier and Harvey 1967). The results of all such calculations – limited only to the ground rotational band – were spectra that were much too compressed as compared to the experimental data. In addition, a prolate HF solution was generally obtained at energies very close to that of the oblate HF solution which has the lowest energy. This predicted a low-lying prolate 0^+ excited state which has no counterpart in the observed spectra. Such results cast a serious doubt on the validity of the HF methods for application to ^{28}Si and other nuclei also in this region.

On the other hand, in recent years some ambitious shell model calculations have been reported which give fairly good agreement between theory and experiment. The work of Soyeur and Zucker (1972) which provided the chief motivation of our

effort, shows clearly that the calculated structure of ^{28}Si is strongly interaction-dependent, and a suitable interaction [in their case the two-body reaction matrix elements obtained by Kahana, Lee and Scott (1969)] can give good agreement with experiment. Shell model calculations of Wildenthal and McGrory (1972) gave equally good fits with a surface-delta-interaction with appropriately chosen parameters. Recently a full d-s shell calculation is also reported by Whitehead and Watt (1972) using Kuo interaction (Kuo 1967).

In a very early calculation Wong *et al* (1969) have indeed shown that in the HF calculations of ^{28}Si the prolate-oblate separation varies strongly with the choice of the interaction, but they have not projected out states of good $\mathcal{J}$ and calculated their energies. Only the moment-of-inertia of the ground-state (oblate) band was calculated and was found about twice as large as the experimental value. Our motivation in this report has been to present a careful HF calculation of the structure of ^{28}Si states, including projection of good $\mathcal{J}$ states. We are particularly interested in the structure of the excited 0^+ states, and the location of the prolate band. We have chosen an interaction that was suggested by Preedom and Wildenthal (1972) from an analysis of the experimental data on nuclei in the $A=22$ region. Different interactions which give equally good results for nuclei with a small number of particles in d-s shell are known to give very different results when applied to nuclei such as ^{28}Si. The calculations of Halbert *et al* (1971) for nuclei of mass $A=18$-22 have shown that a variety of different interactions give almost equally good fits to nuclear properties in this region. On the other hand, Wong *et al* (1969) have shown that different interactions can give very different results for nuclei such as ^{28}Si. Apparently, the interactions are described by sets of 63 matrix elements with minor statistical variations. Hence, unfortunately we do not have a clear understanding of the essential features of the interaction that give rise to different results for complex nuclei. It would be very desirable to analyse the different effective interactions and isolate the components which lead to small or large moments of inertia or prolate-oblate separations, etc. This would then enable us to obtain a better insight into the way in which the effective interaction needs to be twisted when dealing with other nuclei in this region, and also into the effects of renormalisation processes for nuclei with many more than two nucleons outside a closed shell. Some progress in this direction is made (Kulkarni 1972).

2. HF calculation of ^{28}Si structure

We have carried out a complete Hartree-Fock calculation in the space of $(ds)^{12}$ configurations. The deformed self-consistent single-particle orbitals are expanded in terms of the spherical oscillator states $0d_{5/2}$, $1s_{1/2}$ and $0d_{3/2}$, and the energies of these states are taken from the observed structure of ^{17}O as

$$E(d_{5/2})=0{\cdot}0 \text{ MeV}, \quad E(s_{1/2})=0{\cdot}87, \quad E(d_{3/2})=5{\cdot}10 \text{ MeV}$$

Although one expects the $d_{5/2}-s_{1/2}$ separation to be somewhat larger in ^{28}Si compared to that in ^{17}O, Soyeur and Zucker mention that reasonable variation of single particle energies that they have tried do not affect significantly any results.

Preedom and Wildenthal (1972) have proposed a modification of the famous Kuo interaction (Kuo 1967) in the d-s shell that gives an excellent fit to the experimental data for nuclei in $A=18$-22 region. In our calculations we use the set of sixty three matrix elements given by them.

The methodology of Hartree-Fock calculations is well-known in literature (Ripka

Table 1. HF energies and intrinsic quadrupole moments

	Kuo			P–W		
	Prolate	Spherical	Oblate	Prolate	Spherical	Oblate
E_{HF} (MeV)	−94·16	−73·60	−97·88	−74·61	−80·03	−82·01
Q_{HF} (b²)	23·64	0	−22·96	22·72	0	−19·81

1968) and we do not describe it here. We have only considered axially symmetric self-consistent solutions with the total axial projection of angular momentum $K=0$. Such HF solutions corresponding to the lowest oblate and the lowest prolate minima are obtained. We also find a spherical solution in this low energy region, corresponding to $(d_{5/2})^{12}$ $\mathcal{J}=0$. Good angular momentum states are then projected from each of these solutions, and the total Hamiltonian operator is diagonalised in this space to obtain the final eigenfunctions and eigenvalues. Care has been taken to orthogonalise the projected states of the same $\mathcal{J}$ value from different HF solutions. In general the overlaps between such projected states are very small. Some quadrupole moments as well as $B(E2)$ transition probabilities are also calculated.

We find that the HF gap for the oblate solution is quite large, viz., 9·2 MeV, and that for the prolate solution is 2·6 MeV. The spherical solution is also quite stable with a HF gap of 8·0 MeV. The prolate solution is 7·4 MeV above the oblate solution, whereas the spherical solution is only 2 MeV above the oblate one. It is noteworthy that a similar calculation with Kuo matrix elements gives a prolate-oblate separation of only 3·7 MeV, and the spherical solution 24 MeV above the oblate solution. Table 1 gives a comparison of the results of the two interactions for the HF energies as well as the intrinsic quadrupole moments.

The projected spectrum of ^{28}Si calculated with both the above interactions is shown in figure 1, along with the experimental data. It is immediately obvious that whereas the Kuo interaction gives a rather compressed spectrum, the Preedom-Wildenthal matrix elements give the states of the ground state (oblate) band very close to the observed states, including the 8^+ state identified at 15·2 MeV, and predicted at 14·8

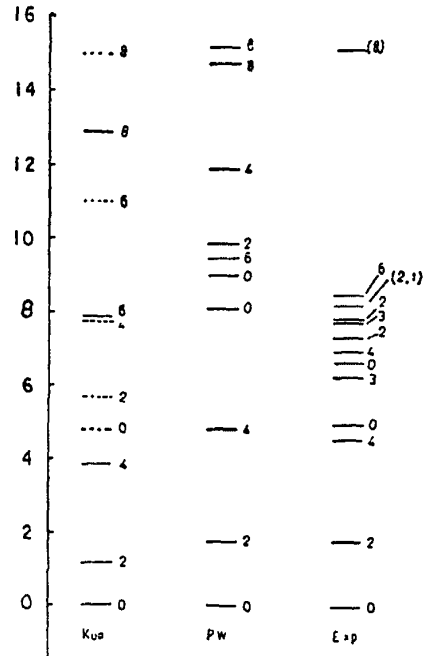

Figure 1. Energy spectra of ^{28}Si. The first column gives the states projected from oblate (full line) and the prolate (dashed line) HF states using Kuo interaction. The second column gives the spectrum obtained by mixing the states projected from the oblate, the prolate and the spherical solutions using PW interaction. The last column gives the experimental spectrum. The numbers on the extreme left denote the energy scale in MeV.

MeV. It is clear also from table 1 that the success of the Preedom-Wildenthal interaction is due to the smaller deformations and smaller moment-of-inertia it produces.

It is well established now that the first excited 0_1^+ state observed at 5 MeV is essentially a beta-vibration based on the ground state. It was first pointed out by Das Gupta and Harvey (1967) that the second excited 0_2^+ state at 6·7 MeV should be identified with the prolate band-head. On the other hand Castle and Parikh (1970) in a variation-after-projection calculation obtained a prolate 0^+ state at 3·5 MeV and a spherical 0^+ state at 4·76 MeV. Experimentally, the situation regarding the 0_2^+ state is ambiguous, and there is no definite evidence regarding its shape. If it is a prolate band-head one should expect a band of states with $\mathcal{J}=2^+$, 4^+, 6^+ etc. above it showing the characteristic strong $E2$ transitions. Huang *et al* (1971) have looked carefully at five excited 2^+ states above the 0_2^+ state but have failed to find any evidence for a rotational band of states based on this state.

Our calculations of the 0^+ states show that for the Preedom-Wildenthal interaction the largely spherical 0^+ state is at 8·15 MeV, whereas the prolate 0^+ occures at 9·05 MeV. For the Kuo interaction, the spherical state is quite high and the prolate 0^+ occures at only 4·8 MeV. The overlap between the oblate and the spherical 0^+ is only 0·11. In view of the experiments of Huang *et al* (1971) one would be tempted to associate the 6·7 MeV 0_2^+ state with the predicted spherical state in this region. It should then be necessary to look for an additional 0^+ state (presumably prolate) at about 9-10 MeV excitation. More experimental investigations on the states in the region 7-10 MeV excitation would be very desirable.

Two points need to be emphasized. The excitation energies of 0^+ states of different shapes are strongly interaction-dependent. Secondly, even with the Preedom-Wildenthal interaction one can construct $K=0$ excited bands from one-particle-one-hole excitations on the ground oblate band, and the energies of these band-heads would be again about 9-10 MeV (oblate HF gap). Such 1p-1h bands would mix into the prolate band and would alter the structure and the properties of the prolate states. We hope to carry out such band-mixing calculations.

3. Electromagnetic properties

The observed electric quadrupole moment of the first excited $\mathcal{J}=2^+$ state is recently given as 17 ± 5 eF^2 (Nakai *et al* 1970). The Preedom-Wildenthal interaction gives a value of 18·75 eF^2, whereas the Kuo interaction gives 21·84 eF^2, indicating a sizeably larger deformation.

Table 2 gives the $B(E2)$ values for transitions connecting states of the oblate band for the Preedom-Wildenthal interaction. In this calculation we follow the standard practice of assuming the effective charges for the protons and the neutrons as 1·5 and 0·5 respectively. The value of the oscillator parameter b used is 1·77 F. The experimental uncertainties in the measured $B(E2)$ values are large, and the values reported in the literature are not always consistent. The observed values of table 2

Table 2. $BE(2)$ values (in units of e^2F^4) for transitions connecting states of oblate band

Initiate State	Final State	$BE(2)$ Case	$BE(2)$ Expt
2^+	0^+	84·03	67 ± 20
4^+	2^+	115·34	79 ± 11
6^-	4^+	115·86	67 ± 36
8^-	6^+	102·56	—

are taken from the paper by Soyeur and Zucker (1972). It is satisfactory that in agreement with the experimental trend as well as the shell-model results, we obtain

$$B(E2; 4^+ \rightarrow 2^+) > B(E2; 2^+ \rightarrow 0^+)$$

The situation regarding $B(E2)$ for the $6^+ \rightarrow 4^+$ transition is quite interesting. Our calculations predict the 6^+ state of the oblate band at 9·3 MeV close to the first observed 6^+ state at 8·6 MeV. The calculated $B(E2)$ for $6^+ \rightarrow 4^+$ transition is as large as that for the $4^+ \rightarrow 2^+$ transition. This seems to be in good agreement with the shell model calculations of Soyeur and Zucker (using 'large space') as well as the experiments quoted by them. The shell model results of Wildenthal and McGrory (1972) give a much smaller value for this transition, and hence they suggest the second 6^+ state calculated by them to be at about 10 MeV (but not experimentally identified) as the candidate for the state of the oblate band. However, their small value may be due to the smaller configuration space used by them. Soyeur and Zucker also obtain a much smaller $B(E2)$ for this transition in 'small space'. It will be of great interest to have accurate measurements of the $B(E2)$ for the 6^+ (8·55 MeV) to the first 4^+ state transition.

4. Conclusions

We have carried out a complete Hartree-Fock calculation of the structure of ^{28}Si in the d-s shell configuration space. It is shown that the energies of various shape excitations are strongly interaction-dependent. The Preedom-Wildenthal interaction gives a good account of the ground oblate band structure. We find a spherical as well as a prolate 0^+ state in the region of 8-9 MeV, and suggest an experimental search for another 0^+ state in this region in addition to the 0^+ state already observed at 6·7 MeV. The absence of a clear cut rotational band based on this 0^+ state (Huang *et al* 1971) prompts us to identify this as the calculated spherical state. On the other hand, Wittner *et al* (1973) have studied ^{24}Mg (^{7}Li, t) ^{28}Si reaction, and on the basis of a large observed cross-section leading to the 6·7 MeV 0^+ state, they have identified it as the prolate band-head. If this is so, the results of Huang *et al* (1971) may find explanation in possible large band-mixing of particle-hole excited states with those of the prolate band. In any case, further experimental data on states of ^{28}Si in this region of excitation are desirable.

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