

Singular pencils of quadrics and compactified Jacobians of curves

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MS received 31 August 1989; revised 3 April 1990

Abstract. Let Y be an irreducible nodal hyperelliptic curve of arithmetic genus g such that its nodes are also ramification points ($\text{char} \neq 2$). To the curve Y , we associate a family of quadratic forms which is dual to a singular pencil of quadrics in \mathbb{P}^{2g+1} with Segre symbol $[2 \dots 2 \ 1 \dots 1]$, where the number of 2's is equal to the number of nodes. We show that the compactified Jacobian of Y is isomorphic to the space R of $(g-1)$ dimensional linear subspaces of \mathbb{P}^{2g+1} which are contained in the intersection Q of quadrics of the pencil. We also prove that (under this isomorphism) the generalized Jacobian of Y is isomorphic to the open subset of R consisting of the $(g-1)$ dimensional subspaces not passing through any singular point of Q .

Keywords. Nodal hyperelliptic curve; compactified Jacobian.

1. Introduction

The geometry of a nonsingular pencil of quadrics Q is intimately connected with some moduli spaces of vector bundles on the nonsingular hyperelliptic curve associated to Q ([2], [3] etc). A few years back Newstead [5] studied the geometry of singular pencils of quadrics in \mathbb{P}^5 . As he has remarked, a very interesting problem will be to extend the above connection between the geometry of a pencil and vector bundles on the associated hyperelliptic curve to the singular case. We have taken here one step in this direction.

Let A and B be matrices corresponding to two nonsingular members of a pencil Q . If the equation $\det(A - xB) = 0$ does not have all roots distinct, then Q is a singular pencil. The simplest case is when this equation has at most double roots. Then one can associate to Q a singular curve Y with the following properties: (i) Y has at the most ordinary double points as singularities; (2) Y is a two sheeted covering over \mathbb{P}^1 with the set of singular points contained in the set of ramification points.

In this paper, we start with an irreducible curve Y with the above two properties and construct a pencil of quadrics Q in \mathbb{P}^{2g+1} , g being the arithmetic genus of Y (see §3). We denote by R the space of $(g-1)$ -dimensional subspaces of \mathbb{P}^{2g+1} which are isotropic for all the members of the pencil Q . Then our main theorem is:

Theorem. *The compactified Jacobian of Y is isomorphic to R .*

We also describe explicitly the subvariety of R corresponding to the generalized Jacobian of Y (Theorem 1, §4). The moduli of rank two bundles on Y will be studied in a subsequent paper.

2. Notations and Preliminaries

Let Y be an irreducible reduced curve with ordinary double points and F a torsion free coherent sheaf of rank one on Y .

DEFINITION 2.1

The degree of F ($\deg F$) is the integer $\chi(F) - \chi(\theta_Y)$ (I.4.1, [4]).

2.2. Since Y is Gorenstein, the dualising sheaf ω_Y is a locally free sheaf of rank one, of degree $2g - 2$. (I.4.2, [4]).

2.3. (a) $\deg(F) < 0$ implies that $H^0(F) = 0$,

(b) $\deg(F) > 2g - 2$ implies that $H^1(F) = 0$ (I.4.6[4]).

2.4. Let \bar{J} denote the compactified Jacobian of Y . Tensorising by a line bundle of degree d , \bar{J} can be identified with the space of torsion free coherent sheaves of rank one and degree d .

By Proposition 10.1, [6], $\bar{J} = \coprod \text{Pic } Y_i$ where \coprod denotes disjoint union over all partial normalizations Y_i of Y . Thus, if Y has only one singular point, $\bar{J} = \text{Pic } Y \amalg \text{Pic } \tilde{Y}$, \tilde{Y} being the normalization of Y .

Henceforth we make the following assumptions. (1) Y is hyperelliptic i.e. there exists a finite degree two map $p: Y \rightarrow \mathbb{P}^1$ (2) The unique node w_0 of Y is a ramification point of p . Let $W_0 = \{w_0, w_1, \dots, w_{2g}\}$ be the set of ramification points of Y . Let i denote the hyperelliptic involution on Y . We define an i -action on a torsion free coherent sheaf E on Y as a $\mathbb{Z}/2$ -action on E which lifts the involution i . We call E i -invariant if $E \approx i^*E$.

Lemma 2.5. (a) *There is a one-one correspondence between i -invariant line bundles of even degree (respectively odd degree) on Y and the set of partitions of $W_0 = \{w_0, w_1, \dots, w_{2g}\}$ into two subsets S, T such that $S \cap \{w_1, \dots, w_{2g}\}$ and $T \cap \{w_1, \dots, w_{2g}\}$ have even cardinality (respectively odd cardinality).*

(b) *There is a one-one correspondence between i -invariant nonlocally free torsion free sheaves of rank 1 and even degree (respectively odd degree) and partitions of $\{w_1, \dots, w_{2g}\} = W_0 - w_0$ into two subsets S', T' of odd (respectively even) cardinality.*

Proof. This lemma can be proved as in the case when Y is nonsingular, so we omit the details.

(a) The correspondence is given by (zero degree case)

$$L = \left\{ \bigotimes_{\omega \in S} \theta_Y(\omega) \otimes \theta_Y \left(-\frac{\#S}{2} \right) \right. \text{ if } w_0 \notin S,$$

$$\left. L = \bigotimes_{\omega \in S \setminus \omega_0} \theta_Y(\omega) \otimes \theta_Y \left(\frac{1 - \#S}{2} \right) \otimes M \right. \text{ if } w_0 \in S, \text{ where } \theta_Y(d) = p^* \theta_{\mathbb{P}^1}(d)$$

and M is the unique squareroot of θ_Y which becomes trivial on pulling back to \tilde{Y} , the normalization of Y . The set S is characterized as the set of points w in W_0 such that i -action on the fibre of L at w is (-identity). The odd degree case can be dealt with similarly.

(b) Let $\pi: \tilde{Y} \rightarrow Y$ be the normalization map. The map $\text{Pic}(\tilde{Y}) \rightarrow T(Y) = \{\text{nonlocally free torsion free sheaves of rank 1 on } Y\}$ defined by $L \rightarrow \pi_* L$ is an isomorphism;

moreover L is i -invariant iff $\pi_* L$ is so. Thus i -invariants in $\text{Pic } \tilde{Y}$ are in bijective correspondence with i -invariants in $T(Y)$. Using this and the correspondence between i -invariants in $\text{Pic } \tilde{Y}$ and partitions of $W_0 - w_0$, we get the result. In case degree = 0, one has

$$L = \bigotimes_{\omega \in S'} \theta_Y(w) \otimes \theta_Y\left(\frac{-1 - \#S'}{2}\right) \otimes \pi_*(\theta_{\tilde{Y}}), \text{ for } s' \subset \{w_1, \dots, w_{2g}\}, \#S' \text{ odd.}$$

S' is the set of points w in $W_0 - w_0$ such that i -action on the fibre of $(\pi^* L / \text{torsion})$ at w is negative of the identity.

3. Quadratic form associated to a singular curve

3.1. Let Y be an irreducible curve with a single node at w_0 . Let $p: Y \rightarrow \mathbb{P}^1$ be a double cover with w_0, w_1, \dots, w_{2g} the ramification points. The image of w_i in \mathbb{P}^1 will again be denoted by w_i , for all i . Consider the divisor $W = 2w_0 + w_1 + \dots + w_{2g}$ in \mathbb{P}^1 . We have $\theta_W = (\theta/m_{w_0}^2) \oplus \left(\bigoplus_{i=1}^{2g} \theta/m_{w_i} \right)$. The multiplication in θ induces quadratic maps A_i on θ/m_{w_i} and A_0 on $\theta/m_{w_0}^2 \approx \theta/m_{w_0} \oplus m_{w_0}/m_{w_0}^2$ given by $A_i(X_i) = X_i^2$, $A_0(X_0, Y_0) = (X_0^2, 2X_0 Y_0)$. Identifying θ_W with k^d , $A = \bigoplus_{i=0}^{2g} A_i$ is a quadratic map from k^d to k^d ($d = 2g + 2$). Now, given an element $b = (b_0, b_{00}, b_1, \dots, b_{2g})$ of k^d , there exists a polynomial f_b of degree $2g + 1$ such that $f_b(w_i) = b_i$ for $i \geq 0$, and the derivative $f'_b(w_0) = b_{00}$. In fact

$$f_b(x) = \sum_{i \geq 0} b_i \mu_{w_i}(x) + \lambda \left(\prod_{i \geq 0} (x - w_i) \right)$$

where

$$\mu_{w_i}(x) = \prod_{j \neq i} (x - w_j) / \prod_{j \neq i} (w_i - w_j) \text{ and } \lambda = \left(b_{00} - \sum_{i \geq 0} b_i \mu'_{w_i}(w_0) / \prod_{i \geq 0} (w_0 - w_i) \right).$$

Then

$$f_b(x) = \sum_{i \geq 0} b_i \lambda_i(x) + \frac{1}{2} b_{00} \lambda_{00}(x),$$

where

$$\lambda_i(w_j) = \delta_{ij},$$

$\lambda_{00}(w_j) = 0$ for all i . Define $\phi_x: k^d \rightarrow k$ by $\phi_x(b) = f_b(x)$. Let $Q_x = \phi_x \circ A$. Then $\{Q_x\}_x$ is a family of quadratic forms on k^d parametrized by \mathbb{P}^1 with values in $\theta_{\mathbb{P}^1}(2g + 1)$ or equivalently a quadratic form Q on the trivial bundle $\mathbb{P}^1 \times k^d$ with values in $\theta_{\mathbb{P}^1}(2g + 1)$. Tracing the above maps, one gets

$$(*) \quad Q_x(X_0, Y_0, X_1, \dots, X_{2g}) = \sum_{i=0}^{2g} \lambda_i(x) X_i^2 + \lambda_{00}(x) X_0 Y_0$$

$$\text{with } \lambda_i(w_j) = \delta_{ij}, \quad \lambda'_i(w_0) = 0, \quad \lambda_{00}(w_i) = 0 \text{ for all } i.$$

3.2. We make a few observations regarding Q . The space $(0, Y_0, 0, \dots, 0)$ is isotropic for Q_x for all x , this is the canonical one dimensional subspace corresponding to $m_{w_0}/m_{w_0}^2$. Also, $Q_{w_i} = X_i^2$, $i = 0, 1, \dots, 2g$; and these are precisely the singular members

of the family. In fact (*) shows that Q is dual (adjoint) to a singular pencil of the form

$$Q_1 = \sum_{i=1}^{2g} X_i^2 + 2X_0 Y_0$$

$$Q_2 = \sum_{i=1}^{2g} a_i X_i^2 + X_0^2 + 2a_0 X_0 Y_0,$$

with $\{a_i\}$ distinct scalars. This is a pencil with Segre symbol $[11 \dots 12]$. We recall that the adjoint of a quadratic form Q is given by the adjoint of the matrix of Q .

3.3. We now give an alternative way of defining Q . We will interpret the maps A_j to have values in $(\theta_{p^1}(d-1))_{w_i}$, $i = 1, \dots, 2g$, and A_0 to have values in $(\theta_{p^1}(d-1)) \otimes \theta/m_{w_0}^2$. The evaluation map $\psi: H^0(\theta_{p^1}(d-1)) \rightarrow \theta_{p^1}(d-1) \otimes \theta_w$ can easily be checked to be an isomorphism. Let e_x be the evaluation map $H^0(\theta_{p^1}(d-1)) \rightarrow (\theta_{p^1}(d-1))_x$. Then $\phi_x = e_x \circ \psi^{-1}$; so that $e_x \circ \psi^{-1} \circ A$ gives Q_x .

3.4. *The general case.* We have restricted ourselves to the case of a single node merely for the simplicity of exposition. The construction in 3.3 works in the case of many ordinary double points too. In that case, for W , we have to take the divisor $\sum_i 2v_i + \sum_j w_j$, where v_i varies over all ordinary double points of Y and $\{v_i, w_j\}_{i,j}$ is the set of ramification points for the double cover $Y \rightarrow \mathbb{P}^1$. The quadratic form Q is then adjoint to the pencil of the form

$$Q_1 = \sum_{i=1}^n X_i^2 + \sum_{j=1}^m Z_j Y_j$$

$$Q_2 = \sum_{i=1}^n a_i X_i^2 + \sum_{j=1}^m (Z_j^2 + 2b_j Z_j Y_j)$$

where a_i, b_j are distinct scalars, m equals the number of ordinary double points; $2m+n=2g+2$, g being the arithmetic genus of Y .

3.5. In this and the next article we want to study a subvariety of the variety of g dimensional linear subspaces of k^{2g+2} which are isotropic for the singular pencil

$$Q_1 = \sum_{i=1}^{2g} X_i^2 + 2X_0 Y_0$$

$$Q_2 = \sum_{i=1}^{2g} a_i X_i^2 + X_0^2 + 2a_0 X_0 Y_0$$

where a_i 's are all distinct scalars. We have a splitting $k^{2g+2} = \bigoplus_{i=1}^{2g} C_i + C_0$ where X_i is the coordinate in C_i for $i = 1, \dots, 2g$ and X_0, Y_0 are the coordinates in C_0 . The subspace of C_0 defined by $X_0=0$ will be denoted by C_0^0 . For simplicity of notation we will often denote $\bigoplus_{i=1}^{2g} C_i$ by k^{2g} .

Let $\text{Grass}_r(\quad)$ denote the grassmannian of r -dimensional subspaces. Define P, P_0 by

$$P = \left\{ V_0 \in \text{Grass}_g(k^d) \mid \begin{array}{l} V_0 \text{ isotropic for the pencil} \\ V_0 = V_1 \oplus C_0^0, V_1 \subset k^{2g} \end{array} \right\}$$

$$P_0 = \left\{ V_1 \in \text{Grass}_{g-1}(k^{2g}) \mid \begin{array}{l} V_1 \text{ isotropic for the pencil} \\ \text{restricted to } k^{2g} \end{array} \right\}.$$

and

Clearly, $P \approx P_0$.

Remark. It is well known that the pencil on k^d induces a dual family on $(k^d)^*$, if a quadratic form on k^d is given by matrix B , the corresponding form on $(k^d)^*$ is given by the adjoint of B . Under the natural correspondence between $\text{Grass}_g(k^d)$ and $\text{Grass}_{g+2}(k^d)^*$ which maps a subspace $V_0 \subset k^d$ to the subspace V of linear forms on k^d vanishing on V_0 , P corresponds to the subvariety P' of $\text{Grass}_{g+2}(k^d)^*$ consisting of subspaces of the form $V = V_1 \oplus C_0^0$ (C_0^0 being the unique subspace of $(C_0)^*$ isotropic for the dual family) such that any generic member of the (dual) family restricted to V has rank 2. Also, if we identify (k^d) with $(k^d)^*$ using the isomorphism given by a generic member of the pencil, say Q_1 , then V gets identified to the orthogonal complement of V_0 with respect to Q_1 .

Lemma 3.6. If V_0 is isotropic for the pencil but V_0 does not belong to P , then

$$V'_0 + k^{2g} = k^d$$

where V'_0 denotes the orthogonal complement of V_0 with respect to a generic form of the pencil.

Proof. Since V_0 is isotropic and the only subspace of C_0 isotropic for a generic form is C_0^0 it follows that $V_0 \cap C_0 = C_0^0$ or $\{0\}$. Hence V_0 is not in P iff $V_0 \cap C_0 = \{0\}$. Taking orthogonal complements with respect to a generic form, the latter is equivalent to $V'_0 + k^{2g} = k^d$.

Remark 3.7. Lemma 3.6 says that V is not in P' iff $\dim V \cap k^{2g} = g$. For V in P' , $\dim V \cap k^{2g} = g + 1$.

4. The generalized Jacobian

With the notations of 3.1, and 3.5, the main result of the section is the following.

Theorem 1. The generalized Jacobian of Y is isomorphic to the complement of P in the space R of g -dimensional vector subspaces isotropic for the pencil.

Since the proof is on similar lines as in §4, [3], we only sketch it briefly with necessary modifications. We will actually show that the generalized Jacobian of Y is isomorphic to the complement of P' (3.5) in the variety R' of $(g+2)$ -dimensional spaces on which Q has rank two generically. We take J as the variety of line bundles of degree g on Y .

4.1. The map $f: J \rightarrow R' - P'$. Let W be the ramification divisor in Y , $\theta_Y(1) = p^* \theta_{p^*}(1)$. Fix a line bundle ξ of degree $2g+1$ on Y . Take α in J . Define $N = (\alpha^2 \otimes \xi)(-g)$. Using 2.3 and the Riemann–Roch theorem, one sees that $h^0(N) = g+2$ and $H^0(N)$ gets embedded in $H^0(N \otimes \theta_W)$ under the restriction map. Now $\xi \otimes \theta_W = \bigoplus \xi_{w_i} \oplus (\xi \otimes \theta/I) =$

$\bigoplus_{i=1}^{2g} C_i \oplus C_0$, length $\theta/I=2$, support $\theta/I=w_0$. Using $\alpha \otimes i^* \alpha = \theta_Y(g)$ and $\alpha_{w_i} = (i^* \alpha)_{w_i}$, one gets $H^0(N \otimes \theta_W) \approx \bigoplus C_i \oplus C_0$. Define $f(\alpha) = H^0(N)$ embedded in $\bigoplus C_i \oplus C_0$ as above. As in §4, [3] one can check that $f(\alpha)$ is in R' . If L is the kernel of the map $N \rightarrow N \otimes \theta/I$, $H^0(N) \cap \left(\bigoplus_{i=1}^{2g} C_i \right) = H^0(L)$ and the latter is of dimension g , so that $f(\alpha)$ is in $R' - P'$.

4.2. *Definition of f^{-1} .* Define a map $h: Y \times k^{2g+2} \rightarrow \theta_W$ by $h=0$ outside W , h/w_i is given by mapping C_i isomorphically onto θ/m_{w_i} and h/w_0 is given by mapping C_0 isomorphically onto θ/I . The kernel K of this map is a vector bundle (Lemma 2.2, [1]). Since Q/K vanishes identically on W , it induces a form q on K with values in $\theta_Y(2g+1)(-2W) = \theta_Y(-1)$ which is everywhere nondegenerate. Take V in $R' - P'$. Let V' be the fibre product of K and $Y \times V$ over $Y \times k^{2g+2}$. As the form restricted to $\bigoplus_{i>0} C_i$ is dual to a nondegenerate pencil, as in the case when Y is nonsingular, one

has $V + \left(\bigoplus_{i \neq j} C_j \right) + C_0 = k^{2g+2}$ for $i > 0$. By Remark 3.7, $V + \left(\bigoplus_{i>0} C_i \right) = k^{2g+2}$.

Therefore the composite of h with the inclusion of $Y \times V$ in $Y \times k^{2g+2}$ is onto, showing that V' is locally free. As in the nonsingular case [3], q restricted to V' has rank two everywhere. Let V'' be the kernel of q on V' . Define $F = (V'/V'')(g+1)$. The quadratic form on F gives an isomorphism $q: F \rightarrow F^*(2g+1)$. There is also an isomorphism $\psi: F \rightarrow F^*(2g+1)$ induced by the canonical alternating form on F with values in $\det F$. Define $N = \text{Ker}((q^{-1} \circ \psi) - Id)$. Then $F = N \oplus i^* N$ with $i^* N \approx \text{Im}((q^{-1} \circ \psi) - id)$ (3.7, [3]). Let $L = (N \otimes \xi^{-1})(g)$. As in [3], to give the inverse of f , we have only to choose a canonical square root of the even degree line bundle L which can be done using Proposition 2.5.

Remark 4.3. The desingularization \tilde{Y} of Y is a nonsingular hyperelliptic curve with ramification points w_1, w_2, \dots, w_{2g} . So, by the theorem in the nonsingular case, the Jacobian of Y is isomorphic to P_0 (defined in 3.5) and hence to P' .

5. The compactified Jacobian

We shall first prove the following theorem in the case of a single node and then indicate the general proof.

Theorem 2. *Let Y be an irreducible curve of arithmetic genus g with only ordinary double points as singularities. Let $p: Y \rightarrow \mathbb{P}^1$ be a double cover such that the set of ramification points contains all double points. Then the compactified Jacobian \bar{J} of Y is isomorphic to the variety R of g -dimensional subspaces of k^{2g+2} which are isotropic for the singular pencil of quadrics of the form*

$$Q_1 = \sum_i X_i^2 + \sum_j 2Y_j Z_j$$

$$Q_2 = \sum_i a_i X_i^2 + \sum_j (Y_j^2 + 2b_j Y_j Z_j)$$

associated to the double cover. Here a_i, b_j are all distinct scalars, j varies over the ordinary double points of Y and i varies over the remaining ramification points.

Proof. We first show that the morphism f^{-1} defined on $R' - P'$ (§4.2) can be extended to whole of R' . Starting with a V in R' we get a rank two coherent sheaf F as in 4.2. If V is in P' , F is not locally free, nor does it split. However, we do have a nondegenerate quadratic form on F with values in $\mathcal{O}_Y(2g+1) \otimes m_{w_0}$ in this case, giving an isomorphism $q: F \xrightarrow{\sim} F^*(2g+1) \otimes m_{w_0}$. Also, one has $\det F$ modulo torsion $= \mathcal{O}_Y(2g+1) \otimes m_{w_0}$. So the canonical alternating form on F gives a morphism (and not an isomorphism) $\Psi: F \rightarrow F^*(2g+1) \otimes m_{w_0}$. Define $N = \ker((q^{-1} \circ \Psi) - Id)$. Notice that this definition of N agrees with that in the case V is in $R' - P'$, in that case we in fact have $F = \text{Ker}((q^{-1} \circ \Psi) - Id) \oplus \text{Im}(q^{-1} \circ \Psi - Id) = N \oplus i^*N$. The rest of the procedure to define f^{-1} works as before, i.e. one can choose canonically a torsion free rank 1 sheaf L such that $L \otimes L$ modulo torsion $= (N \otimes \xi^{-1})(g)$ and define $f'(V) = L$. Thus one gets a morphism $f': R' \rightarrow \bar{J}$ extending f^{-1} . One can check by similar methods that f' restricted to P' is an isomorphism onto $\text{Pic } \bar{Y}$ embedded in \bar{J} as the set J_1 of torsion free rank one sheaves which are not locally free, the embedding being given by the map $L \rightarrow \pi_* L$. Thus we have a bijective birational morphism $f': R' \rightarrow \bar{J}$. Let \tilde{R} and \tilde{J} denote the normalizations of R' and \bar{J} respectively. \tilde{J} is the disjoint union of $\text{Pic } Y$ and two copies of $\text{Pic } \bar{Y}$, so $\tilde{J} \neq R'$. The morphism f' induces an isomorphism of normalizations \tilde{R} and \tilde{J} , let \tilde{f} be the composite of the inverse of this isomorphism with the natural map from \tilde{R} to R' . Since \tilde{f} respects the identification of two copies of $\text{Pic } \bar{Y}$, it factors through \tilde{J} giving $\tilde{f}: \tilde{J} \rightarrow R'$, the inverse of f' . This finishes the proof of the theorem in the one node case.

6. The general case

In the case of m nodes, we have a decomposition $k^{2g+2} = \bigoplus_{i=1}^n C_i \oplus \bigoplus_{j=1}^m D_j$, $\dim C_i = 1$, $\dim D_j = 2$, $n + 2m = 2g + 2$. It can be proved (in a similar way as in the one node case) that the generalized Jacobian of Y is isomorphic to the open subset of R' consisting of those subspaces which intersect $\bigoplus_i C_i \bigoplus_{j \neq k} D_j$, in subspaces of dimension g , for $i \leq k \leq m$. As before, we can also get a birational bijective morphism $\bar{f}: R' \rightarrow \bar{J}$ inducing an isomorphism of normalizations. We identify the normalizations \tilde{R}' and \tilde{J} via this isomorphism. Thus we are again in the situation $\tilde{J} \rightarrow R' \rightarrow \bar{J}$. At a singular point L of \bar{J} , the complete local ring has the form $T \otimes \prod_i k[[X_i, Y_i]]/(X_i Y_i)$, i varying over the p number of nodes at which L is not locally free, T being a regular ring. Thus, the singularities of \bar{J} being product singularities, we can prove $R' = \bar{J}$ using induction on p , i.e. can assume the result for $p < m$. Let U be the complement in \bar{J} of the closed subset S corresponding to torsion free sheaves which are not locally free at all m nodes. By induction, $\mathcal{O}_{R'}/U \simeq \mathcal{O}_{\bar{J}}/U$. Since \bar{J} is Cohen-Macaulay and $\bar{J} - U$ is not a divisor ($m \geq 2$) we have $H^0(V, \mathcal{O}_{\bar{J}}) \simeq H^0(U \cap V, \mathcal{O}_{\bar{J}})$, for any open set V . The commutative diagram

$$\begin{array}{ccc} H^0(V, \mathcal{O}_{\bar{J}}) & \xrightarrow{\sim} & H^0(U \cap V, \mathcal{O}_{\bar{J}}) \\ \downarrow & & \downarrow \approx \\ H^0(V, R') & \rightarrow & H^0(U \cap V, \mathcal{O}_{R'}) \end{array}$$

shows that $H^0(V, \mathcal{O}_{\bar{J}}) = H^0(V, \mathcal{O}_{R'})$ i.e. $\mathcal{O}_{\bar{J}} = \mathcal{O}_{R'}$. Thus $\bar{J} = R'$. Thus we have proved the general theorem.

Acknowledgement

I would like to thank S Ramanan, C J Rego and N Mohan Kumar for useful discussions.

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