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PARABOLIC BUNDLES AS ORBIFOLD BUNDLES

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1. Introduction. Mehta and Seshadri introduced the notion of a parabolic bundle and of parabolic stability on a Riemann surface [MS]. Maruyama and Yokogawa generalized it to higher dimensional varieties [MY].

In this paper we establish a two-way relationship between the parabolic vector bundles and the orbifold vector bundles—these are vector bundles on a variety equipped with a finite group action, together with a lift of the action of the group to the bundle.

Let $Y/\mathbb{C}$ be a smooth projective variety, and let Γ be a finite subgroup of the group of automorphisms of Y such that the projection

$$p\colon Y \to Y/\Gamma =: X$$

is a covering morphism. Let W be a vector bundle on Y equipped with a lift of the action of Γ (i.e., W is an orbifold bundle). Consider the Γ-invariant part of the direct image, namely, $E := (p_* W)^{\Gamma}$, which is a coherent sheaf on X. First we show that the filtration of E, given by its subsheaves of the form $(p_*(W \otimes \mathcal{O}_Y(-i\tilde{D})))^{\Gamma}$, $i \geqslant 0$, where $\tilde{D} \subset Y$ is the (reduced) ramification divisor, defines a parabolic structure on E with $D = p(\tilde{D})$ as the parabolic divisor. (The details are in Section 2c.) Let E_* denote the parabolic structure on E obtained this way.

We then prove that a substantial class of parabolic bundles arises this way. This is done by giving an explicit inverse construction (i.e., from E_* to W). The basic tool used in this inverse construction is the "Covering Lemma" of Y. Kawamata [K, Theorem 17]. (The details are in Section 3.)

Let E be a semistable vector bundle of rank r on a connected smooth projective variety of dimension n over $\mathbb{C}$, equipped with a polarization L. F. Bogomolov [B] proved that

$$c_2(\mathrm{End}(E)) \cup c_1(L)^{n-2} = (2r.c_2(E) - (r-1).c_1(E)^2) \cup c_1(L)^{n-2} \geqslant 0.$$

For the tangent bundle of a smooth projective variety over $\mathbb{C}$ whose canonical bundle is ample, Y. Miyaoka and S. T. Yau both proved a similar (in fact stronger) inequality [M], [Y].

Using the correspondence between parabolic bundles and orbifold bundles mentioned above, we prove an analogue of the Bogomolov inequality for para-

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