

A kinematical basis for power form factors

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Abstract. A kinematical basis is proposed for form factors of the power type associated with multiple derivative couplings, on the basis of a Lorentz contraction effect on the external momenta involved in the transition matrix elements for mesons and baryons as appropriate quark composites. The argument (due to Licht and Pagnamenta) which applies separately to the Breit and c.m. frames for a decay matrix element provides a formal theoretical justification for the *ad hoc* power form factors used by the Delhi group in a series of applications to hadronic processes over the past few years. The radius of interaction finds a natural place in this description simply from dimensional considerations, and its rather small magnitude, less than $0.5 F$, estimated from fits to the data indicates a relatively small role played by structure effects. The Breit frame form factors, which work somewhat better than the c.m. frame ones (effectively used in the earlier studies), give a rather impressive sets of fits to the baryon decays in the $(L+1)$ wave (consistently for both vertical and horizontal) and the $(L-1)$ wave (mainly horizontal). The mesonic decays, the data for which are available mostly for the $(L+1)$ wave, are also fitted with an equal degree of consistency without any extra assumptions.

Keywords. Lorentz contraction; relativistic hadron couplings; power form factors.

1. Introduction

Relativistic form factors for hadron couplings require a more concrete dynamical input than is provided under a mere $SU(6)_w \times O(3)$ framework. Since these form factors are given by the overlap integrals among the (quark) wave functions involved the dynamical input is represented by the knowledge of the latter, e.g. as in the harmonic oscillator model (Feynman *et al* 1971). In the absence of such input, born out of reluctance or otherwise to accept a particular model from the start, they can at best be parametrised phenomenologically, preferably with the inclusion of some general principles.

Over the past few years a fairly systematic study has been carried out, of a variety of (on and off-mass shell) hadronic processes, by Mitra and collaborators (Mitra 1973) using the language of supermultiplet form factors of the power type (in successively improved versions) in an $SU(6)_w \times O(3)$ model of hadron couplings of P and V-mesons (Mitra 1969, Rosner 1973). This approach has found fairly successful applications to two body decays (Choudhury and Mitra 1970, Katyal and Mitra 1970) and reaction processes induced by pionic (Sen Gupta and Gupta 1972) and electromagnetic interactions (Sood and Mitra 1973, Ahmed and Mitra 1973). This warrants a further justification of these form factors at a more fundamental level. In particular, it would be more interesting

if simple kinematical considerations could provide a theoretical basis for power form factors.

The object of this paper is to suggest that a kinematical basis is indeed available to justify such an expectation. In effect, the explicit parametrisation of the power form factor (PFF) used in the cited references is sought to be 'explained' through a kinematic argument due to Licht and Pagnamenta (Licht and Pagnamenta 1970). The LP argument, which is given for the Breit frame can also be adapted to the c.m. frame of the decaying particle, thus motivating two distinct types of Lorentz contraction factors appropriate to these two frames. In particular, the c.m. frame version agrees exactly with that used in the earlier references, except for the replacement $E_k \rightarrow E_{k/m}$. The corresponding form in the Breit frame gives rise to a qualitatively similar, but, quantitatively different structure. To maintain a dimensional balance for the coupling constant, in view of the appearance of multiple derivative couplings, one now requires a dimensional quantity, hopefully universal in character, which can most naturally be interpreted as a radius of interaction on the lines of the Harari model (Harari 1971).

Such a dimensional interpretation of the radius makes physical sense if its magnitude is reasonably small (implying little structure effects), so that the bulk of the energy variation in the form factors comes about through the Lorentz contraction effect thus giving the latter the effective look of a PFF. Though radius form factors have been employed for the investigation of hadronic decays (Faiman and Plane 1972, Kamath and Mitra 1973), the structure details implied by the occurrence of Bessel functions have played an essential part in the analyses. Here we are interested in a relatively passive (dimensional) role of this quantity in the (kinematical) context of Lorentz contraction effects playing the more active role. Moreover, we are concerned not so much with any detailed fits to the decays with a specific model, as to examine with the help of some pertinent data, the kinematical extent to which these ideas are realized in practice.

The plan of this paper is as follows. In section 2 we briefly explain the kinematical basis of the power form factor through the LP argument, but, distinguishing between the Breit and c.m. frames. In sections 3 and 4 we apply the form factors so obtained to several pertinent two-particle ($L \pm 1$) wave decays of the baryons and mesons respectively on the lines of a recent analysis (Kamath *et al* 1974) for the power form factor (now identified as the c.m. type.) The availability of adequate number of 'vertical' ($L + 1$)-wave baryon decays allows us to display these results in a semi-logarithmic plot of $G_L^2/4\pi$ (defined in section 2) against $l (= L + 1)$. The rest of the data for baryons and mesons, being mostly of a 'horizontal nature', have been collected in tables 1, 2 and 3. Finally in section 5 we review the main conclusions of this paper.

2. Theory

The clue to a kinematical understanding of the power form factor comes from the LP paper, which was written a few years ago, but which has caught particular attention in recent times in the context of investigations on the relativistic structures of hadron vertices from the quark point of view (Le Yaouanc *et al* 1973, 1974). The central argument here lies in the use of the Breit frame for the two 'composite' hadrons (A and B) involved in the process $A \rightarrow B + C$, where C is a radiation quantum.

The non-relativistic overlap integral in momentum space has the structure

$$\int d\mathbf{q} \psi_A(\mathbf{q}) \psi_B^*(\mathbf{q} - \alpha \mathbf{k}_e) \quad (1)$$

where \mathbf{k}_e is the momentum of the radiation quantum, \mathbf{q} is a normalized internal momentum (Feynman *et al* 1971) and α is a geometrical factor equal to $\sqrt{2}$ for mesons and (-2) for baryons in the FKR normalization. (For baryons, there is a second internal momentum corresponding to $[2, 1]_s$ symmetry, but it is suppressed in eq. (1) because it plays no active role in the emission process).

The essential structure of the overlap integral for the decay of a resonance of orbital spin L into one of spin zero has been explained in detail in earlier papers (Mitra 1970) and is given by

$$f_L(k^2) k_{i_1} k_{i_2} \cdots k_{i_L} B_{i_1 \dots i_L}^L \quad (2)$$

where $f_L(k^2)$ is a form factor, and $B_{i_1 \dots i_L}^L$ is a symmetric traceless tensor of rank L (Fronsdal 1958, Blankenbecler and Sugar 1968).

The corresponding relativistic structure, following the L - P argument in coordinate space would amount to the following momentum space replacement

$$\mathbf{k}_e = \mathbf{k}_a - \mathbf{k}_b \rightarrow \frac{m_b}{E_{bp}} \mathbf{k}_a - \frac{m_a}{E_{ap}} \mathbf{k}_b \quad (3)$$

where the 3-momentum \mathbf{p} in the Breit frame is related to the c.m. frame \mathbf{k} by

$$\mathbf{p}^2 = m_a^2 \mathbf{k}^2 (2m_a^2 + 2m_b^2 - \mu^2)^{-1} \quad (4)$$

with $\mu = m_e$, $E_{ap} = (m^2 + p^2)^{1/2}$ etc.

Using the Breit frame result $\mathbf{k}_a = -\mathbf{k}_b = -\mathbf{p}$, eq. (3) may be rewritten as, $\mathbf{k}_a \rightarrow \mathbf{k}_e \gamma_p$, where,

$$\gamma_p = \frac{1}{2} \left(\frac{m_a}{E_{ap}} + \frac{m_b}{E_{bp}} \right) \quad (5)$$

expresses the Lorentz contraction effect on the 3-momentum \mathbf{k}_e of the emitted quantum.

Similarly, if one were to adapt the corresponding L - P argument to the rest frame of m_a , there would result the replacement $\mathbf{k}_e \rightarrow \mathbf{k}_e \gamma_k$ where,

$$\gamma_k = m_b / E_{bk} \quad (6)$$

(For details of these and allied derivations, see Indrakumari and Mitra 1975). The substitutions, eqs (5)-(6), have the effect of producing a multiplying form factor in the interaction, eq. (2), of magnitudes $(\gamma_p)^L$ and $(\gamma_k)^L$ for the Breit and c.m. frames respectively.

These form factors have qualitatively, the same features as the PFF $(E_k)^{-L-1}$ that were used in a series of recent applications to photoproduction and electroproduction processes (Ahmed and Mitra 1973, Sood and Mitra 1973). Indeed the Lorentz contraction factor corresponding to the c.m. frame, *viz.* $(\gamma_k)^L$, is identical with the PFF with the substitution $L \rightarrow L + 1$, the extra γ_k factor arising from the P -wave nature of the interaction for $L = 0$. For the Breit frame, the

correspondence is at least qualitatively similar to the c.m. case defined above, so that the net factor in this case is now $(\gamma_p)^{L+1}$. Further there appears an additional factor $\beta = m_a m_b / E_{a,p} E_{b,p}$ from the Lorentz contraction effect on the z -component of the integration variable in the overlap integral. (For details on this item, see Indrakumari and Mitra 1975).

The effect of the mass factor m_b^{L+1} in the above necessitates a re-interpretation of the dimensionless scale factor $(S_F)^{L+1}$ associated with the PFF used earlier (Sood and Mitra 1973, Kamath *et al* 1974) as $S_F = m_b R$, where R should have the significance of something equivalent to the radius of interaction on dimensional grounds. The appearance of an "effective radius of interaction" thus harmonizes with the Lorentz contraction effect through this argument.

While the foregoing considerations are not meant as a substitute for a more full-fledged model of form factors, necessarily of dynamical origin, it is quite conceivable that a major aspect of the variation is already incorporated in these kinematical effects. The form E_k^{-L-1} has recently been employed (Kamath *et al* 1974) for an extensive analysis of the hadronic decay widths at both the vertical and horizontal levels. The value $S_F = 1.16$ of the scale factor used therein, may now be reinterpreted as an effective radius $R = S_F m_b^{-1} = 0.23F$ with $m_b \approx 1$ GeV. This rather small value of R is reminiscent of a structureless interaction justifying a relatively small role of structure effects (presumably of dynamical origin) manifested through suitable Bessel functions of the spherical type used in the literature (Faiman and Plane 1972, Kamath and Mitra 1973). Indeed, this magnitude of R is in qualitative accord with the result of FP but for entirely different reasons.*

In the following two sections (sections 3 and 4) we shall apply these structures to an analysis of some of the decay modes of the baryons and mesons. It will be seen that the Breit frame form factor provides a much better interpretation for the radius on the vertical scale than the c.m. frame one, because of the extra sensitivity of the latter to the variation in m_b on the vertical scale. Further for mesons, the Breit structures will be found to be useful for removing some well-known ambiguities on the choice of the radiation quantum, as well as for providing a more consistent (numerically) basis for the relativistic normalization in meson couplings.

3. Baryon Decays

For the scrutiny of the baryon decays, we keep in mind, the possibility of conforming to the same general principle that was used as a guideline in the earlier phenomenological formulations (Mitra 1973), *viz.*, the reduced coupling constant, after extracting the variation suggested above (section 2), should exhibit constancy in alternate values of L (Regge universality) and successive values of L (exchange degeneracy). According to the derivation given in the preceding section, the

* In the present case, the smallness of the radius, implying an insensitivity to the details of structure, is a passive by-product of the Lorentz contraction effect which accounts for the bulk of the energy variation. The latter exhibits a strong variation on the vertical scale but can make only a marginal impact on the decays of the low lying ($L = 1$) resonances. On the other hand the FP result, *viz.* a small value of R , deduced entirely from the $L = 1$ decays, and that without considering the Lorentz contraction effect, is difficult to understand from the above point of view.

reduced coupling constant $g_L^{(+)}$ in the form factor $f_L^{(+)}$ for $L + 1$ wave decays may be defined as,

$$f_L^{(+)} = g_L^{(+)} [\beta (\gamma_p R_+)^{L+1}; \quad \beta (\gamma_k R_+)^{L+1}] \quad (7)$$

and should hopefully be independent of L following the universality arguments given in the earlier applications. We shall see below the extent to which this conjecture is fulfilled for the $(L + 1)$ -wave baryon decays, of which there are considerable data on the vertical and horizontal scales.

For the $(L - 1)$ wave modes, on the other hand, paucity of data for $L > 2$ necessarily limits this investigation mainly to the horizontal modes. Also for the $(L - 1)$ modes, we incorporate as in the earlier calculation (Mitra 1973);

(i) the GOR factor (Gell-Mann *et al* 1968) which effectively amounts to a multiplication by the factor $(M_L - m)$ on the mass shell,

(ii) the parametrisation is now dominated by the recoil term (Mitra and Ross 1967) or its essentially equivalent version of the 3P_0 model (Micu 1969, Colglazier and Rosner 1971) and L -broken $SU(6)_w$ (Rosner 1973).

Instead of eq. (7) for the $(L + 1)$ wave modes, we now have, for the $(L - 1)$ wave decays,

$$f_L^{(-)} = g_L^{(-)} [\beta (\gamma_p R_-)^L; \quad \beta (\gamma_k R_-)^L] \quad (8)$$

the extra $\gamma_p R_-$ (or $\gamma_k R_-$) factor arising out of the inclusion of the GOR effect. Since the problem of mixing for the $(L - 1)$ states does not fall under the scope of this paper, cases of decay involving $SU(3)$ mixtures, etc., have been ignored in this study.

3.1. $(L + 1)$ wave decays

The L -dependence of the widths can be expressed on the lines of an earlier analysis (Kamath and Mitra 1973) as,

$$\Gamma = c_\lambda \cdot k_L \cdot k_L^{2L+2} \cdot h(k_L) \cdot p_n^{-1} \cdot X_L \quad (9)$$

where

$$X_L = f_L^{(+)^2} g_L^{(+)^2} / 4\pi$$

Figure 1 shows a semi-logarithmic plot of $G_L^2/4\pi \equiv X_L \gamma_{p,k}^{-2(L+1)}$ against $l (= L + 1)$ for the vertical sequences for both the Breit and c.m. form factors $(\gamma_p)^{L+1}$ and $(\gamma_k)^{L+1}$. The slope of the straight line fits, $2 \log R_+$, directly yield the reduced coupling constant via,

$$\log G_L^2/4\pi = 2l \log R_+ + \log g_L^{(+)^2}/4\pi. \quad (10)$$

It is clear from figure 1, that the quality of the fit* is somewhat superior in the

* As mentioned in Section 2 there is a case for a second factor in the form factor arising out of a second (passive) variable of integration. However, the use of this extra factor gives a much poorer fit to the data than the ones given in figure 1. We have therefore ignored this factor, partly taking comfort from the passive nature of the variable associated with it, and partly from the consideration that additional dynamical effects which presumably play some role have also not been considered.

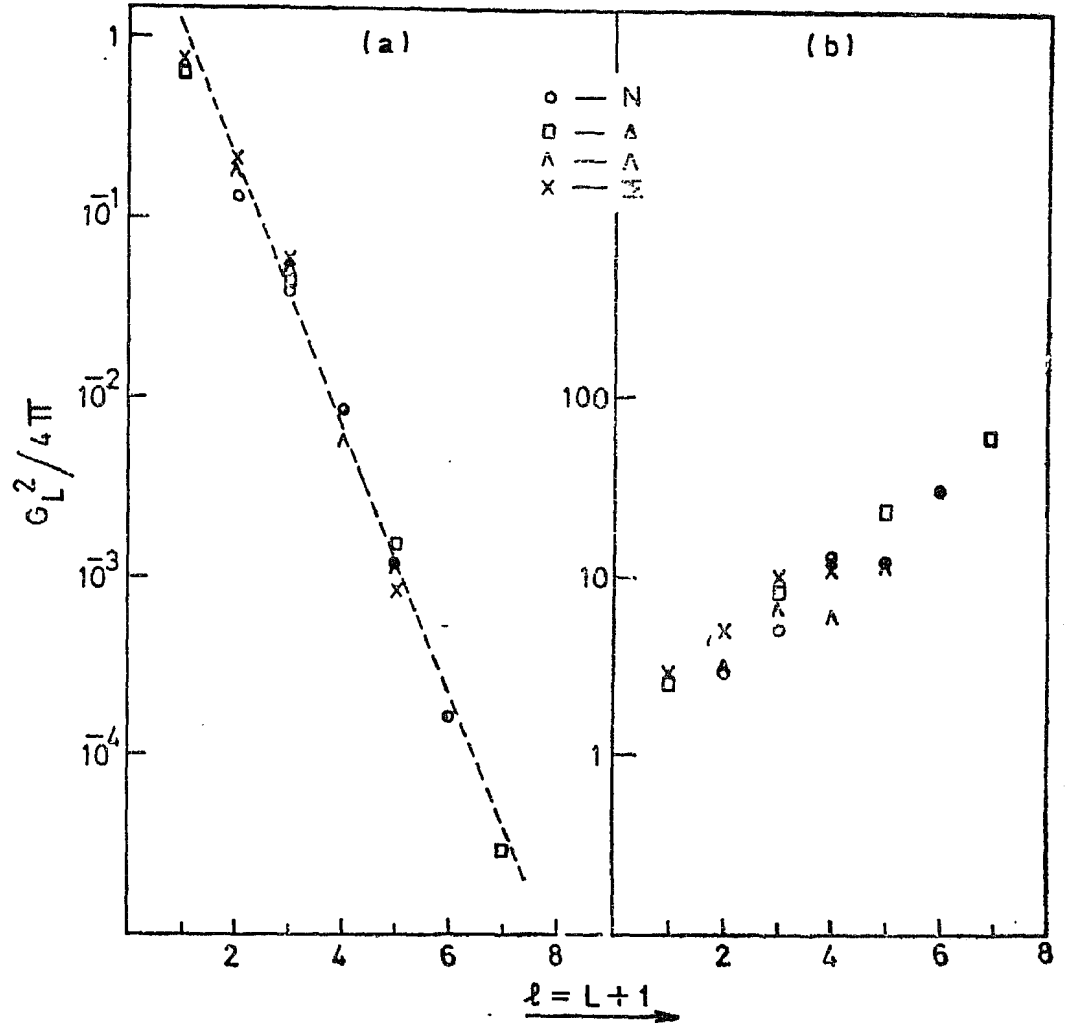


Figure 1. Semi-logarithmic plot of $G_L^2/4\pi$ (defined in text) against $(L+1)$ for the vertical sequences in (a) Breit frame, (b) c.m. frame.

Table 1. List of the $(L+1)$ -wave reduced coupling constants $g_L^{(+)^2}/4\pi$ for the various horizontal data in the Breit frame. These have been obtained by using the $(L+1)$ -wave radius $R_+ = 0.262 F$. We have also included some of the states on the vertical sequences for comparison.

Decay mode	$g_L^{(+)^2}/4\pi$	Decay mode	$g_L^{(+)^2}/4\pi$
$N(1520) \rightarrow N\pi$	0.734	$\Sigma(1385) \rightarrow \Lambda\pi$	0.980
$N(1670) \rightarrow N\pi$	3.094	$\Sigma(1385) \rightarrow \Sigma\pi$	0.957
$\Delta(1670) \rightarrow N\pi$	1.393	$\Sigma(1670) \rightarrow N\bar{K}$	1.849
$\Delta(1890) \rightarrow N\pi$	0.882	$\Sigma(1670) \rightarrow \Lambda\pi$	1.198
$\Lambda(1520) \rightarrow N\bar{K}$	1.033	$\Sigma(1765) \rightarrow N\bar{K}$	1.196
$\Lambda(1520) \rightarrow \Sigma\pi$	1.454	$\Sigma(1915) \rightarrow N\bar{K}$	1.712
$\Lambda(1690) \rightarrow N\bar{K}$	0.153	$\Sigma(1915) \rightarrow \Lambda\pi$	0.368
$\Lambda(1690) \rightarrow \Sigma\pi$	2.200	$\Sigma(2030) \rightarrow \Lambda\pi$	0.376
$\Lambda(1815) \rightarrow \Sigma\pi$	0.955	$\Sigma(2030) \rightarrow \Sigma\pi$	0.147
$\Lambda(1830) \rightarrow \Sigma\pi$	0.656	$\Xi(1530) \rightarrow \Xi\pi$	0.686
$\Lambda(2100) \rightarrow \Sigma\pi$	1.072		

Breit frame, eq. (5), to that in the c.m. case, eq. (6), inasmuch as only the Δ -sequence gives comparable fits in both, while the other sequences appear somewhat scattered in the c.m. frame. On the whole there appears a fair degree of consistency in the determination of the radius from the sequences, the average value being,

$$R_+ \approx 0.26F; \quad g_L^{(+)^2}/4\pi = 1.14. \quad (11)$$

In producing this fit, a decisive role has been played by the factor $(\gamma_n)^{L+1}$, which is especially sensitive on the vertical scale, in fulfilling our expectations on Regge universality and exchange degeneracy, manifested in the constancy of $g_L^{(+)}$ with L .

The horizontal $(L+1)$ -wave modes† have also been analysed in the same spirit as above, and their respective reduced coupling constants are collected in table 1. These have been obtained by using the $(L+1)$ -wave radius, eq. (11), as deduced from figure 1. The values of $g_L^{(+)^2}/4\pi$ in table 1 show a fair degree of unanimity for practically all the decay modes except a few stray cases, viz. $\Lambda(1690) \rightarrow N\bar{K}$, $N(1670) \rightarrow N\pi$ and $\Sigma(2030) \rightarrow \Lambda\pi$, $\Sigma\pi$. The mean value of the reduced coupling constant

$$g_L^{(+)^2}/4\pi = 1.2 \pm 0.4 \quad (12)$$

is again in conformity with the deduction from the vertical sequence.

3.2. $(L-1)$ wave decays

The unmixed $(L-1)$ wave cases collected in table 2 are for the states of $L=1$ and 2 only. As there are no vertical sequences available here, we have chosen to display in table 2, the two dimensional constants $(g_1^{(-)^2}/4\pi) R_-^2$ and $(g_2^{(-)^2}/4\pi) R_-^4$ for $L=1$ and 2 as determined from the observed decays. From these two numbers each of which exhibits a fair degree of consistency within its own supermultiplet region, it is meaningful to obtain estimates of R_- and $g_L^{(-)^2}/4\pi$ which work out as,

$$R_- \approx 0.39F; \quad g_L^{(-)^2}/4\pi = 0.024. \quad (13)$$

This equation formally satisfies the requirements of Regge universality and exchange degeneracy, but the same cannot be confirmed in the absence of data from higher L states. It may be seen that the magnitude of R_- is appreciably higher than that of R_+ , eq. (12), determined for the $(L+1)$ wave. On the other hand, there is little a priori reason for the two to be equal. Also, the odd case of $\Lambda(1405) \rightarrow \Sigma\pi$ is rather poorly represented by this factor, but this appears to be the price for using a uniform set of form factors for both $(L \pm 1)$ wave decays‡.

4. Meson decays

The application of the earlier power form factors to the meson data has been based on the explicit assumption of the similarity of the structures for baryons and

† As in the case of figure 1, we do not give in table 1, the results with the replacement $\beta \rightarrow \beta^2$ since the scatter in the fits for the latter is greatly increased.

‡ This problem had also been encountered in an earlier analysis, but it was sought to be overcome by using an alternate shape of the power form factor.

Table 2. List of the $(L-1)$ wave baryonic dimensional constants $(g_L^{(-)2}/4\pi) R_-^{2L}$ for the supermultiplets $L = 1$ and 2 in the Breit frame. The radius R_- in Eq. (13) has been obtained from a comparison of $N(1520) \rightarrow \Delta\pi$ and $N(1690) \rightarrow \Delta\pi$

Decay mode	$(g_L^{(-)2}/4\pi) R_-^{2L}$
$N(1520) \rightarrow \Delta\pi$	0.0219
$\Delta(1650) \rightarrow N\pi$	0.1318
$\Delta(1670) \rightarrow \Delta\pi$	0.0158
$\Lambda(1405) \rightarrow \Sigma\pi$	0.2221
$N(1690) \rightarrow \Delta\pi$	0.0205
$\Lambda(1815) \rightarrow \Sigma^*\pi$	0.0209
$N(1860) \rightarrow N\eta$	0.0366
$N(1860) \rightarrow \Lambda K$	0.0190
$\Delta(1890) \rightarrow \Delta\pi$	0.0140

mesons (Mitra 1971, 1973). Such an affinity is indeed to be expected in the present theory inasmuch as the non-relativistic wave functions for mesons and baryons effectively involve an emission of the radiation quantum by only one of the internal momentum, as already noted in section 2. However, unlike the baryons, pure mesonic decays, suffer from the ambiguity as to which of the two final state mesons should be chosen as the radiation quantum. While the power and radius form factors used earlier clearly favoured the lighter meson (except when one of them happens to be a vector meson), the present investigation, rather surprisingly, indicates that the heavier meson should always be chosen as the radiation quantum. Again the results unequivocally point to a common normalization factor $(4M_L^2)^{1/2}$ (cf. Becchi and Morpurgo 1966) applicable to all cases irrespective of the nature of the final states mesons (Mitra 1973).

Before discussing the quantitative results we note here that unlike the 'stable' value of m_b in the baryonic decays, the mesonic modes suffer from a wide variation in the masses of the decay products, especially if m_b happens to be a pionic mass. In quantitative terms, it means that the radius and reduced coupling constant for 2-pion final state decays would be considerably different from those determined for other decay modes. A simple way out of this difficulty is to replace the variable mass m_b by a fixed mass near the central value, say, the mass of the ρ -meson. We postulate therefore the replacement

$$\frac{m_a m_b}{E_{ap} E_{bp}} = \beta \rightarrow \beta' = \frac{m_a m_\rho}{E_{ap} E_{b\rho}} \quad (14)$$

before giving the numerical comparison with the data. Table 3, which shows the fits to the data with the above replacement, indeed seems to bear out a fair degree of consistency within each supermultiplet for $L = 0$ and 1. The best values of R_+ and $g^{(+2)}/4\pi$ are obtained from a comparison of the Regge partners $\rho \rightarrow \pi\pi$ and $g \rightarrow \pi\pi$, and they work out to,

$$R_+ \sim 0.45F \quad g^{(+2)}/4\pi = 0.065. \quad (15)$$

Table 3. List of the $(L + 1)$ -wave mesonic dimensional constants $g_L^{(+)^2}/4\pi R_+^{2(L+1)}$ for the supermultiplets $L = 0, 1$ and 2 in the Breit frame. The second of the two final state mesons denotes the radiation quantum.

Decay mode	$(g_L^{(+)^2}/4\pi) R_+^{2(L+1)}$	Decay mode	$(g_L^{(+)^2}/4\pi) R_+^{2(L+1)}$
$\rho \rightarrow \pi\pi$	0.1653	$f' \rightarrow K\bar{K}$	0.2522
$\phi \rightarrow K^+K^-$	0.1628	$K_v \rightarrow \pi K^*$	0.7208
$K^* \rightarrow K\pi$	0.2804	$K_v \rightarrow K^*\pi$	1.1779
$K^* \rightarrow \pi K$	0.1123	$K_v \rightarrow \rho K$	1.1152
$f \rightarrow \pi\pi$	0.5649	$K_v \rightarrow K_\rho$	0.6018
$A_2 \rightarrow \pi\rho$	0.7088	$K_v \rightarrow K\omega$	1.1102
$A_2 \rightarrow \rho\pi$	0.9375	$K_v \rightarrow \omega K$	2.1837
$A_2 \rightarrow \eta\pi$	0.1610	$K_v \rightarrow \pi K$	0.5313
$A_2 \rightarrow \pi\eta$	0.2541	$K_v \rightarrow K\pi$	0.2682
		$g \rightarrow \pi\pi$	1.0938

5. Summary and Conclusions

The object of this paper has been to justify some rather successful applications of power form factors to a variety of hadronic processes by using some simple kinematical arguments which give rise to Lorentz contraction factors in the coupling structures when these are boosted to the relativistic region. As a by-product of this investigation, we find a natural place for the radius of interaction, thus harmonizing the kinematical effect of the Lorentz contraction implicit in the power form factor, with the purely dimensional concept of a radius of interaction, in addition to possible effects of a more dynamical origin, which require a specific model for their treatment. The choice of the factor γ_p (or γ_k) conforms to unit normalization in the non-relativistic limit, so as to bring out the precise amount of the relativistic effect.

The smallness of the $(L + 1)$ wave baryon radius is somewhat reminiscent of the result of Faiman-Plane, although as already remarked, we do not quite understand how such a small value could be inferred by them only on the basis of the $L = 1$ decays, which are not sensitive to the Lorentz contraction effect.

For the $(L - 1)$ wave data, we are somewhat intrigued by the appreciably different value of radius in this case compared to the $(L + 1)$ result. It is however premature to attach excessive significance to this difference on the basis of little vertical data for the $(L - 1)$ wave, and perhaps also in the absence of possible dynamical effects.

For the meson decays, a large variation in the masses (from π to ϕ) has necessitated some precaution against a literal interpretation of the mass m to one of the decay products, unlike the case of baryons, where this problem is not so serious. We have responded to this problem by choosing a central value for m_p , located at say m_ρ . As to the choice of the radiation quantum, our analysis, for the Breit frame, rather surprisingly but consistently, favours the heavier quantum for this role (cf. Kamath *et al* 1974), for c.m. frame results.

A common feature of both the baryon and meson results is that the analysis

in terms of the Breit frame yields consistently better numerical fits to the data than one with the c.m. frame (Kamath *et al* 1974). On the other hand a relatively large difference between the estimated radii of BBM and MMM interactions constitutes an unpleasant feature, unless one remembers the limitations of the basic premises of this investigation, *viz.*, the radiation quantum hypothesis and a lack of any dynamical input. There are theoretical reasons to believe that the radiation quantum hypothesis is itself inadequate (Indrakumari and Mitra 1975) as a basic postulate, and it may well have to give way to a more dynamical assumption such as the quark-pair-creation model (Le Yaouanc *et al* 1973). Since the scope of this (kinematical) investigation, precludes any commitment to these additional (dynamical) dimensions, we content ourselves with the optimistic remark that this rather general analysis provides a more formal understanding of many of the successes achieved with power form factors postulated earlier in a relatively ad hoc fashion.

6. Acknowledgements

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