DIFFRACTION OF LIGHT BY TRANSPARENT SPHERES AND SPHEROIDS: THE FRESNEL PATTERNS

BY SIR C. V. RAMAN AND S. RAMASESHAN

(From the Raman Research Institute and the Department of Physics,
Indian Institute of Science, Bangalore)

Received December 7, 1949

1. INTRODUCTION

THE phenomena with which we are concerned in the present paper are those arising when a beam of light emerges after traversing a sphere of radius large compared with the wave-length of light. Our attention was drawn to them in the course of some studies on the Christiansen experiment with spherules of glass described in a recent paper in these *Proceedings.* To elucidate more fully the effects described in that paper, it appeared to us desirable to examine the optical behaviour of an individual spherule of glass immersed in a liquid of nearly the same refractive index and traversed by light from a distant point source. Very pretty colour effects were observed with white light, which obviously had their origin in the fact that the convergence of the beam after its passage through the particle was widely different for the different rays of the spectrum. This suggested the use of a monochromatic light source, *e.g.*, a sodium vapour lamp. We then noticed that the light emerging from the spherule exhibits very characteristic diffraction patterns; the nature of these depends notably on the shape of the particle, being widely different for the two most interesting cases in which it is respectively a sphere and a spheroid of revolution. The configuration of the patterns changes progressively as the plane of observation is shifted away from the spherule; the difference in the refractive indices of the sphere and of the liquid in which it is immersed determines how rapid this change is. The diffraction patterns may be observed even when this difference is not small; their changes with the shift of the plane of observation are then rapid. The dispersive powers of the glass and the liquid not being the same, the difference in their refractive indices alters rapidly with the wave-length of the light employed. As a consequence, the patterns change quickly with the wave-length. This is readily observed when the light-source is a mercury arc, even without the aid of colour filters for selecting out the different spectral rays emitted by it.

Our observations were in the first instance made with the same tiny spherules of glass one millimetre in diameter as those used in the Christiansen experiment. We subsequently found, however, that essentially similar phenomena can be observed also with spheres which are still smaller, *viz.*, one-tenth of a millimetre in diameter and also with spheres of much larger size. A ball of quartz immersed in benzene shows the effects very well; its birefringence, however, results in two sets of patterns being observed instead of one, their separation varying with the direction in which the light traverses the sphere. It would seem that we have here a very simple and convenient method of observing and exhibiting the optical characters of a crystalline solid.

2. GEOMETRIC THEORY

As we are concerned with spheres whose dimensions are great in comparison with the wave-length, it is justifiable, at least in the first instance, to regard the problem as one of geometrical optics, and then to supplement the results by considerations based on wave-theory. Using elementary methods, we may trace the course of a bundle of parallel rays incident on a transparent sphere and emerging therefrom. The deviation suffered is zero for the axially incident ray; it increases steadily and then much more rapidly as we approach the marginal ray which is incident tangentially on the surface. The deviations of the rays depend on the refractive index of the sphere relatively to that of the sorrounding medium. In the particular case when the relative index is unity, they vanish and the rays emerge again as a bundle of parallel rays. In the cases which interest us, the relative index is a little greater than unity, and the emerging beam would evidently be convergent, but such convergence would be very different for the axial bundle of rays and for the marginal ones. In actual practice, the sphere is immersed in a flat-walled cell completely filled with liquid. In tracing the course of the rays, the further deviations which occur when they emerge from the cell walls would also have to be considered.

The result of the passage of the light through the sphere and the enclosing liquid is most readily visualised by considering the cylindrical bundle of rays incident on the sphere to be divided up to a great number of concentric hollow cylindrical beams. Each such cylinder of rays would converge to a focus on the axis and subsequently diverge, but the focal points would all be different, being farthest from the sphere for the axial pencils and nearest to it for the marginal ones. In other words, instead of all the rays passing through the sphere converging to a single focus on the axis, we would have a continuous line of foci or concentration of intensity along the axial ray.

Further, since each hollow cylinder of rays emerges from the cell as a hollow cone of rays and the convergence of these is different, it follows that the successive cones would intersect each other and form a caustic surface, the cross-section of which by any plane normal to the axis would be a circle. The bundle of rays emerging from the sphere would therefore exhibit a concentration of intensity along its circular periphery. The area enclosed by this would be largest when the beam emerges from the cell and would contract as we recede therefrom, finally collapsing to a point when we reach the focus of the axially incident rays. Except in the limiting case when the relative index is unity, the diameter of the emergent beam would invariably be less than that of the sphere.

The foregoing remarks are illustrated by Fig. 1 below, in which the course of a bundle of parallel rays incident on a glass sphere of refractive index 1·54 immersed in water (refractive index 1·33) and emerging therefrom has been computed and shown. Besides drawing the course of the rays, the form of the wave-surface immediately after emergence from the sphere has also been computed and drawn in. It exhibits the cusp-shaped form which in wave-theory corresponds to the formation of a caustic in geometrical optics.

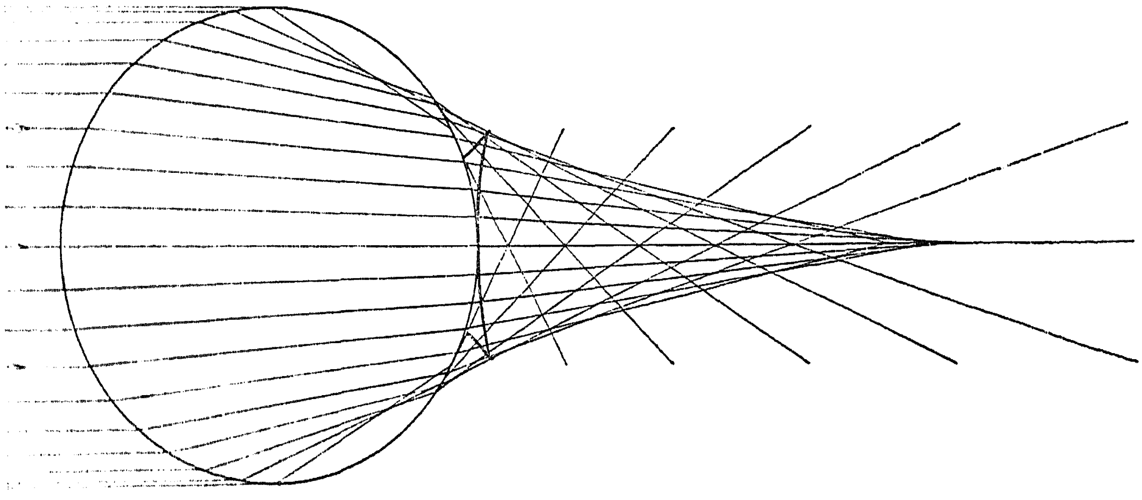

FIG. 1

3. INTERFERENCE PHENOMENA

As is evident from Fig. 1, the rays which have traversed the sphere by different paths intersect each other after emergence from it. It follows that interference effects should be observable. We may, in the first place, remark that wave-optical considerations reinforce the results indicated by the geometrical theory that there should be observable concentrations of intensity of *two* kinds in the light-field, firstly along the circular periphery of the beam emerging from the sphere, and secondly along its axis. At the periphery of the beam, rays which have traversed closely adjacent and therefore nearly equal paths within the sphere intersect one other, while on the axis, rays meet which have transversed identically equal paths but emerge from different points forming a ring on the surface of the sphere. It follows that the axis should everywhere be a locus of maximum intensity, while the periphery should exhibit a maximum of intensity lying close to but not absolutely coincident with it. Corresponding to each of these two loci of maximum intensity, we should have a series of loci of subsidiary minima and maxima of intensity appearing at the points where the intersecting rays differ in optical path respectively by odd and even multiples of half a wave-length. In the case of the interferences running parallel to the circular periphery of the beam, it is evident that the intersecting rays meet at an angle which is small in its vicinity and increases as we move away from the periphery towards the centre of the field. It follows that the interference fringes running parallel to the circular caustic would be very wide at the margin at the field and becomes progressively narrower as we approach the centre of the light beam. The case is altogether different when we consider the interferences of the rays reaching points at and near the axis from points all round the circumference. These rays evidently meet at a fairly large angle which would change very little as we move away from the axis in any given plane of observation. It follows that the interference rings surrounding the axial concentration of intensity would be narrow, but evenly spaced. It is evident also that both the peripheral and axial sets of interference rings should widen out as we move away from the sphere and approach the focal point of the axial rays; for the angles of intersection of the interfering rays then steadily diminish. The superposition of the two sets of interference would obviously become most evident in the same circumstances.

It should not be supposed however, that elementary considerations of the kind indicated above would suffice completely to describe the actual facts of observation. In the first place, the approach is purely qualitative and makes no pretence of giving quantitative indications in respect of the intensity of illumination in the field. Indeed, even if it were to be developed so as to deal also with such questions, we should scarcely expect such a theory to be quite successful. The part of the area of the incident wave-front which gives rise to the experimentally significant phenomena is that which passes through the marginal regions of the sphere. This is a very small part of the whole, especially when the relative index of refraction is only a little greater than unity. Diffraction then necessarily plays a part and determines the observed intensities of illumination in the field. We cannot therefore regard the geometrical approach to the problem as anything more than a useful and easily understood way of interpreting the observed phenomena.

4. METHODS AND RESULTS OF OBSERVATION

For observing the diffraction phenomena produced by a small spherule of glass, the most suitable procedure is to immerse it in a thin flat cell which is completely filled up with liquid and then covered by a glass-plate. The cell is then placed on the stage of a low-power microscope. A convenient light-source is provided by a small aperture backed by a sodium vapour lamp or alternatively a point-o-lite mercury arc. The former is most useful when it is desired to observe the patterns with light of one wave-length, while the latter (unless filters are used) is convenient for observing the patterns with several wave-lengths simultaneously. The light from the aperture is sent up through the cell by the plane reflecting mirror below the stage of the microscope, the condenser having been removed. By racking up the microscope, the patterns formed in the successive planes of observation commencing from the upper surface of the cell up to the focal point of the axial rays or even beyond may be conveniently observed. In the case of the larger spheres, no microscope is needed and the observations may be made with a suitable eyepiece or, better still, by receiving the light emerging from the sphere on a sheet of ground glass.

The experimental studies confirm the results of the foregoing theoretical discussion. The caustic and its accompanying interference fringes at the periphery of the field are readily observable features. The bright spot at the centre of the field indicated by the theory is first seen at some distance from the sphere and increases rapidly in intensity as we approach the focal point of the axial rays. That this spot is an optical image of the light source employed is readily verified by making the latter of triangular shape. The bright spot, if observed with a perfectly spherical particle, has then the same shape and increases progressively in size as we recede further and further from the sphere. It is important to remark that the bright spot continues to be visible along the axis of the beam even beyond the geometric focus of the axial rays. This clearly indicates that a purely geometric theory is insufficient to cover the facts.

To observe the uniformly spaced interference rings surrounding the bright central spot, one should use a very small aperture as the source. The necessity for this is easily understood, since the rings are closely spaced·

Both sets of rings can then be readily seen superposed on each other, especially as we approach the focal point of the axial rays.

Using an aperture backed by a mercury arc (without filters) as source, the caustics corresponding to the indigo, blue, green and yellow rays of the arc are seen well separated. With a quartz sphere, two sets of circular caustics are seen instead of one. That these represent the two refractive indices of the crystal corresponding to the direction of the incident beam is readily shown by placing a polaroid in the path of the incident light and rotating it. One or the other of the two caustics is then periodically extinguished.

It is very common to find particles having a spheroidal shape amongst those used for the Christiansen experiment. With such particles, the phenomena observed are strikingly different from those exhibited by spherical particles. The caustic appearing at the periphery of the emerging light beam is not circular but elliptic in shape, as is to be expected. Instead of the bright central spot given by a spherical particle, we find a light-figure having the shape of the geometric evolute of the boundary of the beam emerging from the spheroid. This is clearly a diffraction effect having its origin at the part of the wave-front which has traversed the interior of the particle near its margin, almost grazing the surface.

SUMMARY

The paper describes and discusses the diffraction effects observed when a beam of light traverses a transparent sphere immersed in a liquid of slightly lower index and emerges therefrom. The two most interesting features are firstly, a concentration of intensity along the periphery of the emerging light beam which is evidently in the nature of a caustic and secondly, a concentration of intensity along the axial ray which is in the nature of a continuous focus. These two features are each accompanied by a set of interference-rings and these appear superposed on each other. Significant alterations appear in these features when the particle has a spheroidal shape. With a birefringent sphere, two sets of caustics are, in general, observed. 12 Photographs illustrate the paper.

DESCRIPTION OF THE FIGURES

FIGS. 1, 2 and 3 in Plate V

Diffraction patterns with sodium light of a spherical glass ball one millimetre diameter immersed in xylene, the plane of observation being progressively removed further and further from the cell containing the sphere. Note the bright circular caustic with the accompanying fringes, and the central bright spot which progressively increases in intensity with increasing distance from the sphere.

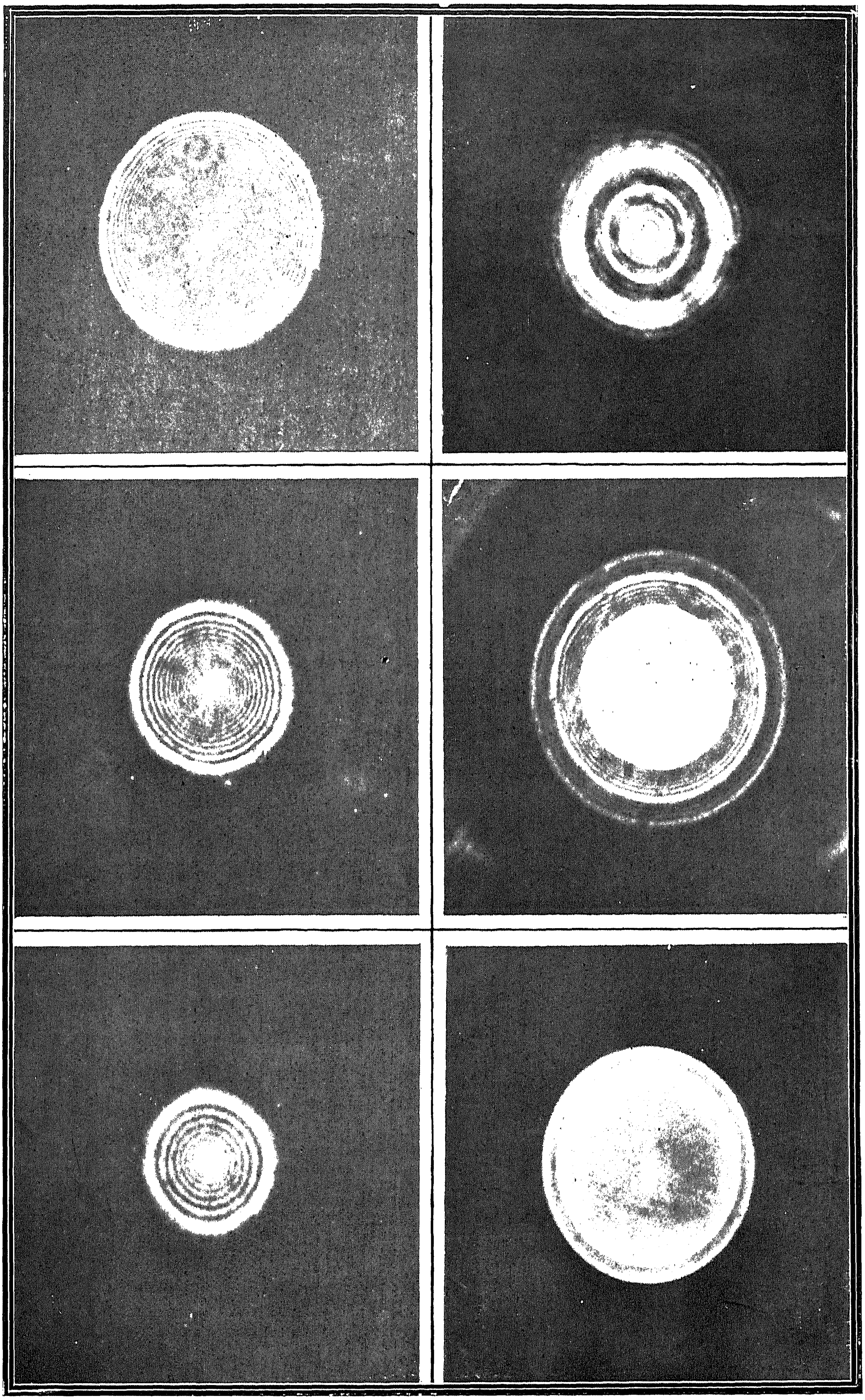

FIG. 1

FIG. 4

FIG. 2

FIG. 5

FIG. 3

FIG. 6

Diffraction by spheres immersed in a liquid.

Sir C. V. Raman and
S. Ramaseshan

Proc. Ind. Acad. Sci., A, vol. XXX, Pl. VI

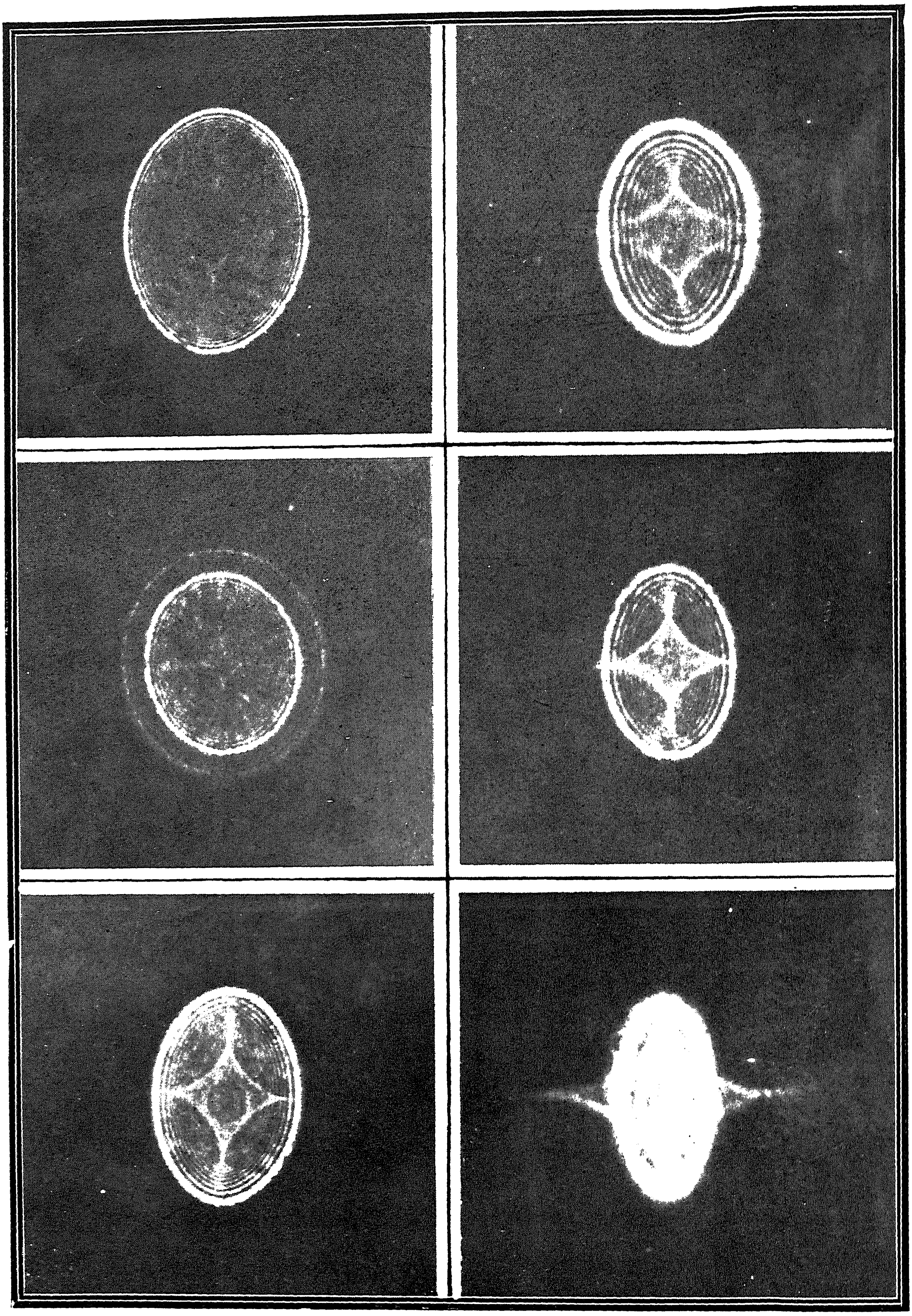

FIG. 7

FIG. 10

FIG. 8

FIG. 11

FIG. 9

FIG. 12

Diffraction by spheroids immersed in a liquid

FIG. 4. Diffraction pattern with sodium light of a spherical ball immersed in nitrobenzene, the plane of observation being rather near the focal point of the axial rays. The pattern is highly magnified in the reproduction to exhibit the two superposed sets of interference-rings.

FIG. 5. Diffraction pattern of a sphere of glass one millimetre in diameter immersed in a nitrobenzene—monobromonaphthalene mixture, with an unfiltered mercury arc as source. The faint outer caustic is due to the λ 4046 radiation. The next bright caustic inside is due to the λ 4358 radiation, while those nearer the centre are due to the green and yellow radiations of the mercury arc.

FIG. 6. Diffraction pattern in sodium light of a quartz sphere of 5 centimetres diameter immersed in benzene. Notice the doubling of the caustic resulting from birefringence. The bright spot at the centre of the pattern is clearly seen.

FIGS. 7, 8, 9, 10, 11 and 12 in Plate VI

Diffraction patterns of a spheroidal ball. All were obtained with sodium light except Fig. 8 which was recorded with a mercury arc and exhibits the caustics due to λ 4046 and λ 4758 separately. The immersion liquid was xylene in all cases except Fig. 12 which was obtained with a nitrobenzene—monobromonaphthalene mixture and has been reproduced on a highly magnified scale.