PHOTOELASTIC CONSTANTS OF SODIUM CHLORATE FROM ULTRASONIC DIFFRACTION

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1. Introduction

Photoelastic constants are generally measured using a static method (Pockels, 1906). One measures the relative retardation produced in a crystal of known orientation by a known stress, using a Babinet compensator or a similar device. From a study of various sections of the crystal, some of the linear combinations of the piezo-optic constants, q_{ij}, can be obtained. In particular, in cubic crystals belonging to the crystal classes T_d, O and O_h, the quantities $(q_{11}-q_{12})$ and q_{44} are obtained and in those belonging to the classes T and T_h, $(q_{11}-q_{12})$, $(q_{11}-q_{13})$ and q_{44} can be determined. Measurements of absolute path retardation by an interference method are required to obtain all the constants individually. The constants q_{11} and q_{12} in the former type and q_{11}, q_{12} and q_{13} in the latter type of cubic crystals mentioned above can thus be obtained separately from such absolute measurements. The elasto-optic constants, p_{ij}, are calculated from the piezo-optic constants knowing the elastic constants of the crystal. Here also, only certain linear functions of the constants can be obtained from relative measurements and absolute path retardations are required for obtaining them all individually.

The possibility of measuring elasto-optic constants of cubic crystals using ultrasonic diffraction was suggested by Mueller (1938). Here also, it is necessary to measure the absolute amplitude of the ultrasonic wave in the crystal if all the constants are to be obtained independently. But, measurements of the state of polarisation of the diffracted beam in the Hiedemann pattern (1935), for different orientations of the crystal, would enable one to obtain p_{12}/p_{11} and p_{44}/p_{11} for crystal classes T_d, O, O_h and the ratios p_{12}/p_{11}, p_{13}/p_{11} and p_{44}/p_{11} for classes T and T_h. For obtaining these ratios, it is unnecessary to measure the absolute magnitudes of either light or sound intensity.

In both cases, absolute measurements are more difficult than relative measurements. However, absolute data have been obtained with the static

method employing the technique of Pockels (1906), *viz.*, of using two identical plane parallel specimens in a Jamin interferometer, or of Ramachandran (1947), *viz.*, obtaining localised interference fringes between the two surfaces of the crystal; but no absolute measurements appear to have been made by the ultrasonic method.

It is interesting to note that relative measurements made by the static method give *linear functions* of the photoelastic constants, while the ultrasonic method gives *ratios*. Combining the two, therefore, it is possible to get all the constants absolutely. This has, in fact, been done by Burstein, Smith and Henvis (1948) and Vedam (1950) for cubic crystals belonging to the classes T_d, O and O_h and for optical glasses, respectively.

Sodium chlorate belongs to the crystal class T and thus has four piezo-optic constants q_{11}, q_{12}, q_{13}, q_{44} and further exhibits optical activity. By means of special techniques, Ramachandran and Chandrasekharan (1951) measured directly $(q_{11}-q_{12})$, $(q_{11}-q_{13})$ and q_{44}; but they could not make measurements of absolute path retardation. However, they obtained q_{11}, q_{12}, q_{13} separately using ultrasonic data measured by one of us (K. V.) which were made available to them. The present paper deals with the principles, technique and results of the ultrasonic measurements. Both the theory of Mueller and the experimental technique have to be modified to suit the optical activity of sodium chlorate.

2. Theory of the Method

The following theory for a rotating crystal is based on Mueller's results, though we do not follow his conventions regarding signs. It is valid for the case when the ultrasonic amplitude is negligibly small. Consider longitudinal waves being propagated along the [100] or [110] direction in a cubic crystal. In both cases, the direction of vibration is along the direction of propagation. Further, if we consider the ultrasonic diffraction of light incident along the [001] direction on the crystal, then the principal axes of the optical polarisability ellipsoid of the strained crystal are respectively the direction of the light beam, the direction of ultrasonic wave-normal (chosen vertical for convenience) and the third perpendicular direction (horizontal). Denote by A and B, the changes in the optical polarisability in the vertical and horizontal directions. Let $2t$ be the thickness of the crystal and $2a$ the total optical rotation produced by the crystal. Let light of unit amplitude be incident with its electric vector at an angle $(\theta - a)$ to the horizontal, measured anticlockwise by an observer looking at the source. At a distance x from the centre of the crystal, its azimuth would be $(\theta + xa/t)$. The vertical and horizontal components of the diffracted beam in the first order

produced by a layer dx at x have the amplitudes

$$K\,A \sin(\theta + x\,\alpha/t)\,dx \text{ and } K\,B \cos(\theta + x\,\alpha/t)\,dx,$$

where K is a constant. On emergence, these two components would be

$$K\,[A \sin(\theta + x\,\alpha/t) \cos(t - x)\alpha/t + B \cos(\theta + x\alpha/t)\sin(t - x)\alpha/t]\,dx$$

and $K\,[A \sin(\theta + x\,\alpha/t) \sin(t - x)\,\alpha/t + B \cos(\theta + x\,\alpha/t) \cos(t - x)\,\alpha/t]\,dx$. The total horizontal and vertical components would be given by integrating the above two expressions from $-t$ to $+t$, and their ratio gives $\cot\phi$ where ϕ is the azimuth of the emergent diffracted beam. It is readily shown that

$$\cot\phi = \frac{(A + B)\sin(\theta + \alpha) + (A - B)\sin(\theta - \alpha)\,(\sin 2\alpha)/2\alpha}{(A + B)\cos(\theta + \alpha) - (A - B)\cos(\theta - \alpha)\,(\sin 2\alpha)/2\alpha}.$$

When $\alpha = 0$, *i.e.*, when the crystal is not optically active, $\cot\phi = (B/A)\cot\theta$ from Eq. (1), as is to be expected from Mueller's theory.

The derivation is not rigorous; but it should be accurate enough for the present purpose, since the difference between the cases when rotation is present and when it is absent is only of the order of 2 to 3%, when the ratio A/B or B/A does not exceed 1·5, as is found in the measurements made.

3. Experimental Method

With a non-rotating crystal, the best conditions are obtained when θ is made $\pm 45°$, so that $\tan\phi = A/B$ directly. One measures the angle γ between the plane of vibration of the directly transmitted light and of the light diffracted in the first order. Then $\phi = \theta + \gamma$ and if $\theta = 45°$, $B/A = \cot(45° + \gamma)$.

The same technique can be adopted here also. θ is made equal to 45° and the azimuth of the incident light is made $(\theta - \alpha)$. The rotation 2α of the crystal is determined, first of all. The emergent direct light will have an azimuth $(\theta + \alpha)$. and the angle γ between this and the light diffracted in the first order is determined as in the previous case. Then $\phi = (\theta + \alpha + \gamma)$ and in the particular case, it is $= 45° + \alpha + \gamma$.

In the non-rotating case, B/A deviates from $\cot(45° + \gamma)$ when the amplitude of the ultrasonic wave is finite. One therefore measures γ for different values of I, the intensity of the ultrasonic beam. Then, plotting $\cot(45° + \gamma)$ against I, the value extrapolated to zero I would correspond to B/A (Vedam, 1951). The same technique is adopted here, and if we call the extrapolated value of γ as γ_0 then

$$\phi = 45° + \alpha + \gamma_0.$$

This value of ϕ is put in Eq. (1) and B/A is evaluated.

4. Results of the Measurements

Crystals of sodium chlorate grow readily in the form of rectangular plates with the thickness along one of the cube axes (say [001]) and the length and breadth along [010] and [100] respectively. One such natural crystal was used for the measurement of p_{12}/p_{11} and p_{13}/p_{11} (*cf.* Table I). In another, a face was ground parallel to (110) and it was used for measuring the ratio $(p_{11}+p_{12}-2p_{44})/(p_{11}+p_{12}+2p_{44})$ and thus evaluating p_{44}/p_{11}. The direction of the incident light beam in all cases was along [001]. The direction of the ultrasonic wave-normal, the appropriate ratio B/A and other details are given in Table I. The graphs of cot $(45°+\gamma)$ *vs* I in the three cases are shown in Fig 1.

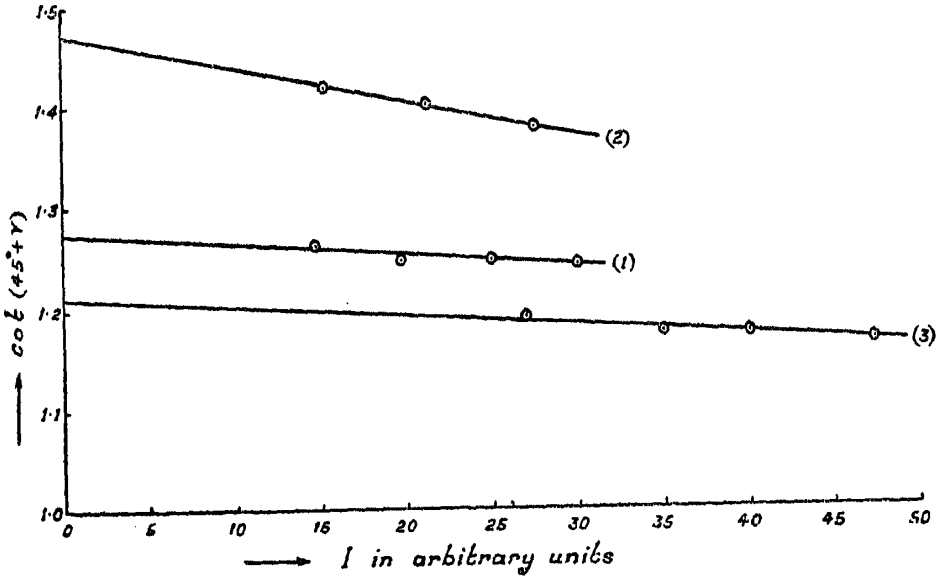

Table I

No.	Direction of ultrasonic wave-normal	Theoretical expression for B/A	Measured value of cot $(45°+\gamma)$	2α	B/A from eq. (1)
1	[100]	p_{13}/p_{11}	1·275	20°	1·285
2	[010]	p_{12}/p_{11}	1·47	20°	1·49
3	[110]	$\frac{p_{11}+\frac{1}{2}(p_{12}+p_{13})-2p_{44}}{p_{11}+\frac{1}{2}(p_{12}+p_{13})+2p_{44}}$	1·21	14°	1·215

We thus have

$$p_{12}/p_{11}=1\cdot49,\ p_{13}/p_{11}=1\cdot285,\ p_{44}/p_{11}=-0\cdot115.$$

In addition to enabling one to obtain p_{11}, p_{12}, p_{13} separately from measurements of relative retardation, the above ratios are also verified to

be consistent with the measurements of Ramachandran and Chandrasekharan, as will be seen from Table II.

TABLE II

Ratio	Static method	Ultrasonic method
$(p_{11}-p_{12})/(p_{11}-p_{13})$	1·68	1·72
$p_{44}/(p_{11}-p_{12})$ ..	0·22	0·23

SUMMARY

Details are given of the theory and technique of measuring photoelastic constants in optically active cubic crystals from ultrasonic diffraction of light. The results thus obtained, in conjunction with the measurements of relative path retardation produced by stressing the crystal, enable one to obtain all the four constants independently.

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