DIFFRACTION CORONÆ DUE TO NON-SPHERICAL PARTICLES

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1. *Introduction*

IT is well known that if a number of opaque spherical particles are interposed in the path of a beam of light, they will give a corona consisting of bright and dark rings. To illustrate the formation of such coronæ, it is usual to employ lycopodium powder dusted on a glass plate which produces two or three bright rings. If, however, the lycopodium spores are observed under a microscope, it is found that their shape is not at all spherical. The shape can best be described as a tetrahedron with a spherical cap. In spite of this deviation from sphericity, it is remarkable that the lycopodium particles can give clear rings, the sizes of which can be verified to obey the circular disk formulæ fairly well.

It is commonly supposed that the formation of the rings is a sort of average effect and that different portions of the particles give rise to different sizes for the rings. Thus, if the shape of the particles does not differ from a sphere by a large amount, then these rings will be of nearly the same size, so that they can be observed. That this explanation is not correct will be seen from the following experiment.

Spores of "*pinus longifolia*" give a corona consisting of at least two bright rings. A photograph of the corona, together with a microphotograph of the spores, magnified about a hundred times, are reproduced in Fig. 4, Plate I. It will be seen from the photomicrograph that the shape of these spores does not at all approach that of a sphere. In fact, their maximum dimension is about double the minimum. Hence, if the formation of the corona is just an average effect, then no ring system must be visible. Actually, rings are visible, although they are not very clear on account of a large background intensity. It is therefore of interest to examine under what conditions a non-spherical particle can give rise to a visible ring system.

2. Diffraction Pattern of a Non-spherical Particle

In order to discuss the above problem, one has to consider somewhat in detail, the formation of a Fraunhofer diffraction pattern. Considering an aperture of arbitrary shape, and neglecting, as is generally done for small angle diffraction, the obliquity factor, it can be shown that the effect of the entire aperture at any point, other than the exact focus, reduces to that of a line distribution of light sources along its edge. This has been discussed by Rubinowicz (1917, 1924) and Laue (1936). But the result can be derived in a simple manner, and the simple derivation is given here for the sake of completeness.

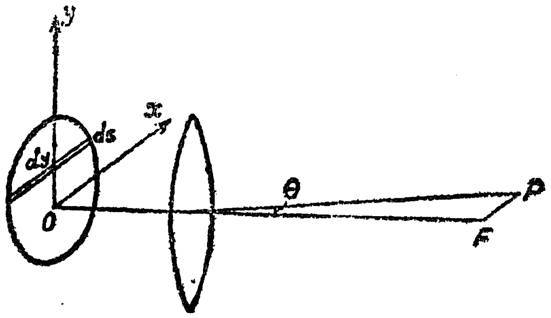

FIG. 1

Suppose that F is the focus of the lens (Fig. 1) and that it is required to find the intensity at a point P in the focal plane, at which the waves diffracted in a direction making an angle θ with that of the incident wave are brought to a focus. Take a set of rectangular axes Ox and Oy in the aperture, so that the x-axis is parallel to FP. If the incident wave is represented by sin Z (amplitude unity), then the amplitude at the point P is

$$\frac{1}{\lambda f} \iint \sin (Z - 2\pi x \sin \theta/\lambda)\, dx\, dy,$$

where the integration is performed over the whole aperture. Now, divide the aperture into a number of strips by means of lines parallel to the x-axis, the width of each strip being equal to dy. Integrating the above expression between the limits x_1 and x_2 which correspond to the extremities of one of these strips, the integral becomes

$$\frac{1}{2\pi f \sin \theta} \int [\cos (Z - 2\pi x \sin \theta/\lambda)]_{x_1}^{x_2}\, dy.$$

Now, calling the portion of the boundary intercepted by this strip as ds, $dy = ds \sin \phi$, where ϕ is the angle made by ds with the plane of diffraction, *i.e.*, with the x-axis. Since the integral of dy vanishes when one makes a complete circuit round the edge, *i.e.*, $\oint dy = \oint ds \sin \phi = 0$, sin ϕ must be considered positive at one end of the strip, and negative at the other end. Hence, the effect of the entire aperture in the direction θ may be written as

$$\frac{1}{2\pi f \sin\theta} \oint \cos\left(Z - 2\pi x \sin\theta/\lambda\right) \sin\phi \, ds.$$

This line integral is taken round the boundary of the aperture, and in it x and $\sin\phi$ are to be regarded as functions of s. Hence, the resultant intensity is

$$I = (P^2 + Q^2)/4\pi^2 f^2 \sin^2\theta, \text{ where}$$

$$P = \oint \cos\left(2\pi x \sin\theta/\lambda\right) \sin\phi \, ds \text{ and}$$

$$Q = \oint \sin\left(2\pi x \sin\theta/\lambda\right) \sin\phi \, ds.$$

Now, it is seen from the above formulæ that the parts of the edge for which ϕ is zero, that is those which run parallel to the x-axis do not contribute anything to the intensity at the chosen point of observation, P. Indeed, we may go further, and state that the only sensible contributions are those made by parts of the edge running approximately parallel to the y-axis, for which ϕ is nearly a right angle. For, the co-ordinate x, and therefore also the phase of the radiations, are stationary for these points, which may be designated as the 'poles' of the point of observation. Thus, in any case in which the aperture has a curved boundary without singularities, the line integral may be replaced by point sources placed at such poles, of which naturally there must be at least two. The diffraction pattern would then be regarded as the interferences of the radiation from these point sources. Geometrically, the positions of these sources are such that the tangents to the boundary at them are parallel, and are all perpendicular to the plane of diffraction. Such points may be said to be "opposed", and they are responsible for the diffracted intensity in the direction considered. The existence of these poles has been observed experimentally by Banerjee (1919) and by Mitra (1919, 1920).

The problem thus reduces to finding such opposed points for a non-spherical particle. Then, the diffraction pattern will approximate to that given by a circular aperture, whose diameter is equal to the distance between these opposed points. It must be noted that when the diffraction corona is given by a large number of particles, the particles themselves are oriented in all directions, so that the pattern will always be circular. Hence, if the distance between the opposed points is a constant over a small region of the boundary, then the rings corresponding to this distance will stand out from the background intensity, the clarity of the rings being greater, the larger the region over which this distance is a constant. If, however, the shape of the particle is completely arbitrary, and the distance is not a constant even for a small region, then no rings will be visible. Thus, the condition for the formation of a ring system is that the distance between the opposed points is a constant over an appreciable region of the boundary.

3. *The Case of Pinus longifolia*

The above criterion for the formation of a ring system by non-spherical particles directly explains why spores of *Pinus longifolia* must give rise to visible rings. The shape of this particle, as already mentioned, deviates very much from a sphere, the maximum and minimum dimensions being about 55 μ and 30 μ, so that if the corona is a sort of average effect, then one should expect no rings at all. However, on the idea of opposed point sources, the formation of the rings is easily understood.

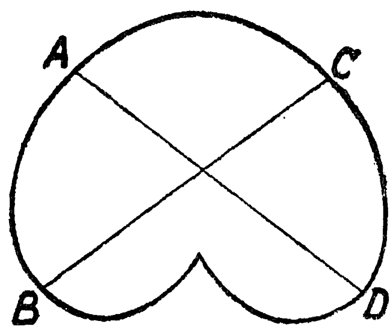

FIG. 2. Shape of *Pinus longifolia*

If one examines the shape of the particle as presented to the beam of light, it is found that it is mostly as shown in Fig. 2. In this cross-section, there is an appreciable portion of the boundary (AB and CD) over which the distance between the two opposed points is very nearly a constant. This, together with the fact that the particles are all orientated at random, is the reason why a visible system of rings is produced. Since the effective width varies rapidly in the other portions of the particle, the background intensity is quite large.

That this explanation of the formation of the rings is correct was verified by measurement. The diameters of the various rings were measured, and knowing the focal length, their angular radii were calculated. The distance AD or BC was determined from the microphotograph, as an average of a large number of measurements. This was found to be 55 μ. Taking this value to be the diameter of a circular aperture, the angular radii of the rings in its diffraction pattern were calculated. These are tabulated in Table I below, and it will be seen from it that the agreement between the value calculated from the diameter, and the one measured is quite close, showing that the rings correspond to the distance AD (or BC), as is to be expected from the theory.

TABLE I

Ring	Angular radius θ	
	Calculated	Measured
1st Min.	0·0121	0·0124
1st Max.	0·0163	0·0169
2nd Min.	0·0223	0·0221
2nd Max.	0·0266	0·0274

4. *Coronæ Produced by Lycopodium*

As already stated, another typical example of a non-spherical particle is *Lycopodium*. The shape of a *Lycopodium* spore can best be described as a tetrahedron with a spherical cap. It is bounded by four sides, three of which are triangular planes while the fourth is spherical and convex. Hence, the cross-section presented by the *Lycopodium* spore to the incident beam of light may be approximately a circle, when the convex face is towards it, or a sector of circle, when one of the plane sides faces the beam. The latter (Fig. 3) is of more frequent occurrence, since the spore can rest on one of its plane sides. In fact, if the microphotograph is examined, it will be found that the circular shape is only of rare occurrence, while the commonest shape for the cross-section is that shown in Fig. 3. Intermediate shapes may also occur, but they resemble Fig. 3 in having a segment of a circle AB, and two lines OA and OB, which may be unequal.

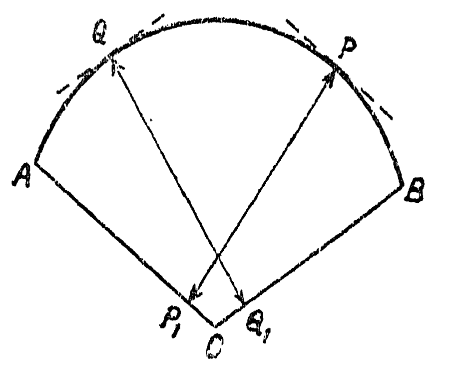

FIG. 3 Cross-Section of Lycopodium Spore

Let us therefore consider what the positions of the opposed points are for the shape shown in Fig. 3. The angle AOB is invariably greater than a right angle, so that there are points P and Q in the segment AB at which the tangents are parallel to OA and OB respectively. Hence, the opposed sources are P and the whole line OA, and Q and the line OB. The distance between these are PP′ and QQ′, which are generally very nearly equal to each other. Hence the distance between the opposed sources is PP′ or QQ′. Obviously, these opposed sources cover a very large portion of the boundary, so that the general background intensity given by the rest of the boundary is quite small. This is why the patterns with *Lycopodium* are much clearer than with *Pinus longifolia*.

There are, of course, variations in the size of the spores. These will give rise to a background intensity, which is discussed in the next section.

Thus, the ring system will correspond to the distance PP′, (or QQ′) *i.e.*, the average value of PP′ for all the particles. That this explanation of the formation of the rings is correct was verified as follows. The distances PP′ and QQ′ were actually measured from the microphotograph my means of a travelling microscope. The negative was rotated so as to bring OA (or OB) parallel to the vertical cross-wire, and the microscope was adjusted so that the cross-wire coincided with it. The microscope was then moved until the cross-wire was tangential to the curved boundary AB. The distance moved by the microscope gave a measure of PP′, and since the magnification was known, the actual distance could be calculated. Both PP′ and QQ′ were measured for a large number of spores (both when they were nearly equal, and otherwise), and the average was determined.

This procedure was employed for two types of *Lycopodium*, *Lycopodium clavatum* and *Lycopodium bisdepuratum*, supplied by Merck, which were available in the laboratory. The values obtained were, for the *bisdepuratum* 33·1 μ and for the *clavatum* 34·4 μ. Taking this to be the diameter of the equivalent circular disk, the theoretical angular radii of the rings in

TABLE II

Ring	*Bisdepuratum*		Ring	*Clavatum*	
	Angular radius			Angular radius	
	Theoretical	Experimental		Theoretical	Experimental
1st Min. ..	0·0206	0·0202	1st Min. ..	0·0194	0·0202
1st Max. ..	0·0277	0·0280	1st Max. ..	0·0260	0·0275
2nd Min. ..	0·0378	0·0361	2nd Min. ..	0·0355	0·0355
2nd Max. ..	0·0454	0·0451	2nd Max. ..	0·0426	0·0436
3rd Min. ..	0·0548	0·0533			
3rd Max. ..	0·0626	0·0615			

the corona were computed. The actual coronæ were also photographed, and the experimental values of these were calculated knowing the focal length of the lens. Enlarged pictures of the coronæ and also the microphotographs (magnification 94) are reproduced in Fig. 5, Plate I.. The theoretical and experimental values of the angular radii are tabulated in Table II below. The agreement between the two is satisfactory, showing that the idea of the opposed points is true for *Lycopodium* also.

5. *Coronæ due to Particles of Variable Size*

It was mentioned in the previous section that variations in the size of the particles will affect the background intensity. The explanation of this is as follows. If there are a large number of spherical particles, whose size is not a constant, but varies slightly, then, the rings given by the different particles will not all be of the same size, but will be different. Consequently, the rings will not be perfectly bright and dark, but there will be some intensity in the dark portions also. This lack of darkness of the dark rings will be greater, the larger the variation in the size of the particles. If the variation is very large, the rings will be blotted out, and there will only be a continuous decrease of the intensity from the centre outwards.

It is also obvious that if the variation in the size of the particles is small then the position of the rings will very nearly correspond to those in the corona due to a sphere of the average size, but their intensities will differ from those in it.

The above results find a confirmation if one studies the coronæ given by the two types of *Lycopodium*. The most striking thing that is observed on looking at the photographs in Fig. 5 is the greater clarity and contrast of the rings given by the *bisdepuratum*. In fact, under identical conditions, three bright rings were visible in the negative of the corona of the *bisdepuratum*, while only two were seen in that of the *clavatum*. Also, the dark rings are darker, and the ring system clearer with the *bisdepuratum* as can be seen from Fig. 5.

The cause for this was investigated. A study of the microphotographs and a large number of observations under the microscope showed that the difference cannot be attributed to any difference in the shape of the particles, for they were both very nearly of the same shape. It was finally found that it must be ascribed to the difference in the range of particle-sizes in the two types. A large number of measurements of the size of the spores showed that the variation in the size was comparatively smaller for the *bisdepuratum* than for the other type. For the former, the size varied mostly between 30 μ and 35 μ, with only very few outside these limits. On the other hand, for the latter, the variation was from 30 μ to 40 μ. This will be clear from the study of the following typical set of readings for the distance between the opposed points.

Clavatum.—Size in μ 32, 34, 31, 37, 37, 32, 33, 33, 30, 34,
37, 33, 34, 34, 35, 36, 37, 39, 36, 37.

Bisdepuratum.—Size in μ 33, 34, 32, 31, 35, 34, 32, 34, 35, 34,
31, 34, 33, 33, 30, 33, 33, 32, 33, 34.

The greater range of variation in the size of the *clavatum* spores clearly explains why the corona is less distinct with it than with the *bisdepuratum*.

The second result, namely that the radii of the rings correspond to a size which is the average has already been verified in the last section.

My best thanks are due to Prof. Sir C. V. Raman for the suggestion of the problem and for the kind guidance he gave me during the investigation.

Summary

It is pointed out that the Fraunhofer diffraction due to an obstacle of arbitrary shape can be replaced by that given by a linear distribution of sources along its boundary. Most of the intensity at any point in the pattern can again be supposed to originate from a finite number (usually two) of point sources, called "opposed points" or "poles", situated on the boundary. In this way, if there are a large number of non-spherical particles distributed at random, then a ring system will be formed whose size will correspond to the distance between the opposed points. If this distance is a constant over an appreciable region of the boundary, then the rings will stand out from the background intensity. This explanation of the formation of the rings, has been verified using spores of *Pinus longifolia* and of *Lycopodium*. It is also shown that variations in the size of the particles affect the clarity of the rings detrimentally, the rings becoming less and less clear as the range of particle sizes increases. This is also verified by using two types of *Lycopodium*.

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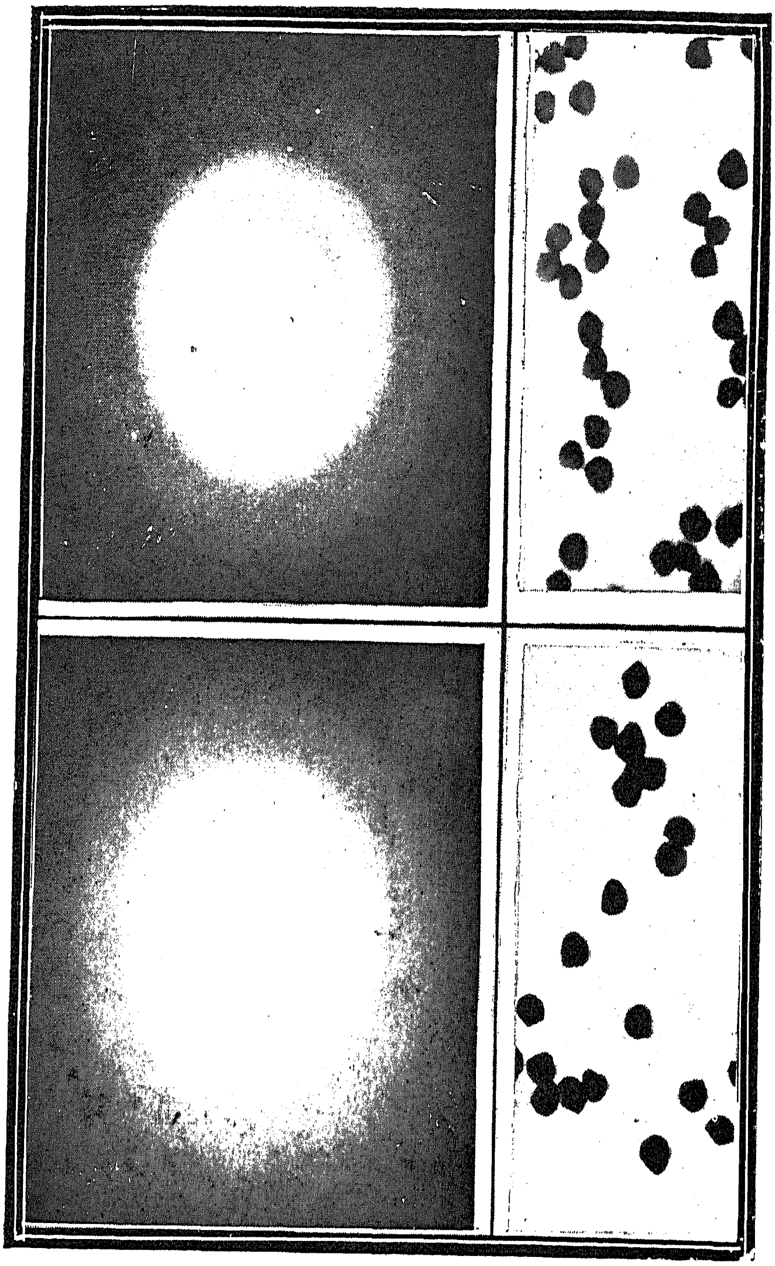

(a) (b)

Fig. 5. Corona and photomicrograph of
(a) *Lycopodium clavatum*; (b) *Lycopodium Bisdepuratum*

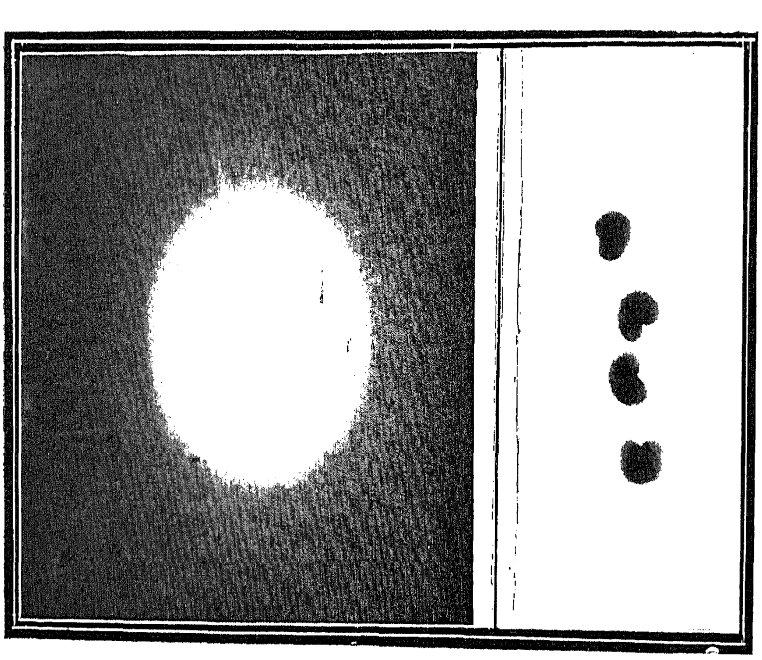

Fig. 4. Corona and photomicrograph
of spores of *Pinus longifolia*