ON A CURIOUS SOLUTION OF RELATIVISTIC FIELD EQUATIONS

In the course of a recent investigation we have come across a metric which is Riemannian (non-flat) by virtue of the condition,

$$B_{\epsilon\mu\nu\sigma} \neq 0,$$

which satisfies the field equations of gravitation for empty space, *viz.*,

$$G_{\mu\nu} = 0.$$

which is free from similarities and for which the pseudo-tensor density of gravitational energy and momentum is *everywhere* zero, that is,

$$t_{\mu}^{\nu} = 0.$$

The metric is

$$ds^2 = -dx^2 (1+kt)^p - dy^2 (1+kt)^q - dz^2 (1+kt)^r + dt^2,$$

where k is an arbitrary constant and p, q, r are constants subject to

$$p + q + r = 2, pq + qr + rp = 0.$$

It follows that if p, q, r are real they must lie between 2 and $-{}^2/_3$. If $q = r = 0$, the space-time becomes flat. One particular case of interest is $p = -{}^2/_3$, $q = r = {}^4/_3$. It is not correct to say that the metric gives a flat space-time when $t = 0$ because the surviving components of the curvature tensor, B_{4114}, B_{4224}, etc., are non-zero even when $t = 0$.

Either as a cosmological model, or as a transitional model for a finite portion of space, that is something like a vacuum pocket into which matter is rushing from the surrounding portions of an extra-galactic nebula, the asymmetric field given by the above metric deserves consideration. We have investigated all possible line-elements for which $g_{\mu\nu} = 0$, $\mu \neq \nu$ and the conditions laid down in the beginning are satisfied. The most general solution is one in which the four surviving g's are certain functions of one and the same variable. But the above solution is the only one we have found worth reporting. In an investigation like this, while there is the danger of discovering a solution of no physical interest, after much mathematical ado, one cannot at the same time overlook the possibility of discovering new gravitational situations.

V. V. NARLIKAR.
K. R. KARMARKAR.
Benares Hindu University,
February 8, 1946.