LETTERS TO THE EDITOR

THE GENERAL LORENTZ MATRIX AND SOME IDENTITIES

THE general Lorentz transformation connecting two inertial frames $S(o\text{-}xyzt)$ and $S'(o'\text{-}x'y'z't')$ and involving six arbitrary parameters of relative velocity and relative orientation may be expressed in the form

$$
\begin{aligned}
x' &= \left[l_1 + \frac{Lu_1(\gamma-1)}{u^2}\right]x + \left[m_1 + \frac{Mu_1(\gamma-1)}{u^2}\right]y + \left[n_1 + \frac{Nu_1(\gamma-1)}{u^2)}\right]z - \frac{\gamma u_1}{c}\,.\,ct \\
y' &= \left[l_2 + \frac{Lu_2(\gamma-1)}{u^2}\right]x + \left[m_2 + \frac{Mu_2(\gamma-1)}{u^2}\right]y + \left[n_2 + \frac{Nu_2(\gamma-1)}{u^2}\right]z - \frac{\gamma u_2}{c}\,.\,ct \\
z' &= \left[l_3 + \frac{Lu_3(\gamma-1)}{u^2}\right]x + \left[m_3 + \frac{Mu_3(\gamma-1)}{u^2}\right]y + \left[n_3 + \frac{Nu_3(\gamma-1)}{u^2}\right]z - \frac{\gamma u_3}{c}\,.\,ct
\end{aligned} \tag{1}
$$

$$ct' = -\frac{L\gamma}{c}x - \frac{M\gamma}{c}y - \frac{N\gamma}{c} + \gamma\,.\,ct,$$

where the velocity of S′ relative to S is (u_1, u_2, u_3) in the co-ordinates f S′ and (L, M, N) in the co-ordinates of S. Thus

$$
\left.
\begin{aligned}
&u_1 l_1 + u_2 l_2 + u_3 l_3 = L, \text{ etc.} \\
&Ll_1 + Mm_1 + Nn_1 = u_1, \text{ etc.} \\
&L^2 + M^2 + N^2 = u_1{}^2 + u_2{}^2 + u_3{}^2 = u^2
\end{aligned}
\right\} \tag{2}
$$

and

$\gamma = (1 - u^2/c^2)^{-\frac{1}{2}}$ and l_1, m_1, n_1, etc., are direction cosines. The transformation is proper and orthochronous, since $\det(g_{ij}) = 1$ and $g_{44} \equiv \gamma \geqslant 1$ for the matrix g_{ij} of the transformation.

One can verify for this transformation that

$$(g_{12} - g_{21})(g_{34} - g_{43}) + (g_{23} - g_{32})(g_{14} - g_{41}) (+ g_{31} - g_{13})(g_{24} - g_{42}) = 0. \tag{3}$$

This may be expressed as

$$\epsilon^{ijkl} g_{ij}\, g_{kl} = 0 \tag{4}$$

in the usual notation. It is also equivalent to

$$\det(g_{ij} - g_{ji}) = 0, \tag{5}$$

In the usual notation the defining property of the Lorentz matrix is

$$g^{T}\eta g = \eta \tag{6}$$

and since η^2 is the unit matrix,

$$g\eta g^{T} = \eta, \tag{7}$$

(6) and (7) may be written as equivalent sets of ten algebraic conditions on g_{ij}. It should be possible to prove (3) from (6). K. B. Prabhu Patkar has recently shown that (3) follows from (6) and in the process obtained three distinct identities of the form

$$g_{ij}g_{kl} - g_{il}g_{kj} + g_{jl}g_{ik} - g_{il}g_{jk} = 0. \qquad i \neq j \neq k \neq l \tag{8}$$

(3) is a consequence of (8).

Even in the general case when the diagonal matrix η has diagonal elements ± 1 so that g_{ij} as defined by (3) is not necessarily a Lorentz matrix, (8) holds good and (3) follows from it. This has been checked by P. P. Kale.

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