THE CANONICAL CO-ORDINATE SYSTEM IN GENERAL RELATIVITY

THE canonical co-ordinate system[1] for which, at the origin, all the first order partial derivatives of $g_{\mu\nu}$ vanish and the second order derivatives are given by a set of hundred equations is well known in the literature of general relativity. It is particularly useful for exploring the neighbourhood of an event in the space-time continuum. We have not seen anywhere the Taylor expansions of $g_{\mu\nu}$ defining the canonical co-ordinate system. The expansions contain explicitly the twenty independent components of the Riemann-Christoffel[2] tensor R_{hijk}. As the metric tensor defines not only the co-ordinate system but the gravitational field itself, we have found the expansions of special interest and service in discussing the purely geometrical, as well as gravitational properties of the relativity metric. A full report is being prepared for communication elsewhere. We have thought it worthwhile to place only the expansions here on record.

We define the twenty independent components of R_{hijk} at (0, 0, 0, 0) by the following equations :

$$R_{1212} = a, \quad R_{1313} = b, \quad R_{1414} = c,$$
$$R_{2323} = d, \quad R_{2424} = e, \quad R_{3434} = f,$$
$$R_{2113} = g, \quad R_{2114} = h, \quad R_{3114} = i,$$
$$R_{1223} = j, \quad R_{1224} = k, \quad R_{3224} = l,$$
$$R_{1332} = m, \quad R_{1334} = n, \quad R_{2334} = o,$$
$$R_{1442} = p, \quad R_{1443} = q, \quad R_{2443} = r,$$
$$R_{1234} = s, \quad R_{1423} = t.$$

If the powers above the second of the coordinates of an event in the neighbourhood of the origin are ignored we have

$$g_{11} = -1 - \tfrac{1}{3}(ay^2 + bz^2 + c\tau^2 - 2gyz - 2hy\tau - 2iz\tau),$$
$$g_{22} = -1 - \tfrac{1}{3}(ax^2 + dz^2 + e\tau^2 - 2jxz - 2kx\tau - 2lz\tau),$$
$$g_{33} = -1 - \tfrac{1}{3}(bx^2 + dy^2 + f\tau^2 - 2mxy - 2nx\tau - 2oy\tau),$$
$$g_{44} = 1 - \tfrac{1}{3}(cx^2 + ey^2 + fz^2 - 2pxy - 2qxy - 2ryz)$$
$$g_{12} = \tfrac{1}{3}\{mz^2 + p\tau^2 + axy - gxz - hx\tau - jyz - ky\tau - (2t+s)\,z\tau\},$$
$$g_{13} = \tfrac{1}{3}\{jy^2 + q\tau^2 - gxy + bxz - ix\tau - myz - nz\tau + (t-s\}\,y\tau,$$
$$g_{14} = \tfrac{1}{3}\{ky^2 + nz^2 - hxy - ixz + cx\tau - py\tau - qz\tau + (t+2s\}yz,$$
$$g_{23} = \tfrac{1}{3}\{gx^2 + r\tau^2 - jxy - mxz + dyz - ly\tau - ozt + (t+2s)\,x\tau\},$$
$$g_{24} = \tfrac{1}{3}\{hx^2 + oz^2 - kxy - px\tau - lyz + ey\tau - rz\tau + (t-s)\,xz\},$$
$$g_{34} = \tfrac{1}{3}\{ix^2 + ly^2 - nxz - qx\tau - oyz - ry\tau + fz\tau - (2t+s)\,xy\}.$$

In the above x, y, z, τ stand for the usual x^1, x^2, x^3, x^4. The algebraic work involved in the above calculation is quite tedious, but the symmetry of the various terms at each stage provides a useful check on the details and simplifies the calculation.

Benares Hindu University, V. V. NARLIKAR.
May 28, 1948. AYODHYA PRASAD.

1. Eddington, A. S., *The Mathematical Theory of Relativity*, 1924, 79. 2. Eisenhart, L. P., *Riemannian Geometry*, 1926, 20.