ON EINSTEIN'S GENERALIZED THEORY OF GRAVITATION

A PAPER, bearing the same title as the present communication, was recently communicated by us to the National Institute of Sciences of India. We have discussed in it the field equations of the generalized theory* and outlined a method of successive approximations for working out the interaction between a gravitational field and an electromagnetic field. The computation of interaction terms is an extremely laborious task and, so far as we know, has not been carried out yet. The results that are given below seem to be new and of considerable interest.

The theory is based on a Hermitian tensor g_{ik}:

$$g_{ik}=a_{ik}+\sqrt{-1}b_{ik},\ a_{ik}=a_{ki},\ b_{ik}=-b_{ki}. \quad (1)$$

We consider a pure gravitational field as given by

$$a_{11}=a_{22}=a_{33}=-(1+m/2r)^4,$$
$$a_{44}=(1-m/2r)^2\,(1+m/2r)^{-2},\ a_{ij}=0,\ i\neq j \quad (2)$$

and a pure electromagnetic field as given by

$$b_{13}=b_{34}=\phi,\ b_{12}=b_{14}=b_{23}=b_{24}=0,$$
$$\phi\equiv A\cos\frac{2\pi}{\lambda}(x-t), \quad (3)$$

in the usual notation. In the complex field unifying the two a_{ij} is modified by ϕ and b_{ij} by m. The exact nature of the interaction of the two fields is certainly very complicated. Our computations show that the usual Maxwell equations are modified as follows:

$$-\frac{\partial E_z}{\partial y}+\frac{\partial E_y}{\partial z}-\frac{\partial H_x}{\partial t}=\frac{2\phi mz}{r^3},$$
$$-\frac{\partial E_x}{\partial z}+\frac{\partial E_z}{\partial x}-\frac{\partial H_y}{\partial t}=0, \quad (4)$$
$$-\frac{\partial E_y}{\partial x}+\frac{\partial E_x}{\partial y}-\frac{\partial H_z}{\partial t}=-\frac{2\phi mx}{r^3},$$
$$\frac{\partial H_x}{\partial x}+\frac{\partial H_y}{\partial y}+\frac{\partial H_z}{\partial z}=-\frac{2\phi mz}{r^3}.$$

The components of the electric current-and-density vector are found to satisfy, to the same degree of approximation, the following equations—

$$\Box\ \rho+\frac{12mxy}{r^5}\ \phi_1=0,$$
$$\Box\ \sigma_x+\frac{12mxy}{r^5}\ \phi_1=0,$$
$$\Box\ \sigma_y+\frac{12m}{r^5}(y^2-z^2)\phi_1=0,$$
$$\Box\ \sigma_z+\frac{24myz}{r^5}\ \phi_1=0, \quad (5)$$

where $\Box$ stands for

$$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial t^2}$$ and ϕ_1 for $\frac{\partial \phi}{\partial x}$. It can be verified that

$$\Box\left(\frac{\partial \sigma_x}{\partial x}+\frac{\partial \sigma_y}{\partial y}+\frac{\partial \sigma_z}{\partial z}+\frac{\partial \rho}{\partial t}\right)=0 \qquad (6)$$

We have similarly obtained the ten gravitational field equations showing how the gravitational potentials a_{ij} are modified on account of the electrostatic and electromagnetic potentials. The details and further deductions will be published elsewhere.

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* Einstein, A., "A Generalized Theory of Gravitation", *Rev. Mod. Phys.*, 1948, **20**, 35. A reference may also be made to the earlier papers in *Ann. Math.*, II (1945), **46**, 578 and (1946), **47**, 731.