THE GENERAL LORENTZ MATRIX AND SOME IDENTITIES

The general Lorentz transformation connecting two inertial frames $S(o-xyzt)$ and $S'(o'-x'y'z't')$ and involving six arbitrary parameters of relative velocity and relative orientation may be expressed in the form

$$x' = \left[ l_1 + \frac{Lu_1 (\gamma - 1)}{u^2} \right] x$$

$$+ \left[ m_1 + \frac{Mu_1 (\gamma - 1)}{u^2} \right] y$$

$$+ \left[ n_1 + \frac{Nu_1 (\gamma - 1)}{u^2} \right] z - \frac{\gamma u_1}{c} \cdot ct$$

$$y' = \left[ l_2 + \frac{Lu_2 (\gamma - 1)}{u^2} \right] x$$

$$+ \left[ m_2 + \frac{Mu_2 (\gamma - 1)}{u^2} \right] y$$

$$+ \left[ n_2 + \frac{Nu_2 (\gamma - 1)}{u^2} \right] z - \frac{\gamma u_2}{c} \cdot ct$$

$$z' = \left[ l_3 + \frac{Lu_3 (\gamma - 1)}{u^2} \right] x$$

$$+ \left[ m_3 + \frac{Mu_3 (\gamma - 1)}{u^2} \right] y$$

$$+ \left[ n_3 + \frac{Nu_3 (\gamma - 1)}{u^2} \right] z - \frac{\gamma u_3}{c} \cdot ct$$

$$ct' = - \frac{Ly}{c} x - \frac{My}{c} y - \frac{Ny}{c} z + \gamma \cdot ct,$$

where the velocity of $S'$ relative to $S$ is $(u_1, u_2, u_3)$ in the co-ordinates of $S'$ and $(L, M, N)$ in the co-ordinates of $S$. Thus

$$u_1 l_1 + u_2 l_2 + u_3 l_3 = L,$$

$$L l_1 + M l_2 + N l_3 = u_1,$$

$$L^2 + M^2 + N^2 = u_1^2 + u_2^2 + u_3^2 = u^2$$

and

$$\gamma = \left(1 - u^2/c^2\right)^{-\frac{1}{2}},$$

and $m_1, n_1, etc.$, are direction cosines. The transformation is proper and orthochronous, since $\det (g_{ij}) = 1$ and $g_{ii} = \gamma^2 > 1$ for the matrix $g_{ij}$ of the transformation.

One can verify for this transformation that

$$(g_{12} - g_{21})(g_{24} - g_{42}) + (g_{13} - g_{31})(g_{34} - g_{43})$$

$$+ (g_{14} - g_{41})(g_{42} - g_{24}) = 0.$$  

(3)

This may be expressed as

$$\epsilon^{(kl)} g_{ij} g_{kl} = 0$$

in the usual notation. It is also equivalent to

$$\det (g_{ij} - g_{kl}) = 0.$$  

(5)

In the usual notation the defining property of the Lorentz matrix is

$$g^{\top} \eta g = \eta$$

(6)

and since $\eta$ is the unit matrix,

$$g g^\top = \eta$$

(7)

(6) and (7) may be written as equivalent sets of ten algebraic conditions on $g_{ij}$. It should be possible to prove (3) from (6). K. B. Prabhu Patkar has recently shown that (3) follows from (6) and in the process obtained three distinct identities of the form

$$g_{ij} g_{kl} - g_{ik} g_{jl} + g_{il} g_{jk} - g_{ik} g_{jl} = 0.$$  

(8)

$i \neq j \neq k \neq l$

(3) is a consequence of (8).

Even in the general case when the diagonal matrix $\eta$ has diagonal elements $\pm 1$ so that $g_{ij}$ as defined by (3) is not necessarily a Lorentz matrix, (8) holds good and (3) follows from it. This has been checked by P. P. Kale.

University of Poona,

V. V. NARLIKAR,

Poona, December 31, 1970.

---

Current Science