Oscillating non-singular relativistic spherical model

Naresh Dadhich*

Inter–University Center for Astronomy and Astrophysics, Post Bag 4, Ganeshkind, Pune 411007, India

A.K. Raychaudhuri

Relativity and Cosmology Research Centre, Jadavpur University, Calcutta 700 032, India

arXiv:gr-qc/9901081v1 28 Jan 1999

February 7, 2008

Abstract

A particular choice of the time function in the recently presented spherical solution by Dadhich [1] leads to a singularity free cosmological model which oscillates between two regular states. The energy-stress tensor involves anisotropic pressure and a heat flux term but is consistent with the usual energy conditions (strong, weak and dominant). By choosing the parameters suitably one can make the model consistent with observational data. An interesting feature of the model is that it involves blue shifts as in the quasi steady state model [2] but without violating general relativity.

04.20, 04.60, 98.80Hw

*E-mail: nkd@iucaa.ernet.in

Following the discovery of non-singular cylindrically symmetric perfect fluid exact cosmological solution of the Einstein equation by Senovilla [3], some spherically symmetric non-singular models have been presented by Dadhich et.al [1,4]. These models have an energy-stress tensor with anisotropic pressure and heat flux but obeying the strong, weak and dominant energy conditions. The metric has a time function which can be arbitrarily chosen subject to the constraint of non-singularity and the energy conditions. It turns out that there exist different such choices which will be discussed in a detailed paper separately. In this letter we shall confine to the choice that gives an oscillatory behaviour of the universe without any singularity.

The authors are not aware of any oscillatory singularity free model in classical general relativity (GR) while there are some oscillatory behaviour models proposed in the recent formulation of of quasi steady state cosmology (QSSC) [2]. As the QSSC models predict the possibility of blue shift, our model would also admit that possibility. Thus should observations in future reveal blue shifts, it may simply indicate that the matter in the uinverse is not perfect fluid and one need not bring in the ideas of non-conservation as in QSSC contradicting GR.

Our oscillatory model is described by the metric [1],

$$ds^2 = (r^2 + P)dt^2 - \frac{2r^2 + P}{r^2 + P}dr^2 - r^2(d\theta^2 + sin^2\theta d\varphi^2) \tag{1}$$

where $P = P(t)$ which can be chosen arbitrarily. The choice $P(t) = a^2 + b^2 cos\omega t$ with $a^2 > b^2$ will render oscillatory behaviour to the model without encountering divergence of any kinematical and physical parameters. In Ref. [1] the choice made was $P(t) = a^2 + b^2 t^2$, which of course did not give oscillatory behaviour.

The energy-stress tensor for imperfect fluid is given by [5],

$$T_{ik} = (\rho + p)u_i u_k - p g_{ik} + \triangle p[c_i c_k + \frac{1}{3}(g_{ik} - u_i u_k)] + 2q c_{(i} c_{k)} \tag{2}$$

where u_i and c_i are respectively unit timelike and spacelike vectors, ρ energy density and p isotropic fluid pressure, $\triangle p$ pressure anisotropy and the term involving q represents heat flux.

We employ the comoving coordinates to write $u_i = \sqrt{g_{00}}\delta_i^0$ and take $c_i = \sqrt{g_{11}}\delta_i^1$. The kinematic parameters; expansion, shear and acceleration for the metric (1) read as follows:

$$\theta = \frac{-\dot{P}r^2}{2(2r^2+P)(r^2+P)^{3/2}},\ \sigma^2 = \frac{2}{3}\theta^2,\ \dot{u}_r = -\frac{r}{r^2+P}. \tag{3}$$

Now applying the Einstein equation, we obtain

$$8\pi\rho = \frac{2r^2+3P}{(2r^2+P)^2} \tag{4}$$

$$8\pi p_r = \frac{1}{2r^2+P} \tag{5}$$

$$8\pi p_\perp = \frac{1}{2r^2+P} + \frac{r^2}{4(2r^2+P)(r^2+P)^2}\left[2\ddot{P} - \frac{(9r^2+5P)\dot{P}^2}{(2r^2+P)(r^2+P)}\right] \tag{6}$$

$$8\pi q = \frac{-\dot{P}r}{(2r^2+P)^{3/2}(r^2+P)}. \tag{7}$$

The pressure anisotropy $\triangle p = p_r - p_\perp$ is given by

$$8\pi \triangle p = \frac{-r^2}{4(2r^2+P)(r^2+P)^2}\left[2\ddot{P} - \frac{(9r^2+5P)\dot{P}^2}{(2r^2+P)(r^2+P)}\right]. \tag{8}$$

Now we choose for the time function

$$P(t) = a^2 + b^2 cos\omega t,\ a^2 > b^2. \tag{9}$$

This lends oscillatory behaviour to the model which oscillates between the two regular states. The oscillation period is $t = 2\pi/\omega$, density is maximum at $t = (2n+1)\pi/\omega$ and it is minimum at $t = 2n\pi/\omega$ for an integer n. The model could have as low and as high density as one pleases by choosing large values for the parameters a and b with the former being as close to the latter from the above. The solution involves three parameters a, b and ω. Besides, when comparing with observational data, one needs a specification of the locale of observation,i.e. the time t_0 of observation and r_0 the radial coordinate of the observer. Since there is abundance of free parameters, it is therefore possible to coast the model as close to the observations as one pleases by suitale choice.

Above all the most interesting feature of the model is oscillatory behaviour indicating like the steady state cosmology no beginning and no end. As in QSSC [2], this model would also predict blue shifts, should they be observed in future, one need not necessarily have to invoke non-conservation of energy but instead an imperfect fluid distribution could as well do without violating GR and the usual energy conditions. This is quite remarkable and interesting feature of our model.

The pressure anisotropy and heat flux fall off as r^{-4} and they vanish at the centre $r = 0$. It is obvious that expansion and heat flux have similar behaviour (note that in Ref. [1], there was a sign error which indicated opposite behaviour), vanishing at $\omega t = 0, \pi$ and attaining maximum at $\omega t = \pi/2, 3\pi/2$. That is in expanding phase heat flows out while the reverse happens for contracting phase. The pressure anisotropy could like density be made as small as one pleases and it changes sign at $t = (2n+1)/2\pi\omega$. Acceleration vanishes at both ends, $r \rightarrow 0, \infty$ and is finite for all t at a given r.

For the oscillatory choice (9), it can be easily checked that $\rho > p_r, p_\perp > 0, (\rho+p_r)^2 - 4q^2 > 0$ and $\rho - p_r - 2p_\perp + [(\rho+p_r)^2 - 4q^2]^{1/2} > 0$ always. This ensures that all the (weak, strong and dominant) energy conditions are satisfied. All the physical and kinematic parameters always remain regular and finite.. The metric is simple enough to see that it is causally stable and geodesically complete.

The present model gives a picture somewhat different from non-oscillating singularity-free models [1,3,4,6]. There the universe has a state of infinite dilution both in the infinite past and future and in between there is a state of maximum contraction. In our case $P(t)$ never becomes arbitrarily large and so there is no infinite dilution but periodically the physical variables oscillate between finite maxima and minima. However as $r \to \infty$, all the physical variables tend to vanish as in other non-singular models. This behaviour is demanded by the theorem [7] that in non-singular models, the space average of all the kinematic scalars and physical parameters must vanish (for a weaker theorem on the vanishing of space time averages [8].

We have thus shown that it is possible to have a truely oscillatory singularity free spherical model within GR without violating conservation of energy and its usual conditions. Such a model would be consistent with blue shifts, should they be discovered in future. The model has a number of free parameters which can be suitably chosen to coast it arbitrarily close to the observations. One may not quite relish the introduction of an imperfect fluid but then we are not violating any stringent physical requirement. Abondoning the cosmological principle of isotropy and homogeneity requires hardly any apology but the introduction of a specially favoured centre of the universe may require some justification. However one cannot get the solution of the Einstein field equation without introducing some symmetry assumption. In any case the solution broadens our horizon about the potentialities of relativistic models.

ACKNOWLEDGMENTS

We wish to thank L. K. Patel and R. S. Tikekar and members of the Relativity and Cosmology Research Centre, Jadavpur University for helpful discussions. We also thank IUCAA for supporting visits of the authors that facilitated this work.

REFERENCES

1. N Dadhich, J. Astrophys. Astr. **18**, 343 (1997).

2. F Hoyle, G Burbidge, J V Narlikar, Mon. Not. R. Astron. Soc. **267**, 1007 (1994).

3. J M M Senovilla, Phys. Rev. Lett. **64**, 2219 (1990).

4. N Dadhich, L K Patel and R Tikekar, Non-singular spherical fluid model with null radiation, submitted.

5. S D Maharaj and R Maartens, J. Math. Phys. **27**, 2517 (1986).

6. E Ruiz and J M M Senovilla, Phys. Rev. **D45**, 1995 (1992), N Dadhich, L K Patel and R Tikekar, Pramana **44**, 303 (1995).

7. A K Raychaudhuri, to be submitted.

8. A K Raychaudhuri, Phys. Rev. Lett. **80**, 654 (1998).