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A NEW SCHEME FOR THE CLASSIFICATION OF BARYON RESONANCES

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ABSTRACT

A scheme based on pure s wave QQ interactions is suggested for the classification of baryon resonances. It yields only the states $[56, (2\ell)^+]$ and $[70, (2\ell+1)^-]$ of $SU(6) \times O(3)$ symmetry, together with their radial (N) excitations which increase in steps of unity. These states, together with N excitations, are precisely those which seem to be required in the latest Dalitz analysis of the large number of resonances recently identified by Lovelace et al. The scheme thus avoids certain representations like $(20, 1^+)$ or $(70, 0^+)$ which appear in a harmonic oscillator classification but which do not seem to be favoured experimentally.

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Our knowledge of the structure of baryon resonances during the last few years has depended largely on the results of phase shift analyses in πN scattering, especially by Lovelace et al. [1], on the one hand, and periodical quark model assignments to these resonances, especially by Dalitz [2),3)], on the other. Very recently a new πN phase shift analysis up to about 2000 MeV c.m. energy has been reported by Lovelace et al. [4),5)] and this indicates a large number of new resonances in this mass region. Simultaneously Dalitz [6], who has made a fresh quark model analysis of baryon resonances, finds most of these new resonances greatly welcome in terms of the "simple quark picture" based on harmonic oscillator wave-functions of low excitations. According to Dalitz,[6] the "simple quark picture" which regards baryons as merely 3Q states, giving only the SU(3) multiplicities $\underline{1}$, $\underline{8}$ and $\underline{10}$, seems to have received fresh support from the recent data of Abrams et al. [7] which indicate that the bump $Z_1(1900)$ in K^+p, observed earlier by Cool et al. [8], is probably <u>not</u> a resonance, but an effect of interference between the amplitudes in several partial waves.

The Dalitz classification [2),3)] which is based on the supermultiplet representations like $(56, 0^+)$, $(70, 1^-)$, $(56, 2^+)$, etc., of the group $SU(6)\times O(3)$ has already been quite successful in the assignment of most low-lying negative parity resonances to $(70, 1^-)$ and several positive parity ones to $(56, 2^+)$. With several more resonances now established [4),5)], Dalitz in his latest analysis has put some "dynamical content" in his earlier scheme, whereby he now makes essential use (i) of totally symmetric (S) functions first suggested by Greenberg [9] in connection with the use of parastatistics for quark symmetries and (ii) the quantum number N of radial excitations which arises naturally in the classification of 3Q states through harmonic oscillator (HO) wave functions [10]. One of the important uses which Dalitz makes of the N quantum number is to assign the long-standing N^* resonance $P_{11}(1470)$ as well as the more recent Δ^* resonance

$P_{33}(1688)$ to the super-multiplet $(56, 0^+)$ with $N = 2$, while the more familiar baryons have $N = 0$. Dalitz prefers this assignment to other alternatives like $(70, 0^+)$ or $(20, 1^+)$, on grounds of economy as well as simple dynamics based on the availability of the maximum number of symmetric QQ pairs. It should be pointed out, however, that according to HO classifications, representations like $(70, 0^+)$ or $(20, 1^+)$ which correspond to the same N excitation (N=2) are predicted to have significant energy overlaps with $(56, 0^+)$ of $N = 2$. Indeed, an earlier analysis in terms of p wave QQ interactions [11),12)] had seemed to indicate the $(20, 1^+)$ state as fairly low in excitation, so that a dynamical argument (at least in terms of such simple models) may not appear very convincing. On the other hand, a less model-dependent argument based on the rate of production of $(20, 1^+)$ states in a pp collision [13)] seems to rule out such states on experimental grounds. The essential argument is that (in the impulse approximation) the spatial symmetry of the produced baryon resonance should obey the "symmetry selection rule" $\Delta(\text{symm}) = 1$ [13)] which cannot be satisfied by the spatially antisymmetric (A) function characterizing the $(20, 1^+)$ state, requiring as it does, two units of symmetry change from the spatial S function of the incoming proton. On the other hand, the cross-sections for the production of $N^*(1470)$ states in pp collisions are large enough to indicate an apparent violation of this rule [14)]. Dalitz' new assignment [6)] through the N quantum number certainly represents a very good way out of this difficulty.

Now while the HO classification represents a broad framework in which to fit in the various resonances, the framework by itself appears much too broad, even for the new, enlarged, catalogue of resonances. Thus, Dalitz [6)] finds that, of the large variety of states offered by the HO classification, only the $(56 ; 0^+, 2^+)$, $(70, 1^-)$ and at most $(70, 3^-)$ with appropriate N excitations, seem to be realized by the experimentally identified resonances, though from the point of view of the HO theory, it would be extremely hard to rule out the other states of equally low N excitations. Another disadvantage of the HO classification seems to be relatively big jumps (in units of two) in the N excitations for the higher recurrences of "desirable" states like, e.g., $(56, 0^+)$. For example, Dalitz [6] finds that the state $P_{11}(1750)$ must be assigned to as much as $N = 4$, if its $(56, 0^+)$ character has to be maintained. Such jumps are not energetically favoured by the HO classification which does allow for lower N excitations associated with other $SU(6) \times O(3)$ assignments. It would therefore be of interest to see if alternative schemes could be devised which would automatically rule out the "unwanted" states.

We wish to point out the possibility of a more economical scheme which brings out precisely the $SU(6) \times O(3)$ states required in the DIII analysis and keeps out the unwanted HO states on dynamical grounds. The scheme is based on a model which is characterized by the assumption of an s wave QQ interaction [15] (i.e., short-ranged, compared with the HO potential), and yields 3Q super-multiplet states of the types $(56, L^P)$ and $(70, L^P)$, of which only the types

$$[56, (2\ell)^+] \text{ and } [70, (2\ell+1)^-], \quad (\ell = 0, 1, 2, \dots), \qquad (1)$$

have attractive three-body kernels, while all other types have repulsive kernels. Even explicit approximate formulae for the energy levels of the attractive states were derived on this model [15], though it would be unrealistic to take the quantitative aspects of the energy levels literally. It is remarkable, however, that the $SU(6) \times O(3)$ structures of the 3Q states with attractive kernels, are precisely those which are strongly favoured by the latest Dalitz analysis [6] of the resonances, and that the "disfavoured" states like $(70, 0^+)$ or $(20, 1^+)$ are either absent or have repulsive kernels. Thus, in a way, this simple model seems to provide a precise dynamical content to Dalitz' latest raison d'être for the states of maximum symmetry allowed within a 3Q system.

As for the existence of the N quantum number within the model, this question was not examined in Ref. [15]. However, a closer study now indicates that this feature is <u>also</u> present in the model in a particularly simple fashion. To be specific, since the model, in its present form [15], assumes a single separable term for the s wave QQ force, the 2Q wave function has no radial nodes. However, a study of Eqs. (6) and (12) of Ref. [15] shows that the "spectator function" $\underset{\sim}{F(P)}$ for the <u>connected</u> three-body system satisfies an integral equation whose kernel is Fredholm and non-degenerate and hence has more than one eigenvalue [16]. Indeed, for a given L value, these successively higher eigenvalues correspond just to the manifestations of the radial quantum number N_r. For a qualitative idea of the three-body model of Ref. [15], the eigenvalue equation for the binding energy parameter B_L associated with the spectator function $F_L(P)$, has the following approximate form (see Appendix of Ref. [15]) in certain units (which need not be specified)

$$B_L F_L(x) \approx \int_0^\infty y^2 dy \; i_L(xy) \exp[-\tfrac{5}{4}y^2] \, F_L(y), \tag{2}$$

where $i_L(x)$ is a (real) spherical Bessel function with imaginary argument (e.g., $i_0(x) = \sinh x/x$). In Ref. [15], only the <u>highest</u> eigenvalues, corresponding to $N_r = 0$, for each L value were considered, and the usual $(56, 0^+)$, $(70, 1^-)$ and $(56, 2^+)$ states associated with these respective eigenvalues for L = 0, 1, 2. In the usual spectroscopic notation, these states would read as 1S, 1P and 1D. However, it is now clear that, for each L value, Eq. (2) yields, without further assumptions, states of successively higher radial excitations in the notation $(N_r+1)L$. Thus in this scheme, the N*(1470) and N*(1750) states would have the spectroscopic names 2S and 3S respectively, all with the $(56, 0^+)$ representation of $SU(6) \times O(3)$. Similarly the model would predict successive radial excitations of the $(56, 2^+)$ 1D state as 2D, 3D, etc. and of the $(70, 1^-)$ 1P state as 2P, 3P, etc. Note that unlike the HO classification, the successive N_r values associated with a given representation of $SU(6) \times O(3)$ are consecutive, rather than alternate.

It should perhaps be emphasized that while the result that only 3Q states of the type (1) are attractive was obtained through the explicit use of a separable potential, the validity of this result is much wider than what might be thought at first sight. Actually the main ingredient that has gone into the result is not so much the separable nature of the potential, as its s wave structure. Thus, e.g., one would still obtain attractive three-body kernels for states of the type (1) even if one used the s wave part of a local (Yukawa) potential. Indeed, it is the symmetry structure of the 3Q state generated by attractive s wave QQ forces, that is playing the crucial role in determining the sign of the three-body kernel. On the other hand, if one used a complete local potential, the various other partial waves present in it would vitiate this neat result, and any such simple conclusion on the sign of the kernel in relation to the $SU(6) \times O(3)$ symmetry of the 3Q states would in general not be possible. For example, the various additional states like $(70, 0^+)$ $(20, 1^+)$ characteristic of an HO potential can be attributed to the presence of higher partial waves present in it. Indeed, as had been found in an earlier study [12], p wave QQ forces yield a great variety of $SU(6) \times O(3)$ representations, such as $(70, 0^+)$ and $(20, 1^+)$, in common with HO potentials.

It would of course be unrealistic to suppose that the positions of the actual resonances can be fitted quantitatively with such a simple model as a rank-one separable potential, so this note is confined mainly to the general aspects dealing with the spectroscopic classification scheme for the resonances, which seems to work rather well. However, it should be pointed out that it is possible to make the model more realistic by merely increasing the rank of the input potential, though the addition of further separable terms, as long as the criterion of s wave forces is maintained. This requirement is especially important in order that the spectroscopic pattern of the energy levels discussed above may not be destroyed by the appearance of "unwanted" states. One should perhaps also mention that a sum of several s wave separable terms can be made to approximate closely to the s wave part of any local potential.

From a theoretical point of view, a short-range potential seems to be preferable to an HO potential if one wants to give a unified treatment for both bound and scattering states. For example, if one ignores relativistic complications for the moment one expects short-range repulsion between two baryons to be a consequence of parastatistics symmetry [2]. Since, on the other hand, such a system characteristically involves both bound and scattering states, a consistent quantum mechanical description can be given only in terms of short-range forces. Indeed it was shown recently [17] that the same model of s wave QQ interaction provides a formal understanding of short-range repulsion between baryons through the use of parastatistics symmetry.

As to the experimental consequences of the scheme suggested in this note, it is of course vital that the higher negative parity baryons like $G_{17}(2200)$ must belong to the $(70, 3^-)$ representation. In this respect the analysis in DIII looks encouraging, though many more resonances need to be discovered before this representation is definitely established.

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R E F E R E N C E S

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