Supersymmetric Phase Transition

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The existence of a second-order phase transition associated with spontaneous supersymmetry breaking in a prototype globally supersymmetric model with a conserved fermion number is demonstrated. The model is solved exactly and it is shown that one of the superfields develops a vacuum expectation value below a critical temperature and that the ground-state energy is nonzero.

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Recently there has been great interest in the study of supersymmetric theories.[1] In such theories, bosons and fermions are treated on the same footing by putting them in the same multiplet and supersymmetry (SUSY) connects the bosonic and fermionic components of such a supermultiplet. This ensures the absence of quadratic divergences in such a theory and avoids the problems associated with having fundamental scalars in unified theories.[2]

Symmetry must be spontaneously broken if nature is described by such a theory.[3] It is generally believed that SUSY is automatically broken at temperatures $T>0$ even if it is unbroken at $T=0$, because bosons and fermions obey different statistics.[3] On the other hand, it has been argued recently[4] that the difference in statistics of the particles belonging to the same supermultiplet is in itself no sign of symmetry breaking and low-T properties of such systems may in fact be the same as the $T=0$ properties. In view of this controversy, it is of great interest to have a model which, while retaining the essence of SUSY, is simple enough to be exactly solvable. In this Letter, we demonstrate the existence of spontaneous supersymmetry breaking at low temperatures for one such model by solving it exactly.

We consider a prototype SUSY theory[5] of a single scalar supermultiplet consisting of two complex scalars, A^+ and A^-, and a Dirac fermion (and their antiparticles). The Lagrangian of the model which conserves fermion number is

$$\mathcal{L} = |\partial A^+|^2 - |MA^+ + g(A^+)^2|^2 + |\partial A^-|^2 - |[M+2g(A^+)^*]A^-|^2 + \bar{\psi}(i\not{\partial} - M)\psi - [2gA^+\bar{\psi}_-\psi_+ + g(A^-)^*\bar{\psi}_+{}^c\psi_+ + \text{H.c.}], \tag{1}$$

where ψ_+ and ψ_- are the usual chiral projections of ψ, and ψ^c is its charge conjugate, M is the common mass, and g is a coupling constant. The particles A^+, A^-, and ψ have fermion numbers 0, -2, and -1, respectively. The Lagrangian is invariant under SUSY transformation (apart from a change by a total space-time gradient):

$$\delta A^+ = \bar{\epsilon}_-\psi, \quad \delta\psi = -[M+2g(A^+)^*]A^-\epsilon_+ - [MA^+ + g(A^+)^2]\epsilon_- - i\not{\partial}A^+\epsilon_- - i\not{\partial}A^-\epsilon_+, \quad \delta A^- = \bar{\epsilon}_+\psi,$$

where ϵ is an infinitesimal anticommuting Majorana spinor.

The grand canonical partition function of the model can be obtained by the standard method in the weak-coupling ($g\to 0$) limit. We get[6]

$$Z(\mu, T) = Z_B(0,T)Z_B(\mu,T)Z_F(\tfrac{1}{2}\mu, \tfrac{1}{2}T), \tag{2}$$

with

$$Z_B(\mu,T) = \prod_{\vec{k}}\{1-\exp[-\beta(E_k-\mu)]\}^{-1}\{1-\exp[-\beta(E_k+\mu)]\}^{-1},$$

$$Z_F(\mu,T) = \prod_{\vec{k}}\{1+\exp[-\beta(E_k-\mu)]\}\{1+\exp[-\beta(E_k+\mu)]\},$$

$$E_k = (k^2+M^2)^{1/2}, \quad \beta = T^{-1}.$$

Here $\vec{k}$ is a vector in d spatial dimensions and we have chosen units such that $\hbar = c = k_B = 1$. The chemical potential μ has been introduced so as to satisfy the constraint of the mean fermion-number conservation, i.e.,

$$\langle 2N(A^-) - 2\overline{N}(A^-) + N(\psi) - \overline{N}(\overline{\psi})\rangle = \text{const} \equiv N, \quad (3)$$

where $N(A^-)$ and $N(\psi)$ are the number operators for the particles A^- and ψ, respectively (the overbars refer to the corresponding antiparticles) and the angular brackets denote the usual statistical-mechanical average. The interactions between various particles have been kept only to the extent of their mutual interconversion and pair productions allowed by the Lagrangian (1). We assume that the system attains equilibrium with respect to all these processes.

Various factors in (2) may be interpreted as follows. The first corresponds to A^+ particles which have $\mu = 0$ because they can be created and destroyed freely. The second and third factors correspond to A^- and ψ (including antiparticles), respectively. Since these can be mutually converted while preserving SUSY, their μ's are related. From (1) we see that reactions of the type $\psi\psi \to A^-$ are allowed, so we must have $\mu(\psi) = \frac{1}{2}\mu(A^-)$.[7] Of course, the particles and the antiparticles have opposite μ's; this is reflected in (2).

From the partition function, one can get all the thermodynamics in the infinite-volume limit. For example, the net fermion number density ρ ($\equiv N$ per unit volume) is given by[6]

$$\begin{aligned} a_d|\rho| &= 2W_d(\alpha, \alpha) + W_d(\alpha, \varphi) + W_d(2\alpha, \alpha+\varphi) - 2W_d(4\alpha, 2\alpha+2\varphi), \\ W_d(\alpha, \varphi) &= b_d \sum_{r=1}^{\infty} r^{-(d'-1)} \sinh(r\alpha - r\varphi) K_{d'}(r\alpha), \end{aligned} \quad (4)$$

where

$$a_d = 2^{d-1}\pi^{d/2}\Gamma(d/2)M^{-d},$$

$$\alpha = M/T, \quad \varphi = (M-\mu)/T,$$

$$b_d = \pi^{-1/2}2^{d'}\Gamma(d/2)\alpha^{-(d'-1)}, \quad d' = \tfrac{1}{2}(d+1).$$

Here $K_d(z)$ is a modified Bessel function of order d.[8] From (2) we see that to keep the partition function real, we must have $|\mu| \leq M$.[9] In fact, we find from (4) that as $T \to \infty$, $\mu \to 0$ and μ increases (for $\rho > 0$) with decreasing T till $\mu = M$ for $T = T_c$ (say). For $T < T_c$, μ sticks at M. To determine if SUSY is broken, we calculate the vacuum expectation value (VEV) of the fields by introducing symmetry-breaking terms and letting them go to zero.[10] The VEV of a given field is given by the square root of the corresponding particle-number density ρ of the field. We find that, for $T < T_c$,

$$\rho(A^+) = (e^{M/T} - 1)^{-1},$$

$$\rho(A^-) = (e^{(M-\mu)/T} - 1)^{-1},$$

$$\rho(\psi) = (e^{(4M-2\mu)/T} + 1)^{-1}.$$

Clearly as $T \to 0$, $\rho(A^+) = 0 = \rho(\psi)$, but $\rho(A^-) \neq 0$. So the SUSY is broken at $T = 0$. As T rises, the ground state will be depleted not only because A^- will go to higher states but also because of the interconversion of these into fermions. So we expect the critical temperature T_c in this case to be lower than that in the corresponding Bose system. We also find that $E_g = \rho(A^-)M \neq 0$ which is a sure sign of SUSY breaking.[11]

The critical properties of the model can be studied as usual. We omit the details and present the results. (i) The phase transition is a second-order one accompanied by a discontinuity in the specific-heat derivative. (ii) The system is in the same universality class as the nonrelativistic Bose system and, therefore, has the same exponents, scaling functions, etc.[10b] (iii) The pressure, energy density, etc. are given by expressions similar to (4) involving infinite sums. As an example, we mention that T_c in the extreme relativistic limit $M \ll T_c$ is given by $T_c = (\frac{2}{5}\sqrt{6}) \times (\rho/M)^{1/2}$ for $d = 3$ as compared with $T_c = \sqrt{3}(\rho/M)^{1/2}$ in the usual Bose gas.[9]

In summary, we have shown spontaneous symmetry breaking (which is due to the appearance of VEV of one of the boson fields and E_g being nonzero) resulting in a second-order phase transition in a prototype model of a single scalar multiplet in the weak-coupling limit. We emphasize that the symmetry breaking is not due to different statistics of the particles but due to the presence (or formation, if initially absent) of bosons in the system.

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[1]For a review, see, e.g., P. Fayet and S. Ferrara,

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[11]See Witten, Ref. 3.