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Duality in physical sciences and beyond

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Abstract. An extended meaning of duality is suggested in the context of development of major themes in physical sciences since Newton. In such a generalization, *five* distinct aspects of duality are sought to be identified and illustrated through concrete examples drawn from various physical concepts, old and new. These are (i) reciprocity, (ii) parallelism, (iii) alternative formulation, (iv) unification and (v) measurement incompatibility. Bohr's view of duality and the Copenhagen Interpretation are discussed briefly in this context. Finally, duality aspects beyond physics are briefly touched upon, the philosophical link being provided by Bohr's Complementarity Principle on the one hand, and recent attempts (notably by Capra) to draw suggestive parallels between modern science and Eastern mysticism on the other.

Keywords. Duality; reciprocity; harmonic oscillator; parallelism; unification; Bohr; complementarity; modern science; mysticism.

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"Be in truth eternal, beyond earthly opposites"—Bhagwat Gita.

1. Introduction

The year 1985–86 marks the dual event comprising sixty years of quantum mechanics, and Niels Bohr's birth centenary. It has just completed a second dual event, *viz* the respective Golden Jubilees of two principal Science Academies in the country. It is therefore a happy coincidence that this year also incorporates the eightieth birthday of Daulat Singh Kothari, who has played a significant role in steering the course of science in Independent India, more especially the "value system" that goes with it. Indeed, on several occasions in the recent past (Kothari 1980, 1985) he has sought to illustrate his concept of the value system in science, essentially through the theme of "physique-psyche" duality. The same theme is also central to Bohr's complementarity principle which acquires a renewed significance in his birth centenary year. Both these reasons are compelling enough, *a priori*, to warrant the theme of duality for the theme of this article, subject to certain severe limitations of a practical nature. For, in the context of quantum mechanics duality essentially stands for the Copenhagen interpretation on which so much rich literature exists already (including the famous Bohr-Einstein controversy) that it would be futile to attempt yet another essay on the subject without an in-depth preparation in advance. On the other hand, to do even a semblance of justice to a topic as profound as duality at the level of the mind, would necessitate delving into the great depths of philosophy, with the rather "poor" equipment that is generally available to an ordinary working physicist trying to cater to the tastes of his compatriots. For both these reasons I have chosen to rely on a somewhat extended—perhaps diluted—definition of duality to bring out the diverse manifestations of the word, mostly in the

The author felicitates Prof. D S Kothari on his eightieth birthday and dedicate this paper to him on this occasion.

domain of the physical sciences and partly beyond, in a more or less pedagogical spirit.

If one takes a mere dictionary view of duality, it is possible to think of a fairly large number of items which can be naturally associated in pairs. Apart from the very trivial ones which are obviously uninteresting, there exist many pairs whose inter-relations are more subtle than mere synonym-antonym types. After all, was it not Niels Bohr again who had once drawn attention to two kinds of truth, (1) the simple truth whose negation is obviously false, and (2) the deep truth whose negation is another deep truth? In the two adjoining tables, I have attempted to draw up partial lists of such related items, the first from a cross-section of physical concepts, personalities and phenomena, while in the second I have taken the liberty to protrude beyond physics, somewhat presumptuously but in keeping with the limited objectives of this paper.

These tables should serve to illustrate the fact that though duality in an extended sense conveys different meanings in different situations, it has recurrently played a key role in the understanding of a vast complex of phenomena in widely different contexts ranging from the physical to the biological sciences and even the dimensions of abstract thought. This it has achieved by evincing an underlying unity between the "dual partners", one which has often been described as a sort of symmetry principle governing the connection. The only other physical concept with a broad-spectrum jurisdiction is perhaps that of "shell-structure" which has so far spanned our "understanding" of the successive stages (at least 3, possibly 4) of compositeness of matter.

Table 1. Dual partners (physics items).

I-Member	*II-Member*	*Legend/Ref.*
Action (A)	Reaction (R)	Newton III
Coordinates (q, θ) Time (t)	Momenta (p, J) Energy (E)	Hamilton, Jacobi
Electricity	Magnetism	Faraday-Maxwell
E; **B** Pressure; stress	**D**; **H** Volume; strain	Cause vs effect
Time (t) Energy (E) Energy (E)	Space (r) Momentum (p) Mass (m)	Einstein's relativity
Fermat principle $\delta \int \mu \, ds = 0$	Maupertius principle $\delta \int p ds = 0$	Parallelism (optics vs mechanics)
e.m. wave	Photon	Planck
e^--wave	Electron	de Broglie
Schrödinger (wave mech)	Heisenberg (matrix mech)	Alternative formulations
Feynman ( )⁓⁓( ) Path integral	Schwinger ($\delta A = 0$) Source theory	language- duality
Resonances (S-channel)	Exchanges (t, u channels)	F.E.S.R. duality (Dolen *et al* 1968; Logunov *et al* 1967; Veneziano 1968)
Observer (apparatus)	Observable (atomic system)	Bohr's duality
Boson	Fermion	Supersymmetry

Table 2. Dual partners (beyond physics).

I-Member	*II-Member*
Theory (interpretation)	Experiment (data)
Subject	Object
Subjective conjectures	Objective analysis
Mind (consciousness)	Brain (nerve complex)
Psyche	Physique
Religion (philosophy)	Science (pragmatism)
Heart (emotion)	Head (reason)
Mysticism (Eastern)	Modern science (Western)
Purusha (Shiva)	Prakriti (Shakti)
Bhakti (faith)	Gyan (reasoned knowledge)
Yin	Yang
Faraday-Bohr (intuitive knowledge)	Maxwell-Dirac (mathematical precision)
Private science (S_1)* (creative instincts)	Public science (S_2)* (expository skills)

* G Holton's nomenclature (Holton 1973)

To illustrate the nature of the underlying "symmetry" implied by a generalized interpretation of duality it is useful to discuss a few items listed in table 1, classified under five different heads representing as many aspects of this rich concept.

2. Reciprocity (mutuality) aspect

For certain situations, the mathematical equations suggest a sort of mutuality or reciprocal relationship between the dual partners. We list 3 examples:

$$\mathbf{A} = -\mathbf{R} \qquad \text{(Newton III)}. \tag{1}$$

The symmetry implied by this relation is that the mutual potential energy of any two particles is a function of their relative distance and not of their absolute positions.

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}; \quad \nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}. \tag{2}$$

(Maxwell's equations in free space)

Indeed such a conjectured relationship between **E** and **B** was apparently the theoretical motivation behind Maxwell's unique proposal for the "displacement current" which brings out the full symmetry of the mutual interdependence of the **E** and **B** vectors, but was to be detected only much later.

$$\partial H/\partial p = \mathrm{d}q/\mathrm{d}t, \quad \partial H/\partial q = -\mathrm{d}p/\mathrm{d}t. \tag{3}$$

These equations, (whose physical content is strictly confined to Newton's laws of motion) bring out equally convincingly the mutuality of the roles of q- and p-variables, a feat achieved through Hamilton's penetrating formulation of the laws of Newtonian mechanics which was subsequently to pave the "golden road" to quantum mechanics at the hands of Heisenberg and Schrödinger.

More examples of mutually interdependent pairs may be found from the mathematical theory of transforms (Fourier, Hilbert) which revealed this type of interdependence in the most succinct manner, together with their diverse physical applications. Thus while the theory of Fourier transforms is at the root of duality between coordinate space (x) and wave number space (k), Hilbert transforms illustrate the value of "analyticity" in the complex plane in bringing out the inter-relationships among the dual partners represented by the real and imaginary parts of a scattering amplitude. Many such examples may be cited in support of the so-called reciprocity principle (RP) of Born (1949) concerning certain pairs of attributes in a physical system whose intimate connections with each other imply reciprocal relationships between them. In some systems the relationship is so strong that it exhibits a high degree of symmetry; e.g., the exactly symmetrical appearance of the (q, p) variables in a harmonic oscillator led Yukawa (1953) to invoke Born's RP to propose his non-local field theory (NLFT), the non-locality arising from the appearance of a *fundamental length* which necessarily characterizes the picture of a harmonic oscillator (as the "price" of this q-p symmetry). The NLFT is of course not an accepted form of wisdom in field theory (there has all along been a predilection for LFT), but if it at all gets to be taken seriously, the harmonic oscillator is bound to provide the main motivating force, and ipso facto, the RP of Born.

3. Parallelism (analogy) aspect

While the above examples illustrate the "reciprocity" aspect of duality, its "analogy" or parallelism aspect is best exemplified by Fermat's principle for optics, vs Maupertius principle for mechanics:

$$\delta \int \mu ds = 0 \Leftrightarrow \delta \int p ds = 0.$$

As is well known, this close parallelism between the respective laws of two very different branches of physics was not only suggestive of the underlying unity of physical laws but was to play a crucial role in the eventual Schrödinger formulation of wave mechanics from its classical "ray" ($h \to 0$) picture, the latter pair serving as the mechanical analog of wave and ray optics respectively.

A fine example of the parallelism aspect of duality is represented by a profound correspondence between classical and quantum mechanics in the form

$$\{A, B\} \Leftrightarrow \frac{1}{i\hbar}[A, B]$$

discovered by Dirac while taking one of his long evening walks during his early Cambridge days (Dirac 1968).

4. Alternative formulation aspect

Still another feature of duality concerns the formal equivalence of certain alternative formulations apparently unrelated to each other, yet having the same physical content. The Heisenberg vs Schrödinger formalisms of quantum mechanics represent precisely such a situation. Though their equivalence is now mere textbook material, their apparent dissimilarity at the initial stages of formulation had helped catalyze the polarization of two strong schools of thought (Holton 1973), viz (i) Heisenberg's algebraic approach emphasizing the corpuscular aspect characterized by discontinuity, versus (ii) Schrödinger's analytic approach in terms of a "wave" equation stressing the element of continuity. Schrödinger's admirers included virtually all the stalwarts (Planck, Sommerfeld, Born .... and of course Einstein) while poor Heisenberg seemed at that time to have only Niels Bohr (and partly Pauli) on his side. The extent of the polarization may be judged by some typical reactions (Holton 1973): The physical part of Schrödinger's theory appeared "disgusting" to Heisenberg, while the former was "discouraged" if not "repelled" by the latter's theory. In an implied support to Schrödinger and admonition to Heisenberg, Einstein declared "that one has to solve the quantum by giving up the continuum, I do not believe". A modern counterpart of a similar dichotomy is contained in the Feynman (1949) (diagrammatic) versus Tomonaga-Schwinger (analytic) (Tomonaga 1946; Schwinger 1949) formulations of covariant QED, and still later in their respective semi-classical formulations of field theory (path-integral method (Feynmann and Hibbs 1965) versus source theory (Schwinger 1973)). At a more impersonal level, a good example of such a "complementary" aspect of duality is afforded by the empirical finite energy sum rules (FESR) wherein the contributions to a high energy scattering amplitude by the direct (s-channel) resonances are supposed to saturate the effect of the corresponding contributions from the exchange (t, u) channels (Dolen *et al* 1968; Logunov *et al* 1967). This form of duality which gave rise to the so-called Veneziano model (Veneziano 1968) in the late sixties received considerable refinement at various hands through diagrammatic methods (quark or duality diagrams, (Harari 1969; Rosner 1969)) on the one hand, and analytic/operator methods (Fubini *et al* 1969) on the other, in both of which the mathematical vehicle for the basic underlying symmetry seemed once again to be provided by the language of the harmonic oscillator. Nambu's string model (Nambu 1970) which gave the first concrete realization of this kind of duality, was the forerunner of the modern string and super string theories.

5. Synthesis (unification) aspect

A major aspect of duality, emphasizing the synthesis of certain pairs of physical concepts is represented by relativity theory which provides an integrated view of the space-time continuum, as opposed to the Newtonian "partition" of their respective foundations. The parallel concepts of wave-vector-frequency, and momentum-energy 4-vectors are linked to space-time by Fourier transformation and canonical conjugation respectively, while their direct link is provided by quantum theory:

$$\mathbf{p} = \hbar\mathbf{k}, \quad E = \hbar\omega.$$

The situation is illustrated in figure 1.

The most profound physical manifestation of the synthesis aspect of relativistic duality is the celebrated mass-energy relation which establishes the formal equivalance of two entirely unrelated Newtonian concepts. In a very similar spirit some recent theories (Pati-Salam 1973; Georgi and Glashow 1974; Fritz and Minkowski 1975) of collective baryon-lepton conservation in the sense of B-L = const., replacing their

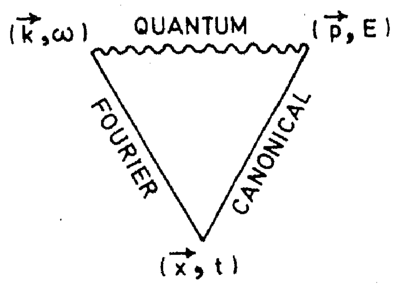

Figure 1. A three-way interlinkage.

individual identities (B = const, L = const) have become quite fashionable. These have their origin in the so-called grand unification principle for all forces (weak, e.m., strong), and they predict proton decay in varying degrees. This principle which still awaits experimental confirmation is an offshoot of the firmly established Glashow-Weinberg-Salam (Glashow 1961; Weinberg 1967; Salam 1968) electroweak theory which is a fine example of unification of two apparently distinct forces on a higher scale of energies.

Perhaps a more dramatic, and historically preceding, manifestation of the unification aspect of duality is the prediction of antimatter, as a result of the successful marriage between the dual partners of relativity and quantum theory under the auspices of Dirac who effectively showed that such a marriage is not possible at the level of single particles but only in the collective context of particles and antiparticles, in other words, in a field theory (Dyson 1952).

As a rather extreme example of this last aspect of duality it is tempting to mention the comparatively recent theories of supersymmetry (Wess and Zumino 1974) purporting to project an integrated view of bosons and fermions hitherto believed to be two distinct fundamental species, totally unrelated to each other. The Bose-Fermi symmetry or SUSY as this new theory is called, has some highly attractive theoretical features such as an ability to cure some vexing problems of infinites (for hitherto unrenormalizable fields), but its predictions are yet to find direct experimental support. The theoretical investment in this field has been extremely rich in recent years, with allied developments on supergravity, superstrings and so on, in contrast to almost zero development on the 'dual' (experimental) front.

6. Measurement incompatibility aspect

We now come to a most significant aspect of duality which provoked Bohr to enunciate his famous complementarity principle, viz the incompatibility of measurements of certain pairs of dynamical variables, known as canonically conjugate pairs (as well as their derivatives). This is the celebrated uncertainty principle (UP) of Heisenberg, mathematically derivable from any consistent formulation of quantum mechanics (Heisenberg or Schrödinger), which expresses such incompatibility in the basic form

$$\Delta q \cdot \Delta p \geqslant \frac{1}{2}\hbar, \quad \text{if} \quad [q, p] = i\hbar,$$

or the more general form

$$\Delta A \cdot \Delta B \geqslant \frac{1}{2}\langle C \rangle, \quad \text{if} \quad [A, B] = iC.$$

Though a strict mathematical consequence of the tenets of quantum mechanics, the physical significance of the UP is profound enough to touch instantly on the philosophical plane. For one thing, it succinctly reveals the paradox of the wave-particle duality which, at the intrinsic physical level, is sought to be justified by the familiar considerations of wave packets, the two-slit experiment, and other gedanken experiments. Such elementary considerations are designed to break the psychological barrier faced by a typical physicist who would find it hard to accept the reality implied by mutually exclusive manifestations of matter. The only "reassuring" thing for him is perhaps the fact that a single experimental arrangement will reveal only one property at a time, and any attempt to discover the second property in the same breath will result in the destruction of the first, and vice versa. This limitation which transcends either the quality of the experimental apparatus or human ingenuity in designing the experiment, stems directly from the mutual interaction between the observer (apparatus) and the observable (the physical property under study). The effect of this apparatus—physical system interaction is negligible on a macroscopic entity (the apparatus) but non-trivial on a microscopic one (of atomic dimensions), so much so that an accurate measurement of one of its attributes precludes a simultaneous knowledge of another.

In this respect this last aspect of duality has no counterpart in the other four categories which were described in the foregoing without conscious reference to the "quantum" limitation, so that the measurement compatibility problem per se, had so far not been an issue in the corresponding descriptions. The incompatibility problem is a typical quantum effect and introduces an interesting duality situation arising out of the observer-observable interaction, which has again no classical analogue. A good example is afforded by the (**E**, **B**) pair of fields listed under category I (reciprocity) in the foregoing. In the quantum background, the measurement of these e.m. field quantities which play the role of observables is intimately linked with that of electric charges which play the role of 'apparatus' (test charges) even though in principle field and charge measurements are two distinct concepts. (The problem was first considered by Heisenberg, and in greater detail by Bohr and Rosenfeld (Bohr and Rosenfeld 1948)). The connection between the measurement fluctuations of these two entities is a good illustration of the observable (field)- apparatus (test charge) interaction which manifests itself at the quantum level (Bohr and Rosenfeld 1948). For the displacement of the test charge (as a result of measurement) produces a back reaction in two different ways, viz (i) the production of a radiation field (due to acceleration of the charge) and (ii) the creation of e^+e^- pairs. In both cases the average value of the reaction varies as the displacement. Though the latter can be compensated, e.g., through a suitable elastic device, or can otherwise be accounted for, there are quantum fluctuations about the average value, since the reaction is transmitted either by a finite number of photons, or by a finite number of e^+e^- pairs. And it is these latter fluctuations which limit the overall accuracy of the field and charge measurements.

7. Bohr's view of duality

Bohr's view of duality has directly to do with the problems of observer-observable interaction at the quantum level, on lines just illustrated in the foregoing. It was first put forward by Bohr in a lecture to the International Physics Congress at Como in

Commemoration of Volta in September 1927. In this he introduced the viewpoint of complementarity to reconcile the characteristic features of individual quantum phenomena with the observational problem "in this field of experience". In particular he emphasized "the impossibility of any sharp separation between the behaviour of atomic objects and their interaction with measuring instruments which serve to define the conditions under which the phenomena appear".

One might wonder what provoked Bohr to espouse this novel philosophy. He had earlier had a full innings shaping the destiny of atomic physics during the turbulent years following his revolutionary proposal on quantized orbits (in 1913), culminating in the Nobel recognition (1922). Being acutely aware of the foundational limitations of the "old atomic theory" he had been watching with obvious concern the subsequent theoretical developments leading to the "new quantum mechanics". In the twin formulation of Heisenberg's matrix-mechanics and Schrödinger's wave mechanics, he sensed the prospects of a practical resolution of the paradox represented by the successes of (continuum) classical electrodynamics on the one hand and the conception of (discrete) stationary states on the other. He had earlier been emphasizing this conceptual conflict in several forms, as if to prepare the physics community in advance for such an anticipated (?) development. And when it finally came in the garb of uncertainty relations, his philosophical mind was ready to abstract its true significance, transcending the mathematical barriers, and giving it a concrete shape in the form of the complementarity principle governing the interaction between (microscopic) observables and (macroscopic) apparatuses of observation.

8. The Copenhagen interpretation (after Stapp 1972)

To appreciate the essential points of Bohr's CP it is first useful to summarize the rules of measurement according to quantum theory. These consist in first preparing a physical system in a specified manner (A), represented by the wave function $\psi_A(x)$, and later "measured" in a specified manner (B), represented by the wave function $\psi_B(y)$. The variables x and y, termed degrees of freedom, characterize the microscopic systems being prepared and measured respectively. The specifications A and B, on the other hand, are couched in the laboratory (macroscopic) language. The transition from the prepared state $\psi_A(x)$ is effected through a "transformation function" $U(y, x)$ which depends on the types of systems prepared and measured respectively, but not on the particular wave functions ψ_A and ψ_B. This leads to the computation of the transition amplitude

$$\langle B|A\rangle = \int \psi_B^*(y)\, U(y, x)\, \psi_A(x)\, dy\, dx,$$

and hence to the probability

$$P(B, A) = |\langle B|A\rangle|^2,$$

that a B-measurement will result from an A-preparation of the physical system. This probability, according to quantum theory, is the predicted limit of the relative frequency of occurrence of the specified result, as the number of systems prepared and measured according to the specifications A and B respectively, goes to infinity. Note that in this definition, the corresponding wave functions of the preparing and measuring devices do not enter. The latter are described operationally in terms of macroscopic entities understood by the experimentalist and the technician.

How does one determine the transformation

$$A \rightarrow \psi_A, \qquad B \rightarrow \psi_B$$

which describe how the technician's preparation (A) of the macroscopic object and recognition (B) of macroscopic responses, gets translated into the mathematical function ψ_A, ψ_B for the corresponding microscopic systems? The problem of constructing this last mapping is the central problem of measurement in quantum theory for which two other approaches, other than the CI have been proposed; these have by and large got superceded by the latter.

The first approach was that of Von Neumann (1955) who advocated the use of wave functions for both the systems, termed the absolute-Ψ approach, whose adherents have included stalwarts like Wigner (1963), Everett (1957), Wheeler (1957) and de Witt (1970), and who are inclined to treat the wave functions of the macroscopic system on par with those of the microscopic ones, often with highly unorthodox consequences. The Copenhagen interpretation, on the other hand, rejects the proposition of an absolute wave function for the whole world and relies instead on the quantum wave function only for the microscopic systems and a simple classical picture for the macroscopic devices. The space-time dispositions of the latter are interpreted by the experimentalist as information about the corresponding microscopic systems, while the quantities $|\langle B|A\rangle|^2$ are themselves regarded as the probabilities of specific responses of the measuring devices under specified conditions. To that end the experimentalist merely *calibrates* his devices in order to handle a total of $N_A \times N_B$ quantities $|\langle B|A\rangle|^2$ in terms of the (much fewer) $N_A + N_B$ unknown functions ψ_A, ψ_B where N_A, N_B represent the different choices of the specifications, A, B respectively. In this way he builds up a catalogue of correspondence between the "experimental" quantities $\langle B|A\rangle$ and the "theoretical" wave functions $\psi_{A,B}$ of the A and B systems. It is this body of accumulated empirical knowledge that effectively serves for the mapping:

$$A \rightarrow \psi_A, \qquad B \rightarrow \psi_B$$

again from a pragmatic, but highly reliable, point of view.

The Copenhagen interpretation also rejects the second (hidden variable) point of view of the existence of "real particles" (localized objects, disturbances, singularities) which do not spread out like waves, so that the quantum probability functions represent the probabilities of such "real" particles being in specified regions. This philosophy has had powerful advocates like Popper and Bunge (1967) and Bohm (1952) (especially the latter) who would seek to reconcile it with the Schrödinger theory through the hypothesis that all particles (in the model universe) are so inexorably linked together (somewhat like Mach's principle?) that a disturbance to any particle is immediately transmitted to all others. In such a picture, the entire collection of "particles" acts like a single complex entity, at total variance with our familiar concept of a particle, since it seeks to transcend even the concept of casuality. However, according to Bell's theorem (Bell 1964), such an underlying reality is incompatible with the statistical prediction of quantum mechanics, if it is sought to be governed by causal dynamical relationships among its spatially separated parts, unless of course certain unknown (meta-physical) connections are envisaged, in which case there would be of course no testable dynamical consequences. (The situation is somewhat reminiscent of several, by now metaphysical, theories which were once advanced to protect the Ether concept, before the advent of special relativity.) Limited experimental evidence so far also appears to rule out such "hidden variable" theories.

Having rejected both the (absolute wave function and real-particle) theories, the CI relies heavily on the pragmatic conception of truth (William James type) which may be summarized by the following statements:

(a) The quantum theoretical formulation must be interpreted *pragmatically*.
(b) Quantum theory provides for a Complete Scientific account of atomic phenomena.

The pragmatic aspect (a) is fully incorporated in the measurement programme outlined above, one whose basic philosophy is governed by Bohr's own attitude as expressed in his famous essays. Some of the more pertinent statements (Stapp 1972) are in order:

(α) The task of science is both to extend the range of our experience and reduce it to order.
(β) In physics, . . . our problem consists in the coordination of our experience of the external world.
(γ) In our description of nature, the purpose is not to disclose the real essential of phenomena but only to track down as far as possible relations between the multifold aspects of our experience.
(δ) The description of the experimental arrangement and the recordings of observation must be given in plain language, suitably refined by the usual terminology. This is a logical demand (for us to be able to) tell others what we have done, and learnt.
(ε) Strictly speaking the mathematical formalism of quantum mechanics and electrodynamics merely offers rules of calculation for the deduction of expectation about observations obtained under well defined experimental conditions specified by classical concepts.

The "completeness" aspect (b) of quantum theory, according to CI, is more subtle and gave rise to the famous Bohr-Einstein controversy. We shall not delve into this point into any depth except for summarizing Stapp's version (Stapp 1972) of Bohr's point of view, viz that well-defined objective specifications on a given phenomenon under investigation are not restrictive enough to determine uniquely the course of the individual processes, yet no further breakdown is possible because of the inherent "wholeness" of the process symbolized by h. This wholeness has no classical analogue which would have recognized the measuring instruments and the atomic objects as separate entities. Instead, the inseparability of the atomic object from the entire phenomenon renders a statistical description unavoidable. This way of reconciling the pragmatic character of quantum theory with the claim of completeness, is based on "quantum thinking". Its ultimate validity must be judged by its *afortiori* success, which includes coherence and self-consistency.

9. Duality-aspects beyond physics

So far this account of duality has been confined to its tangible facets in so far as these played a role in the historical growth of physics through the ages since Newton. The basic philosophy behind this growth is summarized by the phrase "Cartesian partition" which while recognizing the element of the human psyche in the evolution of physical ideas had preferred to let it remain in the background without publicly appearing to influence the "contingent plane" of empirical and analytical statements (Holton 1973). Newton himself had been keenly aware of the duality between the "psyche" and the "physique" but was inclined to project only the latter without much encouragement to the former. His predecessor, Kepler, on the other hand, had relied more heavily on the thematic concepts of the universe as a "mathematical harmony" and as a "central theological order", though his analytical tools were hardly effective. And Newton's decisive influence on western scientific thought had much to do with the uneasy balance between a materialistic pursuit of science and an idealistic devotion to philosophy that had characterized the thematic development of physics till the early part of this century (Rosenfeld 1963) when Einstein and Bohr came on the scene. Einstein's deep philosophy behind his unified view of space-time continuum, on the one hand, and Bohr's abstraction of the principle of complementarity as a necessary consequence of the new quantum mechanics on the other, marked such a radical departure from the Western attitude to science prevalent till then, that these had the effect of a "wind of change" on a relatively close and still atmosphere. In particular the CP, which is more relevant to the theme of this article, set the Western community of physicists on the formidable task of reorienting their attitudes as a result of intrusion of dialectics into their traditional modes of thinking. Interestingly enough, Bohr's exposure of the same philosophy before the Japanese community met with little resistance to their traditional Eastern thought (as recounted by Yukawa to Rosenfeld) (Rosenfeld 1963).

What is this extra ingredient in Eastern thought with which Bohr's CP philosophy found such a ready resonance, even though it had appeared so unorthodox to the Western School? This brings me to the last phase of this paper, typified by table 2, where I have listed a few items of duality which are not fathomable with the (relatively mundane) method of physical science. In this I must be as brief as possible but a total exclusion of this dimension would violate the very spirit of this highly provocative subject.

For any science in its formative stages the traditional method of limited hypothesis, checked against vigorous experimentation and vice versa, has usually proved much more effective than unfettered speculation of ideas with no comparable degree of experimentation to provide the balance. However there comes a stage in its development when this relatively mundane method fails to do adequate justice to the intellectual aspirations of the scientific thinker. A very similar state has been reached in modern physics where the unification of opposite concepts represents precisely such an aspiration where the experimental support often lags so far behind the theoretical ideas that faith in the latter must, in the interim, be sustained through considerations of a thematic nature, again long before eventual experimental confirmation if at all. Such opposite concepts abound at the sub-atomic level where particles are both continuous and discontinuous; and force and matter are but different aspects of the same phenomena. In all these examples it turns out that the "framework of opposite concepts, derived from our everyday experience is too narrow for the world of subatomic particles" (Capra 1976).

Some of these situations have already been illustrated under the unification aspects of duality in the foregoing. In each case, the unification occurs on a higher plane, e.g. space and time become a single entity only in a four-dimensional continuum; wave and particle manifestations of an electron/photon get integrated only at the quantum level; matter and antimatter require a further synthesis of relativity and quantum theory for a self-consistent description, and so on.

These seemingly irreconcilable concepts whose unification is thus achieved at a less tangible and much deeper level of reality, often have a strange but convincing analogue in Eastern Mysticism. Fritjof Capra (1976), the author of a remarkable book named "The Tao of physics", and himself a high energy physicist of the Berkeley School, has systematically documented a large class of such examples (through extensive quotations from the appropriate religious, philosophical and scientific authorities) in his exploration of the parallels between modern physics and Eastern mysticism, and revealed a profound harmony between these highly abstract concepts in physics and very similar ideas emanating from the basic tenets of diverse religious traditions of the East. Little wonder then, that Bohr's philosophy* though firmly rooted in a "materialistic" formulation of science, represented an abstraction at a plane of thought high enough to find a ready parallel in Eastern mysticism. Capra (1976) has brought out this analogy rather succinctly by comparing the status of an electron according to quantum theory (in the words of Robert Oppenheimer) with very similar sentiments expressed in the Upanishads concerning a form of reality transcending the narrow framework of the opposite concepts of existence vs non-existence,

Oppenheimer

"If we ask, e.g., whether the position of the electron remains the same, we must say *no*; if we ask whether electron's position changes with time, we must say *no*; if we ask whether the electron is at rest, we must say *no*; if we ask whether it is in motion, we must say *no*".

Upanishads

It moves, it moves not
It is far, and it is near
It is within all this
And it is outside of all this.

This example illustrates the dilemma of the physicist on the one hand and the mystic on the other, when faced with a reality which lies beyond opposite concepts. He must adopt a higher plane of thinking, transcending the narrow framework of orthodox logic. The natural language for such pursuits presumably takes the form of mathematics (at continually higher levels of abstraction) in the case of physical sciences, and the deeper and deeper levels of meditation for exploring the realms of mystical experience.

A very similar message comes from the Chinese symbolism of the archetypal poles *Yin* and *Yang*, two extremes which are not static opposites, but are constantly engaged in a dynamic interplay which brings about their unity (TAO) on a higher plane. This has a simple physical analog in the example of a circular motion and its linear projection (Capra 1976): The continuous oscillation between the two opposite points (figure 2) is a characteristic only of the linear projection, while the more complete two-dimensional circular motion shows no such fluctuation.

* In the words of Heisenberg, "Bohr was primarily a philosopher not a physicist, but he understood that natural philosophy, in our day and age, carries weight only if its every detail can be subjected to the inexhorable test of experiment."

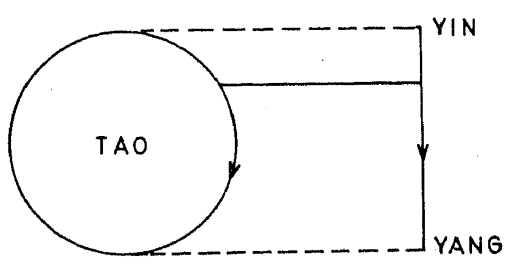

Figure 2. The dynamical synthesis of opposites.

The Buddhist philosophy also revolves about an identical point of view, "wherein the unity of all things becomes a vivid experience" (Capra 1976). This is illustrated by the Zen Poem (Capra 1976):

At dusk the clock announces dawn,
At midnight the bright sun.

As a final example, the Jain Philosophy of Syadvada has a remarkable parallel with the wave particle duality in its *fourth* mode of manifestation of reality, viz inexpressibility or "Avayakta", as succinctly described by Kothari (1985) in a recent article.

Opposites are abstract concepts belonging to the realm of thought, and hence, are relative. The very act of focussing attention on any one concept causes the creation of its opposite. In the words of Lao Tzu "when all in the world understand beauty" to be beautiful, then ugliness exists; when all understand "goodness" to be good, then evil exists (Capra 1976).

Mystics through their meditation, transcend this realm of intellectual concepts, and in so doing they become aware of the polar relationship of all opposites. Physicists grope for a glimpse of the same through their language of mathematics.

Western philosophers have been keenly aware of this very duality, as vividly illustrated by Emerson's thesis on the hidden law of compensation, wherein "an inevitable dualism bisects nature so that each thing is a half, and *suggests* another things to make it a whole."* Today, theoretical physicists are increasingly feeling its impact as they probe deeper and deeper into the mysteries of the sub-atomic world (experimentally down to 10^{-16} cm but theoretically all the way to Planck's length), having received its first taste in Bohr's complementary principle, provoked experimentally by wave-particle duality. There is no going back on this truth, irrespective of the source, be it modern physics or mysticism, of its inspiration.

* "... Gain is loss, less is more ... Love and you shall live; Nature hates monopolies and exceptions. An inevitable dualism bisects nature, so that each thing is a half, and suggests another thing to make it a whole; man and woman; in, out; rest, motion; sweet has its sour; evil has its good; for everything missed there is something gained, and vice versa ... All things are moral, Justice is not postponed. The dice of God are always loaded. The world looks like a mathematical equation which balances itself. Every secret is told, every crime punished; every virtue rewarded .....

There are two sides (good and evil) to everything. In the nature of the soul is the compensation for the inequalities of condition ... ."

—Ralph Waldo Emerson.

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Note added in proof

It is particularly gratifying to learn that the year 1986 also happens to coincide with the eightieth birthday of Hans Bethe whose impact on the entire gamut of physics in the twentieth century would instantly register on the worldwide physics community. This article would remain incomplete without a reference to the lifelong philosophy of Bethe, which is of direct relevance to its theme of duality. The Bethe philosophy in physics may be summed up in the two key words of *simplicity* and *thoroughness*, which fit in rather naturally as dual partners, as another item of Table 2 in this very order. This dual theme has been in evidence in most of Bethe's papers and articles right from the thirties, be it

(i) his celebrated articles[a] on the subject of nuclei (then in its infancy), which provide a clue to the pioneering ideas that led to his famous discovery of energy production in heavenly bodies; or
(ii) his delightful paper[b] on effective range theory which brought out so succinctly the basic simplicity of the entire mechanism; or
(iii) his remarkable 2-page explanation[c] of the Lamb shift, which told the layman what the thing is all about, in advance of the Revolution that followed; and so on.

The Bethe-Salpeter equation,[d] which is still in wide use for the physics of strong interactions, is still another example of the same central theme.

(a) Bethe H A *et al. Rev. Mod. Phys.* **8** (1936); **9** (1937) 69, 245
(b) Bethe H A *Phys. Rev.* **76** (1949) 38
(c) Bethe H A *Phys. Rev.* **72** (1947) 339
(d) Salpeter E E and Bethe H A *Phys. Rev.* **84** (1951) 1232