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Studies in biogas technology. Part II. Optimisation of plant dimensions

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Abstract. In this paper, the design basis of the conventional Khadi and Village Industries Commission biogas plants has been elucidated. It has been shown that minimisation of the cost of the gas holder alone leads to the narrow and deep digesters of conventional plants. If instead, the total capital cost of the gas holder plus digester is minimised, the optimisation leads to wide and shallow digesters, which are less expensive. To test this alternative, two prototype plants have been designed, constructed and operated. These plants are not only 25–40% cheaper, but their performance is actually slightly better than the conventional plants.

Keywords. Biogas plants; dimensions; design; optimisation; modifications; gas production; yield; capital costs; economies of scale.

1. Introduction

One of the first 'surprises' confronting newcomers to the field of biogas technology is the absence of a theoretical approach to the detailed dimensioning of biogas plants. There is, of course, the trivial calculation of the volume V' of the digester viz.,

$$V' = v_{sl}\, t_d, \tag{1}$$

where v_{sl} is the volume of slurry with which the plant is charged daily, and t_d is the detention time of the slurry in the plant. But little has been said about the diameter and height of the gas holder (in floating cover-cum-gas-holder type of plants) and of the digester.

In practice, however, a wide variety of plants have been described—from shallow and long horizontal digesters to narrow and deep vertical plants. The well-known design propagated by the Khadi and Village Industries Commission (KVIC) is of the latter type. For instance, the conventional biogas plant with a rated capacity of 200 cubic ft biogas per day (5·66 m³/day), which has been studied in the programme of work described in this paper (cf. part I) has the following dimensions: diameter and height of gas holder—6 ft (1·83 m) and 4 ft (1·22 m) respectively; and diameter and depth of digester—6½ ft (1·98 m) and 16 ft (4·88 m) respectively. Enquiries regarding why conventional digesters are made so deep bring forth informal arguments regarding the density stratification inside digesters, the need to bury them deep in the ground in order to insulate them from the ambient temperature, etc. Since such answers are inconsistent with the findings of this study (cf. part I of this paper), and since the dimensions of biogas plants are a major factor in capital costs, it was considered essential, firstly, to understand the basis, if any, underlying the dimensions of conventional plants, and secondly, to develop a rationale for optimising the dimensions of biogas plants. The results of these efforts are described below.

2. Optimisation based on minimising gas holder cost

In the conventional plants of KVIC design, the mild steel (floating) gas holder accounts for a substantial percentage (about 40%) of the total cost of the biogas plant. If R, R_s and R_r are the capital costs of the gas holder, its sides and its roof respectively

$$R = R_s + R_r,$$

$$= \pi D h t \rho u + (\pi D^2/4)\, t \rho u, \tag{2}$$

where D is the diameter of the gas holder, h its height, t its thickness, ρ its density and u, its unit cost in Rs/kg (taking into account the cost of steel, transport, fabrication, welding and painting). Further, the height h of the gas holder is given by

$$h = 4V/\pi D^2 = 4\gamma C/\pi D^2, \tag{3}$$

where V is the volume of the gas holder and γ is the maximum fraction of the daily gas production, i.e., actual plant capacity C, which is intended to be stored in the gas holder. Combining (2) and (3), the result is

$$R = (4\alpha\gamma C/D) + (\pi \alpha D^2/4), \tag{4}$$

where $\alpha = t\rho u.$ (5)

If R is considered to be the objective function which is sought to be minimised, i.e., if R is differentiated with respect to D and the result is set equal to zero, the diameter D_K corresponding to this minimum-cost gas holder is

$$D_K = (8\gamma/\pi)^{1/3}\, C^{1/3}, \tag{6}$$

i.e., D_K increases linearly with $C^{1/3}$, the slope being given by $(8\gamma/\pi)^{1/3}$. Further, the height of such a gas-holder is

$$h_K = 4V_K/\pi D_K^2 = 4\gamma C/\pi D_K^2. \tag{7}$$

and its cost is given by

$$R = (3a\pi^{1/3}\,\gamma^{2/3})\,C^{2/3}, \tag{8}$$

as may be shown by combining equations (4) and (6).

It is interesting that if γ is required, as it is in the conventional Indian plants, to have a value of 60% of the rated daily gas production (corresponding to a gas storage of 12 hr overnight gas production plus an excess storage capacity of 20% of this 12 hr production), it turns out (figure 1) that the least-square line drawn through the D vs $C^{1/3}$ plots for the conventional Indian plants coincides with the plot of equation (6) for $\gamma = 0{\cdot}6$. Evidently, the diameters of the gas holders of these plants have been chosen so as to minimise the costs of the gas holders.

Further, the design of the conventional plants is such that the diameter of the digester pit is equal* to that of the gas holder plus about 6 in. (~15 cm) to facilitate free up and down floating of the gas holder as it fills with gas and empties. That is, for design purposes, it can be assumed to be within about 10% that $D'_K = D_K$ where D'_K is the diameter of the digester pit**.

In such a design, the depth h'_K of the digester pit becomes

$$h'_K = 4V'/\pi D'^2_K = 4V'/\pi D^2_K. \tag{9}$$

But from equation (1),

$$V' = (m_{sl}/\rho_{sl})\,t_d, \tag{10}$$

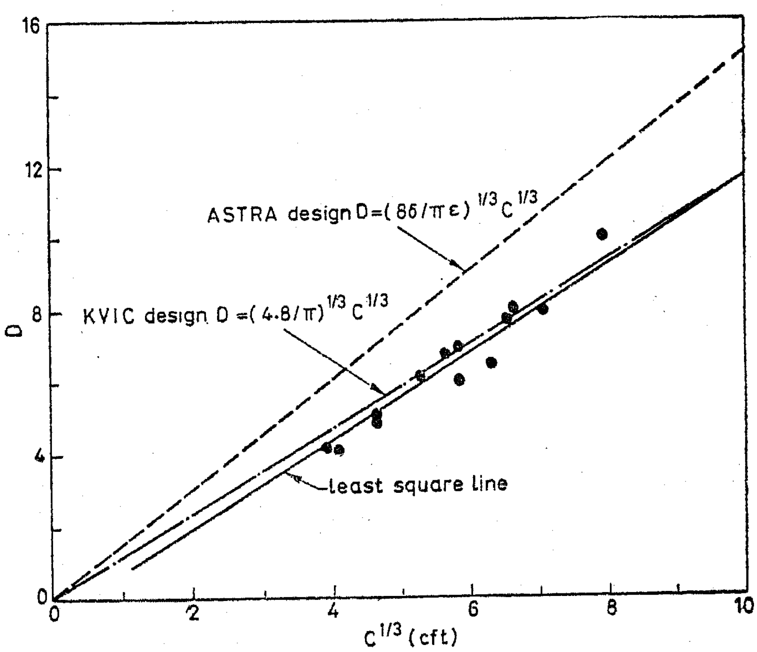

Figure 1. Variation of diameter with cube root of capacity

*This is to ensure the anaerobic seal for the fermentation process.

**Throughout the rest of part II of this paper, primed dimensions refer to the digester and unprimed ones to the gas holder.

where ρ_{sl} and m_{sl} are the density of the slurry and the mass of the slurry which is charged daily. Or, since $m_{sl}=2m_{WD}$ because of the 1 : 1 (by weight) mixture of m_{WD} and m_W, the masses of wet dung and water charged daily, and $m_{WD}=C/Y$ where C and Y are the daily gas yield of the biogas plant and its gas yield per unit weight of wet dung respectively,

$$V' = 2Ct_d/\rho_{sl}\, Y = 2\,\phi\, C, \tag{11}$$

where $$\phi = t_d/\rho_{sl}\, Y. \tag{12}$$

Hence $$h'_K = 8\,\phi\, C/\pi\, D_K^2, \tag{13}$$

and $$h'_K/D'_K = \phi/\gamma. \tag{14}$$

Conventional biogas plants are designed on the basis of $t_d = 52$ days, $\gamma = 0{\cdot}6$, $\rho_{sl} \approx \rho_{water} \approx 1$ g/cm³ and $Y = 34$ cm³ biogas/g wet dung (≈ 3 cubic ft biogas/lb total solids), and therefore since $\phi = 1{\cdot}53$,

$$h'_K/D'_K \approx 2{\cdot}5, \tag{15}$$

which implies *narrow and deep digester pits.*

For example, according to this approach to the design of conventional plants, a 200 cubic ft/day plant requires, according to equations (6), (7) and (13), a gas holder diameter and height of about 6·5 ft (2 m) and 3·5 ft (1 m) respectively and a

Table 1. Comparison of conventional and modified plants

	200 cubic ft/day (5·66 m³/day) plants		60 cubic ft/day (1·70 m³/day) plants	
	KVIC	ASTRA	KVIC	ASTRA
Gas holder diameter (m)	1·83	2·44	1·35	1·68
Gas holder height (m)	1·22	0·61	0·76	0·46
Gas holder volume (m³)	3·21	2·85	1·09	1·02
Digester diameter (m)	1·98	2·59	1·45	1·75
Digester depth (m)	4·88	2·44	2·72	1·52
Digester depth-diameter ratio	2·46	0·94	1·88	0·87
Digester volume (m³)	15·02	12·85	4.49	3·65
Capital cost of plant (Rs)	8100	4765	3250	2355
Relative costs	100	58·80	100	72·50
Daily loading (kg fresh dung)	150	150	50	50
Mean temperature	27·60	27.60	*	27·60
Daily gas yield (m³/day)	4·28±0·47	4·89±0·60	*	1·93±0·38
Actual capacity/rated capacity	75·6%	86·4%	*	113·7%
Gas yield (cm³)/g fresh dung	28·5±3·2	32·7±4·0	*	38·5±7·6
Improvement in gas yield	—	+14·2%	*	*

*Since a 60 cubic ft/day plant of KVIC design has not been studied under identical conditions as the corresponding ASTRA plant, values have not been inserted for the performance of the former type of plant.

digester of diameter 6·5 ft (2 m) and 16·5 ft (5 m) respectively. These dimensions are sufficiently close to the dimensions recommended by the designers (cf. table 1) to conclude that the basis for the design of conventional plants has been successfully elucidated here.

Evidently, the diameters of the gas holders of conventional plants are chosen by minimising the gas holder cost; then, the diameters of the digesters are set equal to (actually, slightly greater than) the diameters of the gas holders; and finally, the digester diameters along with the volumes of these pits, determine the depths of the digesters. The whole procedure leads to the narrow and deep digesters characteristic of conventional plants. Further, the fact that these characteristics have been explained quite simply with cost-minimisation arguments indicates that the dimensioning of conventional plants has little to do with stratification and temperature influences which incidently have not been observed in the present study (cf. part I of this paper). The latter conclusion is confirmed by the fact that shallow horizontal digesters function quite satisfactorily without being as deep as the conventional digesters.

3. Optimisation based on minimising capital cost of plant

If cost minimisation is the crucial factor in optimising the dimensions of biogas plants, then it is clear that the total capital cost of the gas holder *and* the digester, must be taken into account and not the former alone. In other words, the optimisation must be based on minimising R (cap) given by

$$R\,(\text{cap}) = R + R', \tag{16}$$

where R and R' are the capital costs of the gas holder [cf. equation (2)] and digester respectively.*

The cost of the digester involves four contributions (i) R'_b, the cost of the base of the digester pit, (ii) R'_s, the cost of its sides, (iii) R'_p, the cost of the central partition wall separating the digester pit into the inlet and outlet sides, and (iv) R'_e, the cost of excavating the digester pit. Thus,

$$R' = R'_b + R'_s + R'_p, + R'_e. \tag{17}$$

The civil engineering costs associated with these four contributions may be elaborated further in the following way (using the assumption $D' \approx D$, i.e., the diameters of the digester and gas holder are almost equal)

$$R' = \frac{\pi D^2 t' u'}{4} + \pi Dh' t' u' + Dh' t' u' + \frac{\pi D^2 h' u_e}{4}, \tag{18}$$

where h' is the depth of the digester, t', the thickness of the masonry (assumed for simplicity to be the same for the base, sides and partition wall), u', the unit cost of

*The total capital cost here excludes the cost of inlet and outlet tanks, pipes, etc.

the masonry in Rs/unit area, and u_e, the unit cost of excavation in Rs/unit volume. Using the symbol

$$\beta = t' u', \tag{19}$$

and expressing h' in terms of $D' = D$ with the aid of equation (13), the result is

$$R' = (\pi D^2 \beta/4) + (8 \beta \phi C/D)\left(1 + \frac{1}{\pi}\right) + 2\phi u_e' C. \tag{20}$$

Combining this expression with equation (4), the total capital cost of the digester plus gas holder becomes

$$R\,(\text{cap}) = \frac{4C}{D}\left\{\alpha\gamma + 2\beta\phi\left(1 + \frac{1}{\pi}\right)\right\} + \frac{\pi D^2}{4}(\alpha + \beta) + 2\phi u_e' C$$

$$= (4\delta C/D) + (\pi\epsilon D^2/4) + 2\phi u_e' C, \tag{21}$$

where $$\delta = \alpha\gamma + 2\phi\beta\left(1 + \frac{1}{\pi}\right), \tag{22}$$

and $$\epsilon = \alpha + \beta. \tag{23}$$

In minimising the objective function R (cap), it shall be assumed, *as a first approximation*, that the unit civil engineering costs of masonry construction and excavation are independent of depth*, i.e., $u' \neq f(h')$ and $u_e' \neq f(h')$.

On this basis, by differentiating R (cap) with respect to D and setting the result equal to zero, the diameter** $D_A' = D_A$ corresponding to the minimum total capital cost turns out to be

$$D_A' = D_A = (8\,\delta/\pi\,\epsilon)^{1/3} C^{1/3}, \tag{24}$$

and the height to depth ratio for a digester pit optimised in the above manner is

$$h_A'/D_A' = h_A'/D_A = \phi\,\epsilon/\delta. \tag{25}$$

For the same values of γ and ϕ that were used for conventional plants [cf. equation (15)], and using present costs of steel fabrication and masonry construction to assign the values: α = Rs 22/ft² (Rs 237/m²) and β = Rs 6·75/ft² (Rs 73/m²), it turns out

*Even if the variation of unit costs with depth is taken into account, e.g., by writing $u' = u'^0 + k_1 h$ to correspond with civil engineering rates, the basic conclusions are unaffected, though the mathematics gets slightly more complicated, as is shown in appendix 1.

**The subscript A is used to indicate that the diameter, D_A', depth, h_A', etc correspond to the modified design of biogas plants developed in this work as part of the ASTRA programme of the Indian Institute of Science.

that $\epsilon =$ Rs 28·75/ft^2 or Rs 309/m^2 and $\delta =$ Rs 39·4/ft^2 or Rs 424/m^2. For these values,

$$h'_A/D_A \approx 1{\cdot}1, \tag{26}$$

i.e., the optimised dimensions correspond to *wide and shallow digesters with depths almost equal to the diameters,* when the total capital cost of gas holder and digester is minimised.

The gas holder height is determined, as stated earlier, by the gas storage that is required—cf. equation (7)—i.e.,

$$h_A = 4\,\gamma\, C/\pi\, D_A^2. \tag{27}$$

The differences in the dimensions of conventional plants [cf. equations (6), (7), (13) and (14)] and those of plants based on minimising the total capital cost [cf. equations (24), (25) and (26)] lead to differences in the comparative costs of the plants. The total cost of conventional narrow and deep plants can be obtained from

$$R_K\,(\text{cap}) = R_K + R'_K, \tag{28}$$

by introducing expressions (8) and (20) for R_K and R'_K respectively and using equation (6) to substitute for D in equation (20). The result is

$$R_K\,(\text{cap}) = \pi^{1/3}\,\gamma^{2/3}\,\{3\alpha + \beta\,[1 + (4\phi/\gamma)\,(1 + 1/\pi)]\}\,C^{2/3} + 2\phi u'_e C, \tag{29}$$

which can be further simplified thus:

$$R_K\,(\text{cap}) = (\pi/\gamma)^{1/3}\,(\epsilon\gamma + 2\delta)\,C^{2/3} + 2\phi u'_e C. \tag{30}$$

On the other hand, when the capital cost of the gas holder *plus* digester is minimised, this capital cost is given by substituting for D in equation (21) with the aid of equation (24):

$$R_A\,(\text{cap}) = (27\pi\epsilon\delta^2)^{1/3}\,C^{2/3} + 2\phi u'_e C. \tag{31}$$

A comparison of the capital costs of the two types of plants can be obtained by inserting some numerical values. For example, with $\gamma = 0{\cdot}6$, $\epsilon =$ Rs 28·75/ft^2, $\delta =$ Rs 39·4/ft^2, $\phi = 1{\cdot}53$ (for $t_a = 52$ and $Y = 34$), the expressions are

$$R_K\,(\text{cap}) = 166{\cdot}7\,C^{2/3} + 1{\cdot}2\,C, \tag{32}$$

$$R_A\,(\text{cap}) = 155\,C^{2/3} + 1{\cdot}2\,C, \tag{33}$$

showing that the latter type of plants are about 7% cheaper even assuming that civil engineering costs of excavation and construction, i.e., u'_e and u' (i.e., β, ϵ and δ) are independent of depth, which is not the case. If this depth-dependence of civil engineering costs is taken into account, a greater cost-reduction is achieved.

4. Performance of modified plants

To test the new approach to optimising the dimensions of biogas plants developed above, it was decided to build plants with modified dimensions and to compare their performances with conventional plants of KVIC design.

In this process, the value to be chosen for the detention time t_d and therefore, the volume of the digester pit [cf. equation (1)] was also considered. The detention time affects both the capital and operational costs of a biogas plant: a longer detention time leads to greater gas yield from a given input of volatile solids, but also to a greater digester volume and therefore greater capital cost; a shorter detention time results in a cheaper digester, but also to a smaller gas yield and therefore greater loss of operational revenues. To be rigorous, the detention time must be determined by minimising the sum of the capital and operational costs, or maximising the return from the plant given by the net operational revenue minus the capital charges. This approach to choosing the detention time—apparently not attempted hitherto—will be described in a subsequent publication (Subramanian & Reddy, to be published.)

In the present work, an empirical alternative was adopted. The detention time was chosen from the results of previous workers who have shown that for the mean temperatures prevalent at Bangalore, i.e., about 25°C, a detention time of about 35 days is valid (Meynell 1976).

Using the values $\gamma = 0{\cdot}5$, $\alpha =$ Rs 22/ft^2, $\beta =$ Rs 6·75/ft^2, $t_d = 35$ days, $\rho_{sl} = 1{\cdot}03$ g/cm^3, $Y = 34$ g/cm^3, i.e., $\phi = 1$, $\epsilon = 28{\cdot}75$ and $\delta = 28{\cdot}8$, a 200 cubic ft/day (5·66 m^3/day) plant was designed with the aid of equations (24), (25) and (27). In addition, a 60 cubic ft/day (1·7 m^3/day) plant was also designed on exactly the same basis, except that $\gamma = 0{\cdot}6$. The dimensions of these ASTRA plants are given in table 1, which also includes for comparison purposes the dimensions of the conventional plants of KVIC design described in part I and in the literature.

The ASTRA plants were constructed in January 1979 and charged in February 1979. Owing to difficulties in the procurement of dung supply, a rigorous comparison of the performances of the conventional and modified plants could be made only from 7 April 1979.

The results of this comparison given in table 1 lead to the following conclusions.

(i) The ASTRA plants are significantly cheaper than the conventional plants—the cost reduction of 25–40% being achieved by optimisation of dimensions and choice of realistic detention times.

(ii) The performances of the ASTRA plants are as good as—in fact, slightly better than—those of the conventional plants. For example, for the same daily loading of 150 kg fresh dung into the 200 cubic ft/day plants, the ASTRA plant gives a 14% greater gas yield per unit weight of input material despite it being 40% cheaper.

(iii) The shallow digesters of the ASTRA plants are more convenient from the civil engineering point of view, and are a great advantage in situations where the water table is high. Besides, the wider plants reduce the strength requirements of foundations.

5. Conclusions

(i) The ASTRA plants described above have been designed, constructed and operated on the assumption that stratification does not occur to any significant extent in biogas plants, i.e., there is uniformity of temperature and density inside the digesters. This assumption has been validated by the actual performances of the ASTRA plants which are in fact slightly better than the performances of conventional plants.

(ii) In view of the fact that there are no technical reasons, for example, stratification, which necessitate narrow and deep digesters, the optimisation of plant dimensions must be based merely on cost minimisation grounds alone.

(iii) For given materials and techniques of construction of the gas holder and digester and given conditions of operation, three levels of optimisation of plant dimensions are possible:

Level 1: *Optimisation:* Dimensions based on minimisation of gas holder cost—this leads to conventional plants of the KVIC design.
Level 2: *Optimisation:* Dimensions based on minimisation of capital cost of gas holder + digester—this leads to the ASTRA plants described in this part of the paper.
Level 3: *Optimisation:* Dimensions based on a detention time corresponding to minimum total capital + operating costs—this may lead to further modifications if the detention time thus derived differs from that chosen in the Level 2 optimisation.

(iv) If the detention time has been chosen on the basis of previous work, then Level 2 optimisation leads to three simple expressions for the plant dimensions

$$\text{(a)}\ D_A \approx D'_A = (8\,\delta/\pi\,\epsilon)^{1/3}\,C^{1/3}, \tag{24}$$

$$\text{(b)}\ h_A = (4\,V/\pi\,D_A^2) = (4\,\gamma\,C/\pi\,D_A^2),\quad (\gamma = \text{fraction of daily gas yield to be stored}), \tag{27}$$

$$\text{(c)}\ h'_A = 4\,V'/\pi\,D_A^2 = (\phi\,\epsilon/\delta)\,D_A. \tag{25}$$

As a zeroth approximation, even simpler thumb-rules can be stated for biogas plants with masonry digesters and floating mild-steel gas holders.

$$\text{(a)}\ D \approx D' \approx 1{\cdot}5\,C^{1/3}, \tag{34}$$

$$\text{(b)}\ h \approx 0{\cdot}3\,C^{1/3}, \tag{35}$$

$$\text{(c)}\ h' \approx D'. \tag{36}$$

(v) The expression for the cost of biogas plants [cf. equation (31)] show that plant costs do not increase linearly with plant capacity. In other words, there are definite economies of scale in biogas plants. As a first approximation excluding excavation costs, the total cost of N plants each of capacity (C/N) is $N^{1/3}$ times the cost of one plant of capacity C.

(vi) These same expressions also define a clear-cut strategy for cost reduction in biogas plants. The crucial parameters which need to be reduced are t' (thickness of masonry in digester), γ (fraction of daily gas yield which is stored), u and u' (the costs per unit area of gas holder and digester respectively), and t_d (the detention time). The reduction of u and u' is best achieved through the use of alternative materials and/or techniques of construction. In contrast, the reduction of t_d must be accomplished by operation of biogas plants under optimum conditions, for example, at the optimum temperature of 35°C (for mesophilic bacteria) instead of at lower ambient temperatures. A simple way of increasing the temperature inside the digester is described in part IV of this paper.

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Appendix 1

Calculation of dimensions and costs assuming $u' = f(h')$

The calculation of dimensions and costs taking into account the dependence of the u', the unit cost of masonry construction, upon h', the digester depth is achieved by introducing a depth-dependence into the parameters δ and ϵ into equation (21), viz.,

$$R\,(\text{cap}) = (4\delta\, C/D) + (\pi\, \epsilon\, D^2/4) + (2\,\phi\, u'_e\, C).$$

Writing $u' = u'_0 + k_1\, h'$, the parameters become

$$\beta = t'\, u'_0 + k_1\, t'\, h' = \beta_0 + k_1\, t'\, h',$$

$$\epsilon = \alpha + \beta_0 + k_1\, t'\, h' = \epsilon_0 + k_1\, t'\, h',$$

$$\delta = [\alpha\, \gamma + 2\,\phi\, (1 + 1/\pi)\, \beta_0] + (2\,\phi\, (1 + 1/\pi)\, k_1\, t']\, h' = \delta_0 + k_2\, h'.$$

Introducing these values into equation (21), using equation (13) for h', and ignoring the excavation cost by virtue of its negligible contribution to the total cost, the result is:

$$R\,(\text{cap}) = (\pi\, \epsilon_0/4)\, D^2 + (4\delta_0\, C)\, 1/D + (32k_2\, \phi\, C^2/\pi)\, (1/D^3) + 2k_1\, t'\, \phi\, C.$$

Differentiating this equation and setting the result equal to zero,

$$d\, R\,(\text{cap})/dD = (\pi\, \epsilon_0/2)\, D - (4\delta_0\, C)\, 1/D^2 - 3\,(32k_2\, \phi\, C^2/\pi)\, 1/D^4 = 0,$$

or $$(\pi\, \epsilon_0/2)\, D^5 - (4\delta_0\, C)\, D^2 - 3\,(32k_2\phi\, C^2/\pi) = 0.$$

This equation can be solved by the Newton-Raphson method for a $C = 200$ cubic ft/day (5·66 m^3/day) plant using the values $\alpha = 22$, $\beta_0 = 6{\cdot}75$, $\epsilon_0 = 28{\cdot}75$, $\delta_0 = 40{\cdot}41$, $\phi = 1{\cdot}53$, $k_1 = 0{\cdot}25$ and $t' = 1$.

The result is $D^* = 3{\cdot}15$ m from which it follows that $h'^* = 2{\cdot}22$ m and $h'^*/D^* = 0{\cdot}7$, in comparison with the first-approximation $[u' \neq f(h')]$ values of $D_A = 2{\cdot}73$ m, $h'_A = 2{\cdot}97$ m and $h'_A/D_A \approx 1{\cdot}1$. Hence, by taking into account the dependence of u' on h', the optimised digesters become even more wide and shallow.

The cost differences arising from the functional dependence of u' on h' can be calculated from an alternative form of equation (21), viz., (after ignoring excavation costs)

$$R\,(\text{cap}) = \underbrace{[(4\,\alpha\,\gamma\,C/D) + (\pi\,\alpha\,D^2/4)]}_{\text{Gas holder}} + \underbrace{[(\pi\,D^2\beta/4) + Dh'\,\beta\,(\pi + 1)]}_{\text{Digester}}. \tag{A.1}$$

If $\beta = \beta_0 + k_1 t' h'$ and $\beta_0 = 6{\cdot}75$, $k_1 = 0{\cdot}25$, $t' = 1$ and $h' = 7{\cdot}28$ ft (2·22 m), the capital cost for $\alpha = 22$ and $\gamma = 0{\cdot}6$, is Rs 6250. This should be compared with Rs 5400 (excluding excavation costs) obtained by assuming $u' \neq (h')$, $\beta = \beta_0 = 6{\cdot}75$ and equation (31) for R (cap).

If instead of assuming that u', and therefore β, is a continuous function of h', it is assumed that u' is a staircase function of h', so that $u' = 6{\cdot}75$ for $0 < h' \leqslant 6$ ft and $u' = 10{\cdot}75$ for 6 ft $< h' \leqslant 12$ ft, and that D and h' are obtained by assuming no depth-dependence of u', i.e., $D = 2{\cdot}73$ m and $h' = 2{\cdot}97$ m, it turns out that R (cap) is Rs 6232. Hence, by assuming this staircase function for u', it is possible to calculate D and h' without taking into account the depth-dependence of u' of h', and still obtain a biogas plant cost which is within 0·3% of the cost obtained by assuming $u' = u'_0 + k_1 t' h'$ and calculating D and h' with this assumption.

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