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Complete mass spectra of $q\bar{q}$ and $Q\bar{q}$ mesons with improved Bethe-Salpeter dynamics of confinement

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Abstract. The Bethe-Salpeter (BS) dynamics of harmonic confinement developed by ANM and collaborators over the last three years and already applied with considerable experimental success to various hadron spectra and coupling structures has been significantly improved through (i) a more exact treatment of a certain momentum-dependent operator $\hat{Q}_q$ appearing in the BS equation, using the techniques of SO (2, 1) Lie algebra, and (ii) a sharpened definition of the QCD Coulomb term, so as to yield unambiguous values for different flavour sectors. The resulting mass spectra of light ($q\bar{q}$) meson towers and semi-heavy ($Q\bar{q}$) quarkonia which are most sensitive to the improved treatment of $\hat{Q}_q$, reveal excellent agreement with experiment, one in which only slight changes in the reduced spring constant ($\tilde{\omega}$) and quark masses (m_q) over the earlier parametrizations are involved. These changes are however found to have a negligible effect on the (already good) numerical values of the other predictions (electroweak and pionic couplings) depending on the $q\bar{q}$ and qqq wave functions. A critical assessment of the strong and weak points of this method is made *vis-a-vis* other related approaches.

Keywords. Bethe-Salpeter dynamics; harmonic confinement; mass spectra; mesons.

1. Introduction

It is generally believed (despite significant progress of lattice gauge theories on the numerical front) that the Bethe-Salpeter (BS) equation represents a most natural and practical form of dynamics to describe the quark structure of hadrons. Now while for heavy quarkonia the BS equation reduces to the more orthodox Schrödinger equation together with small corrections of $O(V^2/C^2)$, the former differs qualitatively from the latter for hadrons made up of light (u, d, s) quarks. Indeed it has been argued (Mitra 1981) that a closed form BS equation is in all fairness the more appropriate form of dynamics for *uds* spectroscopy which is intrinsically relativistic in character. (Even a ($Q\bar{q}$) system, where Q is heavy and $\bar{q}$ light, requires an intrinsically relativistic framework.) For light (or semi-light) meson spectroscopy, there is evidence of the use of the Dirac equation and the associated languages of scalar *vs* vector (Critchfield 1975; Gunion and Li 1975), 3-scalar versus 4-scalar, mixtures of scalar and vector (Smith and Tassie 1975; Critchfield 1975; Rein 1977; Jena 1983), etc potentials to simulate an effective confining interaction. Since the Dirac equation (or the Breit equation) is an incomplete response to the dynamics of a basically relativistic system (Bethe and Salpeter 1957), it is not clear as to what extent issues such as the (scalar versus vector) nature of the confining potential have physical relevance. The Bethe-Salpeter equation, on the other hand, being intrinsically four-dimensional in formulation (Bethe and Salpeter 1957), avoids such controversies in a natural manner (since the kernel must necessarily be a 4-scalar), and therefore puts the connection between the Lorentz-character of the potential and the problem of confinement in a much clearer perspective.

For some time we have been involved in the formation (Mitra 1981) of a Bethe-Salpeter dynamics for $q\bar{q}$ and qqq hadrons in the instantaneous approximation (Levy 1952; see also Fishbane and Namyslowski 1980 for a perspective and a complete list of earlier references on the instantaneous approximation) and its applications to their mass spectra (Mitra and Santhanam 1981a, b; Kulshreshtha *et al* 1982), electroweak couplings (Mitra and Kulshreshtha 1982; Mitra and Mittal 1984) as well as their pionic interactions (Kulshreshtha and Mitra 1983; Mittal and Mitra 1984). Under conditions of harmonic confinement, the BS equation has been explicitly solved (Mitra and Santhanam 1981 a, b) except for the presence of (i) a momentum-dependent term ($\hat{Q}_q$) which has required a certain non-perturbative approximation (Mitra and Santhanam 1981 a, b) and (ii) The coulomb (one-gluon exchange) interaction for which a perturbative treatment has proved adequate. The model is characterized by a reduced spring constant $\tilde{\omega}$ common to all flavour sectors, and the mass (m_q) of the flavour sector under study (Mitra and Santhanam 1981a, b; Kulshreshtha *et al* 1982; Mitra and Kulshreshtha 1982; Kulshreshtha and Mitra 1983; Mitra and Mittal 1984; Mittal and Mitra 1984). The spectral predictions for $q\bar{q}$ and qqq systems have each been expressed by a universal formula of the type (Mitra and Santhanam 1981a, b; Kulshreshtha *et al* 1982) $F(M) = N + \text{const}$, where N is the total HO quantum number for each system and $F(M)$ is a certain known function of the actual hadron mass. The comparison of the theory (Mitra and Santhanam 1981a, b; Kulshreshtha *et al* 1982) with the observed mass spectra has been made somewhat indirectly, *viz* by computing $F(M)$ for the observed masses of meson and baryon spectra of different species, and checking against the expected behaviour of $F(M)$, especially its universality for different members of a common SU(6) multiplet (spin and flavour) and the unit spacing rule ($\Delta F = 1$) for successive N-values. A detailed agreement has been found on both these counts for a wide class of hadronic spectra (non-strange and strange), thus providing a good (*albeit* indirect) test of the model in respect of meson and baryon spectra of several flavour combinations. Simultaneously several other items of rather impressive agreement of a more direct nature have been found in respect of

(i) electroweak couplings of $q\bar{q}$ systems illustrated by $f_{\pi,k}$; $r_{\pi,k}$ and $V \to e^+e^-$ decays (Mitra and Kulshreshtha 1982)

(ii) electromagnetic couplings of qqq baryons, especially magnetic moments, proton charge radius and $\Delta \to N\gamma$ helicity amplitudes (Mitra and Mittal 1984)

(iii) pionic couplings of hadrons, highlighted by $g_{\omega\rho\pi}$ $\Gamma(\rho \to \pi\pi)$ (Kulshreshtha and Mitra 1983), $\Gamma(\Delta \to N\pi)$ and $G_{NN\pi}$ (Mittal and Mitra 1984)

These applications have been formulated in the (four-dimensional) language of Feynman diagrams in which the key-ingredient to the BS vertex structure is the wave-function of the hadron concerned. All these constitute rather good tests of those features of the model which depend sensitively on the hadronic wave function. The only inputs used so far for the entire analysis are (Mitra and Santhanam 1979 a, b; Kulshreshtha *et al* 1982; Mitra and Kulshreshtha 1982; Mitra and Mittal 1984; Kulshreshtha and Mitra 1983; Mittal and Mitra 1984).

$$\tilde{\omega} = 0{\cdot}15 \pm 0{\cdot}01,\ m_{ud} = 0{\cdot}28 \pm 0{\cdot}02,\ m_s = 0{\cdot}35 \pm 0{\cdot}02 \tag{1}$$

all in the GeV units.

The main object of this paper is to have a fresh look at the problem of mass spectra with a view to providing a direct quantitative comparison of the predicted and observed masses with (marginal) modification in the parameters (1) if necessary; and to check on the other predictions noted above, which depend on the hadronic wavefunctions of the states concerned. To this end we have found it necessary to effect a three-fold improvement in the earlier formalism:

(a) A more exact treatment is now given of the operator (Mitra and Santhanam 1979 a, b; Mitra and Kulshreshtha 1982)

$$\hat{Q}_q = 4\mathbf{q}^2 \cdot \nabla^2 + 8\mathbf{q} \cdot \nabla_q + 6 \tag{2}$$

(b) A reassessment is made of the (perturbative) coulomb contribution to $F(M)$ and thence to M itself—wherein the variation of α_s with the hadron mass under study is postulated to be as follows:

$$\alpha_s(M^2) = \frac{12\pi}{(33-2f)\ln(M^2/\Lambda^2)}; \; \Lambda \approx 250 \text{ MeV}. \tag{3}$$

Since the α_s-values predicted by this explicit (*albeit adhoc*) formula in the different flavour sectors are in rather close accord with those employed in most phenomenological calculations (Pennington 1983; Rosenberg 1982; Blumenfeld *et al* 1982), this effective device removes an earlier arbitrariness in the α_s values hitherto employed (Mitra and Santhanam 1981a, b; Kulshreshtha *et al* 1982) for different hadronic mass zones. The M-dependence of α_s of this nature has also been suggested in the bag model (Carlson *et al* 1983).

(c) Meson states with unequal mass kinematics seem to go better with the (modified) ansatz recently employed to investigate R-hadronic states (Mitra and Ono 1983), *viz*

$$\omega_{q\bar{q}}^2 = \tau_{12} m_{12} \tilde{\omega}^2; \; \tau_{12} = 4m_1 m_2/m_{12}^2 \tag{4}$$

which is more in accord with the original suggestion (Mitra and Santhanam 1981a, b) than with the one employed by Kulshreshtha *et al* (1982) and Mitra and Kulshreshtha (1982) without the τ_{12}-factor.

Since the basic formulation for $q\bar{q}$ dynamics is already available in the literature (Mitra and Santhanam 1981a, b; Mitra and Kulshreshtha 1982) in sufficient details, we shall not go over it again but merely sketch the necessary steps clarifying only the newer aspects bearing mainly on item (a). These are found to result in the effective replacement of the quantity

$$Q_N = \tfrac{1}{6}(N^2 + 3N - 3) - (N + \tfrac{3}{2})^2 \tag{5}$$

in the earlier mass formula (Mitra and Santhanam 1981a, b; Mitra and Kulshreshtha 1982) by an analogous function Q'_N behaving asymptotically as $\frac{1}{2}Q_N$. Section 2 outlines the essential steps of the new construction in terms of a non-compact group SO (1, 2) whose eigenvalues are bounded from below only. Section 3 gives the results for meson mass (M) spectra for successively heavier quarks upto the (D, F) region, obtained from a numerical inversion of the equation $F(M) = $ constant for each state, where $F(M)$ is now calculated in accordance with the above improvements (a), (b), (c) over the previous constructions (Mitra and Santhanam 1981a, b; Kulshreshtha *et al* 1982; Mitra and Kulshreshtha 1982; Kulshreshtha and Mitra 1983; Mitra and Mittal 1984; Mittal and

Mitra 1984). Very good fits are obtained with the input parameters (in GeV):

$$\tilde{\omega} = 0{\cdot}14,\; m_{ud} = 0{\cdot}30,\; m_s = 0{\cdot}40,\; m_c = 1{\cdot}66,\; m_b = 5{\cdot}3, \tag{6}$$

which differ only slightly from the values given by (1), employed earlier (Mitra and Santhanam 1981a, b; Kulshreshtha *et al* 1982; Mitra and Kulshreshtha 1982; Kulshreshtha and Mitra 1983; Mitra and Mittal 1984; Mittal and Mitra 1984). The resulting modifications in the various other quantities depending explicitly on the hadronic wave functions which have been calculated so far (Mitra and Kulshreshtha 1982; Kulshreshtha and Mitra 1983; Mitra and Mittal 1984; Mittal and Mitra 1984) are also found to be reproduced equally well. Section 4 concludes with a summary and a brief comparison with some related approaches. Due to the relative insensitivity of the difference between the input parameter sets (1) and (6) to the baryon states, a corresponding reassessment on the baryon masses has not been undertaken in this paper.

2. Improved solution of $q\bar{q}$ BS equation

We start with the BS equation (without the coulomb term for unequal mass kinematics in the instantaneous approximation *viz* (Mitra and Kulshreshtha 1982)*

$$2M[\mathbf{q}^2 - \tfrac{1}{4}\tau_{12}(M^2 - m_{12}^2)]\Psi(\mathbf{q}) = \omega_{q\bar{q}}^2[\tau_{12}M^2\nabla_q^2 + \hat{Q}_q - 4(2\mathbf{J}\cdot\mathbf{S} - 3) + \frac{2}{m_1 m_2}\mathbf{q}^2 - \frac{(m_1^2 + m_2^2)M}{m_1 m_2 m_{12}}(4\mathbf{q}\cdot\nabla_q + 6)]\Psi(\mathbf{q}), \tag{7}$$

while $\hat{Q}_q$ is given by (2), and the other symbols are as defined in Mitra and Santhanam (1981 a, b) and Mitra and Kulshreshtha (1982). Eliminating the $(4\mathbf{q}\cdot\nabla_q + 6)$ term in the usual manner leads to the simplified form

$$[\mathbf{q}^2\gamma^2 - \tfrac{1}{2}M\tau_{12}^2\tilde{\omega}^2 m_{12}\nabla_q^2 - \frac{\tau_{12}m_{12}\tilde{\omega}^2}{2M}\hat{Q}_q - G(M)]\phi(\mathbf{q}) = 0, \tag{8}$$

where use has been made of (4), and

$$\gamma^2 = 1 + \left[\frac{(m_1^2 + m_2^2)^2}{2m_1^2 m_2^2} - 1\right]\frac{\tau_{12}m_{12}\tilde{\omega}^2}{m_1 m_2 M}, \tag{9}$$

$$G(M) = \tfrac{1}{4}\tau_{12}(M^2 - m_{12}^2) - \tfrac{1}{2}\tau_{12}m_{12}\tilde{\omega}^2 M^{-1}(8\mathbf{J}.\mathbf{S} - 12), \tag{10}$$

$$\phi(\mathbf{q}) = \exp(-\tfrac{1}{4}(m_1^2 + m_2^2)m_{12}m_1^{-2}m_2^{-2}M)\psi(\mathbf{q}). \tag{11}$$

As explained in Mitra and Santhanam (1979a, b), $\phi(\mathbf{q})$ is the true wavefunction satisfying the normal condition of probability conservation (which $\psi(\mathbf{q})$ does not). To recall briefly why this vital property got lost in the transition from the four-dimensional BS wave function $\Psi(q_\mu)$ to the three-dimensional (instantaneous) function $\psi(\mathbf{q})$, such problems in general reflect the foundational inadequacies inherent in a *truncation* from

* The slight difference of the coefficient of $\mathbf{q}\cdot\nabla_q + 3/2$ in (7) from the corresponding one in equation (8) of Mitra and Kulshreshtha (1982) stems from neglecting the effect of the $T_1 I_1$ term in the present case, unlike that of Mitra and Kulshreshtha (1982). This reassessment results from the present approximation $\omega_1/\omega_2 \approx m_1/m_2$ which leads to $I_1 \approx 0$ and seems to be more justified than $\omega_1 \approx \omega_2$ considered in Mitra and Kulshreshtha (1982). In either case, the results for $m_1 = m_2$ remain unaffected.

a four-dimensional to a three-dimensional framework. A good example of precisely this kind of situation is provided by the celebrated FKR model (Feynman *et al* 1971) in which an attempt to suppress (by hand) the time-like excitations (because of the wrong sign of the exponent associated with the "relative-time" degree of freedom) had resulted in an overestimate (greater than unity) of the total probability, a malady which in turn needed an effective form-factor (again adjusted by hand) for a possible remedy.

Further processing of (8) requires the use of the standard creation and annihilation operators a_i, a_i^+ defined by

$$a_i = 1/\sqrt{2}[q_i\beta^{-1} + \partial q_i \beta],\ a_i^+ = 1/\sqrt{2}[q_i\beta^{-1} - \partial q_i \beta] \tag{12}$$

in terms of which (13) may be rewritten (in field-theoretic notation) as

$$[N + 3/2 - 1/4\xi \hat{Q}_q - F_1(M)]\,|\phi\rangle = 0, \tag{13}$$

where

$$N = a_i^+ a_i,\quad \xi = 8m_{12}\tau_{12}\tilde{\omega}^2 M^{-1}\Omega_M^{-1}, \tag{14}$$

$$\hat{Q}_q = (a_i a_i + a_i^+ a_i^+)^2 - 6 - (2N+3)^2, \tag{15}$$

$$F_1(M) = \tau_{12}\Omega_M^{-1}(M^2 - m_{12}^2) - 8m_{12}\tau_{12}\tilde{\omega}^2 M^{-1}\Omega_M^{-1}(2\mathbf{J}.\mathbf{S} - 3), \tag{16}$$

$$\Omega_M = 4\gamma\omega_{q\bar{q}}(2M\tau_{12})^{1/2} = 8\beta^2\gamma^2. \tag{17}$$

A further transformation involving the use of the operators Q_α ($\alpha = 1, 2, 3$) defined by

$$2Q_3 = N + 3/2;\ Q_\pm = Q_1 \pm iQ_2, \tag{18}$$

$$2Q_+ = a_i^+ a_i^+,\ 2Q_- = -a_i a_i, \tag{19}$$

which shows that the number operator N appearing in (18) is directly connected with Q_3, while the other two quantities a_i^{+2}, a_i^2 are related to $Q_\pm$. The new quantities Q_α satisfy the commutation relations

$$[Q_3, Q_\pm] = \pm Q_\pm,\ [Q^+, Q^-] = 2Q_3 \tag{20}$$

reminiscent of an SU(3) algebra, but it is really a non-compact SO(2,1) (because of the relative definitions (19) of $Q_\pm$) which is nevertheless locally isomorphic to SO(3). In terms of Q_α-operators, (15) may be re-expressed as

$$\begin{aligned} -\hat{Q}_q &= +16Q_2^2 + 6 + 16Q_3^2 \\ &= 8(C + Q_3^2) + 6 + 8(Q_2^2 - Q_1^2), \end{aligned} \tag{21}$$

Where C is the Casimir operator defined by

$$C = Q_1^2 + Q_2^2 + Q_3^2 = Q_+Q_- + Q_3^2 - Q_3 \tag{22}$$

and the non-diagonal part $\sim (Q_2^2 - Q_1^2)$ connects states differing by two units of excitation. The latter may be ignored in view of the large energy denominators involved in such transitions (Mitra and Santhanam 1981a, b), and also the fact that this term is in the first place merely a part of a non-leading term $\hat{Q}_q$ in the BS dynamics. Thus (13) is reduced to an effectively diagonal form, leading to the algebraic equation

$$F_1(M) - \xi(2C + 2Q_3^2 + 3/2) = N + 3/2 \tag{23}$$

for a determination of M in terms of the eigenvalues of C and Q_3. These eigenvalues may be formally depicted as $u(u+1)$ and m respectively, in the usual quantum mechanical language of angular momenta, but their values are not of the integral and/or half-integral types. Instead their spectra are now governed by the generalized Lie theory of G (1, 0) algebra, which classifies them into four different types (see, e.g. Miller 1968). The type which is relevant to the present case corresponds to a rising Q_3 spectrum in unit steps, but bounded from below, *viz*

$$(Q_3)_{\text{eig}} = m = -u+k, \qquad k = 0, 1, 2 \ldots \tag{24}$$

where u is the parameter characterising the eigenvalue $u(u+1)$ of C. To determine the value of u one must invoke the compatibility of (24) with (18), leading to the following connection between the m and N quantum numbers

$$m = \tfrac{1}{2}N + \tfrac{3}{4}, \quad N = 0, 1, 2, \ldots \tag{25}$$

Distinguishing the two cases of even and odd N, a comparison of (24) and (25) gives the following results for the Casimir operator in the two cases

$$\text{(a)}\ N = 2n\text{: } u = -\tfrac{3}{4};\ u(u+1) = -\tfrac{3}{16} \tag{26a}$$

$$\text{(b)}\ N = 2n+1\text{: } u = -5/4;\ u(u+1) = +5/16. \tag{26b}$$

Both results are consistent with the generalized SO(2, 1) Casimir eigenvalues of the form (Ghirardi 1972; Gardero and Ghirardi 1972)

$$C = \lambda - \tfrac{3}{16}, \quad (\lambda = 0, \tfrac{1}{2}), \tag{27}$$

thus providing further confirmation of the SO(2, 1) nature of the algebra of the Q_α-operators on hand.

As a result of these manipulations, the (approximate) eigenvalues of $\frac{1}{4}\hat{Q}_q$ given by (21) are expressible as

$$Q'_N = \langle \tfrac{1}{4}\hat{Q}_q \rangle = -3/2 - 2u(u+1) - 1/2(N+3/2)^2 \tag{28}$$

where $u(u+1)$ alternates between $-3/16$ and $+5/16$ for even and odd values of N respectively. The quantity Q'_N now substitutes for the quantity Q_N, equation (5), which had appeared in the earlier treatment (Mitra and Santhanam 1981 a, b; Mitra and Kulshreshtha 1982). Substitution of (28) in (23) leads to the explicit solution

$$F_{\text{HO}}(M) = F_1(M) + \xi Q'_N = N + 3/2, \tag{29}$$

whose formal similarity to the earlier form (Mitra and Santhanam 1981a, b) can be recognized except for the replacement $Q_N \to Q'_N$. Inclusion of the one-gluon exchange (coulomb) term in a perturbative fashion adds a further correction to the left side of (29) of amount

$$\begin{aligned} F_{\text{coul}} &= \frac{8\,\alpha_s M}{3\,\Omega_M} \int \frac{1}{r} \left| \phi_L(r) \right|^2 \mathrm{d}^3 r \\ &= \frac{16}{3} \frac{\beta_L}{\sqrt{\pi}} \frac{M\alpha_s}{\Omega_M} \delta_L \end{aligned} \tag{30}$$

where

$$\phi_L(r) = \sqrt{4\pi}\, N_L Y_{LM}(\theta\phi)(\beta r)^L \exp(-\tfrac{1}{2}\beta^2 r^2), \tag{31}$$

$$N_L^{-2} = 2\pi \beta_L^{-3} \Gamma(L+3/2);\ \delta_L = \frac{\Gamma(3/2)\Gamma(L+1)}{\Gamma(L+3/2)}, \tag{32}$$

and $\alpha_s(M^2)$ is given by (3). Also according to the findings of an earlier analysis in respect of $q\bar{q}$, qqq, $qq\bar{q}\bar{q}$ systems (Mitra 1982), the sum of the HO and coulomb contributions to $F(M)$ accounts for almost the entire zero point energy (ZPE) for all these systems (Kulshreshtha *et al* 1982; Mitra 1982) except for an overall shortfall of one unit only in each case. In other words for $q\bar{q}$, qqq, $qq\bar{q}\bar{q}$ systems the effective ZPE values should be taken as 1/2, 2 and $3\frac{1}{2}$ respectively (Mitra 1982). Thus the final equation for the determination of the mass M for a $q\bar{q}$ system is

$$F_{\mathrm{HO}} + F_{\mathrm{coul}} = N + 1/2, \tag{33}$$

where the two numbers on the left side are given by (29) and (30) respectively.

3. Mass spectra of $q\bar{q}$ and $Q\bar{q}$ states

To obtain the explicit masses (M) of meson states from (33), we proceed stepwise from light flavours, starting from the ($q\bar{q}$) composites of (u, d) quarks (assumed equally massive), successively to those of (u, d, s) and then on to semi-heavy quarkonia ($Q\bar{q}$).

For a practical determination, it is most convenient to choose the input parameters (m_q, $\tilde{\omega}$) beforehand, and for a given (NJLS) state to "hunt" for the M-value that will reproduce the right side value, of (31) *viz* $N + 1/2$. Since moreover the change $Q_N \to Q'_N$ is small and should leave almost unaffected all but the lowest ($N = 0$) states, only small changes from the original input values (1:1) are envisaged.

As to the coulomb term (30), it is fully determined by the QCD parameter Λ and the flavour number (f) according to the defining function (3) for α_s. As to the M dependence of α_s, this function is well behaved for all $M > \Lambda$ but becomes singular for $M \sim \Lambda$ and negative for $M < \Lambda$, the last behaviour representing the physical limits on the practical usefulness of such a parametrization for α_s. This affects only the lightest meson (the pion) for which we must advocate the omission of the perturbative coulomb term. Apart from this (small) exception, we stick meticulously to our general rule for treating the pion as a $q\bar{q}$ composite on par with all other mesons, as recently discussed elsewhere (Mittal and Mitra 1984). This is a far more conservative point of view than most other approaches advocating the treatment of the pion as an effectively elementary entity (Chodos and Thorn 1975; Brown and Rho 1979; Thomas *et al* 1981). From a more general point of view too, the concept of a perturbative one-gluon exchange correction to the mass of an unusually light hadron as the pion may well be of questionable validity.

With these precautions, the theoretical values of M obtained for different $q\bar{q}$ states are listed in table 1. The input ($\tilde{\omega}$, m_q) values are given in (6), which differ very little from the earlier values (1), the differences arising as a result of the replacement $Q_N \to Q'_N$, as well as the modified definitions of $\omega^2_{q\bar{q}}$ (for unequal masses) and α_s with respect to the earlier treatment. The present set (6) of (m_q, $\tilde{\omega}$) parameters are also in close accord with a recent input employed for the determination of R-hadron masses (Mitra and Ono 1983), after normalizing such parameters to a limited fit to $q\bar{q}$ and $Q\bar{q}$ masses ($L = 0$ states only). This last approach (Mitra and Ono 1983) used the definition (4) of $\omega^2_{q\bar{q}}$ for unequal masses, in common with this paper, but did not (till then), have the facility for the replacement $Q_N \to Q'_N$ which is particularly sensitive for $N = 0$. This sensitivity for $N = 0$ is borne out by the fact that even a formal solution of (31) for the (unusually low) pion mass seems to exist only for the new value Q'_N but not for the older quantity Q_N.

Table 1. Predicted meson masses for different flavour sectors. Experimental masses are given with particle symbols.

Meson	(N, J, L, S)	α_s	M(GeV)
$\pi(140)$	(0, 0, 0, 0)	−1·135	0·141
$\rho(770)$ }	(0, 1, 0, 1)	0·543	0·827
$\omega(783)$ }			
$\delta(980)$ }	(1, 0, 1, 1)	0·427	1·145
$B(1235)$ }	(1, 1, 1, 0)		
$D(1284)$	(1, 1, 1, 1)	0·411	1·215
$f(1270)$ }		0·390	1·324
$A_2(1320)$ }	(1, 2, 1, 1)		
$\rho'(1600)$	(2, 1, 0, 1)	0·355	1·560
$\pi_2(1660)$	(2, 2, 2, 0)	0·359	1·530
$\rho_2(1685)$ = g	(2, 3, 2, 1)	0·341	1·682
$A_4(2040)$ = h	(3, 4, 3, 1)	0·310	2·033
$\rho_4^a(2300)$	(4, 5, 4, 1)	0·291	2·330
$A_6^b(2510)$	(5, 6, 5, 1)	0·276	2·630
$\eta(549)$	(0, 0, 0, 0)	0·852	0·536
$\eta'(958)$	(0, 0, 0, 0)	0·497	0·923
$\phi(1020)$	(0, 1, 0, 1)	0·467	1·005
$E(1420)$	(1, 1, 1, 1)	0·405	1·400
$f'(1515)$	(1, 2, 1, 1)	0·396	1·455
$K(496)$	(0, 0, 0, 0)	0·867	0·529
$K^*(892)$	(0, 1, 0, 1)	0·5005	0·916
$Q_1(1280)$	(1, 1, 1, 0)	0·437	1·236
$Q_2(1400)$	(1, 1, 1, 1)	0·422	1·305
$K^{**}(1430)$	(1, 2, 1, 1)	0·402	1·418
$K^{***}(1780)$	(2, 3, 2, 1)	0·355	1·789
$K^{****}(2086)$	(3, 4, 3, 1)	0·324	2·155
$D(1865)$	(0, 0, 0, 0)	0·347	1·869
$D^*(2010)$	(0, 1, 0, 1)	0·334	2·016
$F(1970)$	(0, 0, 0, 0)	0·338	1·973
$F^*(2140)$	(0, 1, 0, 1)	0·327	2·115
$B^c(5283)$	(0, 0, 0, 0)	0·247	5·284
$B^*(?)$	(0, 1, 0, 1)	0·246	5·341

[a] Alper *et al* (1980); [b] Binon *et al* (1983);
[c] Behrends *et al* (1983).

Table 1 exhibits the quality of the fits to the masses of various mesons which seems to leave little to be desired in terms of the overall pictures. Because of the assumed isospin degeneracy ($m_u = m_d$), (ρ, ω), (f, A_2), (ω, ρ_2) etc states are degenerate in this model. Similarly B and δ mesons, each of $N = 1$ are predicted to be degenerate in view of the equality of their $\mathbf{J}.\mathbf{S}$ values and the absence of any other theoretical signature in (33) to distinguish between them. Therefore the only meaningful check for these cases is a comparison of their average mass, *viz* 1107 MeV, with the theoretical prediction (1145 MeV), which is quite reasonable. All other cases agree with data within 2% for $S = 1$ states and 5% for $S = 0$ states. Especially impressive are the fits to the masses of ρ and A_2 towers (upto $N = 5$), and likewise for the K^* series.

For the η, η' states, the following assignments have been used for the calculation

$$|\eta\rangle = \left|\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\right\rangle \cos(\theta + \alpha) - \left|S\bar{S}\right\rangle \sin(\theta + \alpha), \tag{34}$$

$$|\eta'\rangle = \left|\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})\right\rangle \sin(\theta+\alpha) + \left|S\bar{S}\right\rangle \cos(\theta+\alpha), \tag{35}$$

where (Particle Data Group 1982)

$$\theta = -10.8°;\ \tan\alpha = \sqrt{2}. \tag{36}$$

As a result, the $F(M)$ values in (33) and the preceding ones read as follows;

$$\begin{pmatrix} F_\eta \\ F_{\eta'} \end{pmatrix} = F(S\bar{S})\begin{pmatrix} \sin^2(\theta+\alpha) \\ \cos^2(\theta+\alpha) \end{pmatrix} + F\left(\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})\right)\begin{pmatrix} \cos^2(\theta+\alpha) \\ \sin^2(\theta+\alpha) \end{pmatrix} \tag{37}$$

with the appropriate m_q masses used for computing the F-functions with indicated quark flavour.

Fits to the masses of heavy-flavoured $(Q\bar{q})$ states depend on the assumption of a mass m_Q for each of these two sectors (c, b). The input value $m_c = 1{\cdot}66$ GeV fits the (D, F) sector remarkably well, indeed better than those in the light meson sector, without any change in the values of m_q $(q = u, d, s)$ already determined from the lighter meson zones. The same is true of the B-meson with an input $m_b = 5{\cdot}3$ GeV.

In view of these high quality fits to the $Q\bar{q}$ mesons, it is tempting to predict the masses of their (N, L)-excited states as well as other B-like mesons. These are recorded separately in table 2 for some low lying excitations $(N = 1)$. We have not attempted to calculate the explicit masses of the heavy quarkonia $(c\bar{c}, b\bar{b})$, for reasons discussed in §4.

As for the baryon mass spectra a fresh assessment has not been found necessary for the following reasons (i) The correction to the qqq equation arising from a more refined treatment of the baryonic analogues of the $\hat{Q}_q$ operator is negligible even for $N = 0$ states. (ii) The baryonic $F(M)$ functions have already exhibited (Mitra and Santhanam 1981a, b; Kulshreshtha *et al* 1982) a high degree of regularity on theoretically expected lines with the input parameters (1), and are generally less sensitive to slight changes in these values than are the $q\bar{q}$ or $Q\bar{q}$ spectra. (iii) The slight change of inputs for $(\tilde{\omega}, m_q)$ from (1) to (6) is within their respective parametric uncertainties implicit in the earlier treatment (Mitra and Santhanam 1981a, b; Kulshreshtha *et al* 1982; Mitra and

Table 2. Predicted states of low lying $Q\bar{q}$ mesons upto $L = 1$. The unstared and stared states in the first column represent $S = 0$ and $S = 1$ respectively.

Meson $(Q\bar{q})$	(N, J, L, S)	α_s	M (GeV)
cu	(1, 1, 1, 0)	0·305	2·473
$(cu)^*$	(1, 2, 1, 1)	0·297	2·613
cs	(1, 1, 1, 0)	0·300	2·554
$(cs)^*$	(1, 2, 1, 1)	0·293	2·694
(bu)	(1, 1, 1, 0)	0·238	5·942
$(bu)^*$	(1, 2, 1, 1)	0·237	6·013
(bs)	(0, 0, 0, 0)	0.245	5.412
$(bs)^*$	(0, 1, 0, 1)	0.244	5.468
(bs)	(1, 1 1, 0)	0.237	6.031
$(bs)^*$	(1, 2, 1, 1)	0·236	6·102

Table 3. Various physical quantities bearing on electromagnetic and weak couplings of mesons.

Physical quantity	Present calculation	Old calculation (Mitra and Kulshreshtha 1982)	Experimental
f_π (MeV)	92·13	89·1	93
f_K (MeV)	86·95	85·0	100?
f_D (MeV)	71·11	89·6	?
r_π (fm)	0·78	0·77	0·663–0·73[a]
r_K (fm)	0·52	0·50	0·53 ± 0·05
λ_+^e	0·027	0·026	0·029 ± ·004
$\Gamma_{\rho\to e^+e^-}$ (keV)	6·25	6·43	6·54 ± 0·5
$\Gamma_{\omega\to e^+e^-}$ (keV)	0·686	0·72	0·76 ± 0·07
$\Gamma_{\phi\to e^+e^-}$ (keV)	1·283	1·31	1·27 ± 0·1
$g_\rho^2/4\pi$	2·203	2·14	—

[a] Daly *et al* (1982) and Adylov *et al* (1977)

Kulshreshtha 1982; Kulshreshtha and Mitra 1983; Mitra and Mittal 1984; Mittal and Mitra 1984).

We close this section with the results of a recalculation of the physical quantities which depend on the hadronic wave functions (Mitra and Kulshreshtha 1982; Kulshreshtha and Mitra 1983; Mitra and Mittal 1984; Mittal and Mitra 1984) as a result of the change of parametrization from (1) to (6), and the revised definition (4) of the reduced spring constant $\tilde{\omega}$ for unequal mass kinematics. For the various electroweak couplings of meson states, these results are summarized in table 3 together with the results of the earlier calculation (Mitra and Kulshreshtha 1982) and the corresponding experimental values for comparison. As may be seen from the table, the "good" features are practically unaffected, rather exhibiting a slight overall improvement if anything.

For the electromagnetic couplings of qqq states, the complete formalism is much more elaborate (Mitra and Mittal 1984) than for $q\bar{q}$ states and has been described in detail in a parallel publication (Mitra and Kulshreshtha 1982; Kulshreshtha and Mitra 1983). As such we record here only the (slight) modifications arising out of the change of parametrization from (1) to (6). For the charge radius of the proton we now get

$$\langle r_p^2 \rangle^{1/2}\ 0{\cdot}87\,\text{fm (present) and } 0{\cdot}86\,\text{fm (Mitra and Mittal 1984)}$$

to be compared with the experimental value of 0·87 (±0·01) fm. Similarly for the magnetic moments* of baryons the results of the present calculation with parameters

* The calculational procedure for unequal mass kinematics which requires the use of a sort of weighted average over the different ways in which the constituent quarks (of unequal mass) can interact with the electromagnetic field has been outlined separately in the Appendix of Mitra and Mittal (1984). The same procedure is adopted here, except for a slight redefinition of the weighting procedure which may be symbolically compared as follows. In Mitra and Mittal (1984), the magnetic moment has been defined as

$$\mu = \sum_{i=1}^{3} A_i \Big/ \sum_{i=1}^{3} B_i$$

where $i(=1, 2, 3)$ labels the different ways of combining the constituents. The present procedure corresponds to the definition

$$\mu = 1/3 \sum_{i=1}^{3} (A_i/B_i)$$

For the precise definition of A_i, B_i, see Mitra and Mittal (1984).

Table 4. Modification in the magnetic moment predictions for baryons (in n.m. units) arising out of the change in inputs from equations (1) to (6)

Baryon	Present	Mitra and Mittal (1984)	Expt.
p	2·771	2·796	2·793
n	−1·847	−1·854	−1·913
Σ^+	2·245	2·626	$2·33 \pm 0·13^{a, c}$
Σ^-	−0·748	−0·876	$-0·89 \pm 0·14$
Λ	−0·77	−0·57	$-0·614 \pm 0·005$
Ξ^0	−1·5103	−1·518	$-1·236 \pm 0·01^{a, b}$
Ξ^-	−0·755	−0·751	$-0·75 \pm 0·07$

[a] Quoted in Bohm *et al* (1982); [b] Cox *et al* (1981);
[c] Aukenbrandt *et al* (1983)

(6), as well as those of a parallel calculation (Mitra and Mittal 1984) with parameters (1) are recorded in table 4 together with the experimental data. Except for the Λ-case, the overall agreement (with no free parameters) is almost unaffected.

As for the various pionic couplings, the results so far obtained with the input (1) confined to equal mass kinematics and are practically unaffected by the change (1) to (6). For completeness, we record these results only

$$G^2_{NN\pi}/4\pi = 13·02 \text{ MeV (Mittal and Mitra 1984) (expt: } 14·6 \pm 1),$$

$$\Gamma(\Delta \to N\pi) = 104·6 \text{ MeV (Mittal and Mitra 1984) (expt: 110–120).}$$

4. Discussion and conclusions

The foregoing represent the results of a comprehensive reassessment of a Bethe-Salpeter dynamics of harmonic confinement of light quarks under development for the last three years. Since the initial proposals (Mitra 1981), the techniques have been continuously refined (Mitra and Kulshreshtha 1982; Kulshreshtha and Mitra 1983; Mitra and Mittal 1984) and the applications considerably diversified to meson and baryon spectra (Kulshreshtha *et al* 1982), their electroweak couplings and their strong (pionic) interactions under a single integrated framework, in which a universal spring constant ($\tilde{\omega}$) representing the confining mechanism and the quark masses (m_q) of the concerned hadrons play central roles. Since other specialized applications such as R-hadron spectra (Mitra and Ono 1983) and proton decay (Mitra and Ramanathan 1983, 1984), have been in progress and several more are envisaged, some of the weaker links in the framework have needed strengthening in order to provide some overall credibility to the entire approach. It is with this end in view that a major improvement in the solution of the $q\bar{q}$ BS equation has been undertaken in this paper through a more exact treatment of the $\hat{Q}_q$ operator and a more systematic handle on the coulomb term. Such a treatment of the $\hat{Q}_q$ term has led to a considerable refinement in its eigenvalue, resulting in the eventual replacement $Q_N \to Q'_N$, without otherwise affecting the formal structure of the original equation $F(M) = N + \text{const}$. However, this refinement which is numerically significant for $N = 0$, has proved to be of considerable value in the inversion of the equation $F(M) = N + 1/2$, leading to an explicit value of M in excellent agreement with experiment all the way from very low mass mesons to excited states as high as $N = 5$. While the overall shortfall of one unit in the zero point energy is still not understood (and must at this stage be regarded as a free parameter), the same amount of shortfall for most other hadrons (qqq, $qq\bar{q}\bar{q}$) (Kulshreshtha *et al* 1982; Mitra 1982) is nevertheless suggestive of a rather general inadequacy in the approach which should be susceptible to a separate examination of the foundations (instantaneous approximation, break-up into the coulomb and confining terms, etc) without violating the numerical results noted above.

Another feature of our results is that for light (uds) quark composites ($q\bar{q}$) the fits to their mass spectra are on the whole less accurate (2–5 %) than the almost perfect fits to the semi-heavy ($Q\bar{q}$) mesons, which have been obtained at the cost of a fresh input for the heavy quark mass (m_Q) characterizing each flavour region (c, b). However, since the ($Q\bar{q}$) states are so far available only for $L = 0$, it may be premature to draw any firm conclusions in regard to the success of this model in respect of their excited states, though table 2 formally records such predictions for low lying states. More serious is the problem of $Q\bar{Q}$ mesons which are much richer in spectroscopy than are the $Q\bar{q}$ states. As had indeed been discovered earlier (Mitra and Santhanam 1981a, b), this model seems inadequate for the $c\bar{c}$ spectra and even more so for $b\bar{b}$ spectra, reflecting the growing importance of the coulomb term *vis a vis* the confining term. The success of our BS model up to $Q\bar{q}$ systems suggests that it works for two light quarks ($q\bar{q}$) and even up to one heavy quark (Q) and one light quark ($\bar{q}$). In each case the constituents maintain a 'safe' distance from each other so as to conform the HO dominated character of the formalism with the coulomb term treated perturbatively.

However for a $Q\bar{Q}$ system, the constituents are close enough to each other so as to necessitate a more exact treatment of the coulomb term, something which the present formalism does not yet facilitate. For such systems, on the other hand, the simpler NR Schrödinger equation is in principle adequate, as long as the important requirement of exact treatment of the coulomb term is not lost sight of. In this respect since excellent treatments already exist in the literature for the spectroscopy of heavy quarkonia (Ono and Schoberl 1982; Bertelman and Ono 1981), the present approach is best regarded as largely complementary to these, both in regard to methodology as well as to the systems under investigation. To reconcile the two approaches would presumably require a dynamical mechanism capable of predicting the momentum transfer (Q^2) dependence of both the quark mass $m_q(Q^2)$ and the reduced spring constant $\tilde{\omega}(Q^2)$.

As to the ansatz (3) on the M^2-dependence of the QCD charge α_s, it has a natural meaning only in the time-like region ($Q^2 = -t$) of the $q\bar{q}$ annihilation channel, but not when used as a q-q or q-$\bar{q}$ potential (as in this application), where the legs refer to quarks in different initial and final states (when looked in the direction of the annihilation channel). Our defence of this ansatz for a term which has been employed only perturbatively is largely phenomenological, insofar as it reproduces the range of α_s-values normally used in potential calculations, but it seems to receive an added support from a recent bag-model derivation of a very similar formula (Carlson *et al* 1983).

Other relativistic treatments that exist in the literature for light quark spectroscopy, include (i) the bag model (Chodos *et al* 1974) or its variants (Zhu Wei 1982), and (ii) the lattice gauge models (Hamber 1982; Luscher 1983; Hamber 1983). So far there does not seem to be enough evidence of an equally quantitative set of fits to the masses over so many distinct flavour sectors in either of these models. In particular, the pion in bag model treatments often needs a special status (Chodos and Thorn 1975; Brown and Rho 1979; Thomas *et al* 1981) while the lattice gauge models (Hamber 1982; Luscher 1983; Hamber 1983) do not yet seem to exhibit adequate stability in the numerical results (Hamber and Parisi 1983; Bernard *et al* 1983) when the lattice size goes to zero. More importantly, these classes of models are not easily amenable to applications other than hadronic mass spectra, especially to a wide range of processes relating to the electroweak and hadronic couplings of mesons and baryons. On the other hand, the BS model lends itself most naturally to such relativistic applications through the elegant (four-dimensional) language of field theory and Feynman diagrams (Mitra and Kulshreshtha 1982; Mitra and Mittal 1984).

It is pertinent once again to ask in what manner if any our BS frameork compares with standard QCD. To the extent that the colour and spin dependence of the q-q or q-$\bar{q}$ kernels are those of perturbative QCD, it is a fair inference that the present approach is strongly QCD-oriented, for the only difference lies in the spatial dependence. In this connection it is useful to remember that perturbative QCD is good only for the shortest distances while its intermediate energy manifestations are often clouded in ambiguous approximations. Attempts to circumvent such regions by defining suitably 'scaled' functions, *e.g.* in connection with the electromagnetic form factor of the deutron (Brodsky *et al* 1983) can only be at the cost of the very nature of the information implicit in the physics of intermediate and medium high energies (1–5 GeV) characterizing the confining region that the QCD theory is designed to unearth in the first place. Judged from such angles, our effective HO assumption on the spatial dependence of the pairwise q-q or q-$\bar{q}$ kernels, while retaining other QCD features (w.r.t. spin and colour) represents in all probability a fair degree of implementation of the basic QCD spirit, at least until such time as more reliable and practical tools of QCD calculations become available for the confinement region of hadron dynamics. This conclusion is greatly reinforced by the entire sweep of agreements on a wide range of hadronic properties (from mass spectra to coupling structures) obtained within a unified framework of harmonic confinement, *albeit* at the cost of a rather fundamental constant ($\tilde{\omega}$) as a basic input. In retrospect, the BS formalism for harmonic confinement has turned out to be a viable calculational programme for hadron dynamics with a wide sweep of experimental successes, a conclusion which has been considerably strengthened by the distinct success now achieved in this paper on the absolute mass predictions of whole towers of $q\bar{q}$ states, as well as those of the available $Q\bar{q}$ states employing a more refined formalism for the operator $\hat{Q}$ than was hitherto possible (Mitra and Santhanam 1981a, b; Mitra and Kulshreshtha 1982). Because of its basic simplicity and considerable ease of application to wide ranging phenomena, without sacrificing fully relativistic features, this model competes very favourably with contemporary approaches (Chodos *et al* 1974; Zhu Wei 1982; Hamber 1982; Luscher 1983; Lipps 1983; Hamber and Parisi 1983; Bernard *et al* 1983) whose applicational facilities still seem to be severely limited at the present stage of theoretical development. Several other applications of a more sophisticated kind (*e.g.* L-excited hadron couplings, high energy meson-baryon reactions and electromagnetic masses of hadrons) are under way.

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