THE CONSISTENCY OF EINSTEIN'S NEW RELATIVITY WITH THE GEODESIC POSTULATE

EINSTEIN, INFELD and HOFFMANN[1] have recently obtained a solution of the problem of n bodies from the field equations

$$G_{\mu\nu} = 0, \qquad \ldots \qquad \ldots \quad (1)$$

no use being made either of the geodesic postulate or of the energy-momentum tensor. It is well-known that (1) stands for only six independent equations. Four conditions can therefore be chosen so that the co-ordinate system is fixed and all the ten components of the metric tensor, $g_{\mu\nu}$, are known. The procedure of these authors is to build up $g_{\mu\nu}$ to higher degrees of approximation in stages contravening the four co-ordinate conditions by introducing $4\ n$ functions. When these functions are put equal to zero the equations (1) are satisfied and the $4\ n$ equations so obtained reduce substantially to the required $3\ n$ equations of motion. A full exposition of this method has been given elsewhere.[2] For two particles of masses m_1 and m_2, separated by a distance r, at η^m, ξ^m at time t the equations are of the type

$$\ddot{\eta}^m - m_2 \frac{\partial(1/r)}{\partial \eta^m} = m_2 \Big\{ \Big[\dot{\eta}^s \dot{\eta}^s + \frac{3}{2} \dot{\xi}^s \dot{\xi}^s - 4 \dot{\eta}^s \dot{\xi}^s - \frac{4 m_2}{r} - \frac{5 m_1}{r} \Big] \frac{\partial}{\partial \eta^m} \Big(\frac{1}{r} \Big) + \Big[4 \dot{\eta}^s \Big(\dot{\xi}^m - \dot{\eta}^m \Big) + 3 \dot{\eta}^m \dot{\xi}^s - 4 \dot{\xi}^s \dot{\xi}^m \Big] \frac{\partial}{\partial \eta^s} \Big(\frac{1}{r} \Big) + \frac{1}{2} \frac{\partial^3 r}{\partial \eta^m \partial \eta^s \partial \eta^t} \dot{\xi}^s \dot{\xi}^t \Big\} \qquad \ldots \qquad \ldots \quad (2)$$

Here m, s, t are suffixes running over the values 1, 2, 3, and the dummy-suffix convention is valid for them. A dot denotes as usual a differentiation with regard to t. The last equation gives the motion of m_1. In it m_1 appears only in one term on the right-hand side. If we put $m_1 = 0$ in (2) we get one term less and the equations of motion of a body of negligible mass are obtained.

The motion of a body of negligible mass is derived here without the use of the geodesic postulate; the geodesics of the field of m_1 and m_2 can also be obtained in the limiting case $m_1 = 0$. The question in which one is interested is this. Will the equations of motion for the case $m_1 = 0$ as derived from (2) be identical with the corresponding equations derived from the geodesic postulate applied to the field satisfying (1)? If one studies the procedure of Einstein and his collaborators there is nothing to indicate that the two should be identical; and in fact their work is guided by the supposition that the two results need not be identical. On carrying out the necessary calculations we obtain the surprising result that the equations of motion of Einstein's new relativity such as (2) are fully in accord with the geodesic postulate at least up to the second order of the masses. The calculations in question are lengthy and they will be published elsewhere. It looks as if the result is not accidental for the number of terms involved in the equations is large. The two methods of deriving the equations, although so different apparently, might be logically interconnected.[3]

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[1] Einstein, Infeld and Hoffmann, *Ann. Math.*, 1938, **65**, 5, 39.

[2] Narlikar, V. V., *J. Bombay Univ.*, 1939, **51**, 8.

[3] Narlikar, V. V., and Singh, J., *Phil. Mag.*, 1937, **628**, 23.