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Upper bound on the mass of the lightest neutralino in a general supersymmetric theory with grand unification

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We derive an upper bound on the mass of the lightest neutralino in a supersymmetric theory containing an arbitrary number of singlet, doublet, and triplet Higgs superfields under the standard model gauge group. Assuming grand unification of the gauge couplings, whereby the theory reduces to the minimal supersymmetric standard model with an arbitrary number of singlets, the bound can be expressed in terms of the gluino mass. Including radiative corrections, the upper bound on the mass of the lightest neutralino is 62 GeV for a gluino mass of 200 GeV, which increases to 178 GeV for a 1 TeV gluino.

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In supersymmetric theories with R-parity conservation [1], the lightest neutralino state is expected to be the lightest supersymmetric particle (LSP). In the minimal supersymmetric standard model (MSSM) at least two Higgs doublets H_1 and H_2, with hypercharge (Y) -1 and $+1$, respectively, are required to give masses to quarks and leptons, and to cancel triangle gauge anomalies. The fermionc partners of these Higgs doublets mix with the fermionic partners of the gauge bosons to produce four neutralino states $\tilde{\chi}_i^0$, $i = 1, ..., 4$, and two chargino states $\tilde{\chi}_i^{\pm}$, $i = 1, 2$. In the nonminimal supersymmetric model containing a Higgs singlet and two Higgs doublets of the minimal model, the mixing of fermionic partners of neutral Higgs and gauge bosons produces five neutralino states. The neutralino states of the minimal [2–4] and the nonminimal model [5–7] have been studied in great detail, because the lightest neutralino, being the LSP, is the end product of any process involving supersymmetric particles in the final state.

In this paper we consider the neutralino mass matrix in a general supersymmetric theory containing an arbitrary number of singlet, doublet, and triplet Higgs superfields under the standard model gauge group. We obtain an upper bound on the mass of the lightest neutralino, and a lower bound on the mass of the heaviest neutralino state in such a general supersymmetric theory. These bounds depend on the soft-supersymmetry-breaking gaugino masses and the vacuum expectation values of the doublet and triplet, but not the singlet, Higgs fields. This is in contrast with the situation that obtains in the Higgs sector of such a theory [8], where the (tree-level) upper bound on the lightest Higgs boson mass is controlled by the vacuum expectation value of the doublet Higgs fields and dimensionless parameters only. Nevertheless, if we assume the simplest form of grand unification of such a general supersymmetric theory, whereby the triplet and the extra doublet Higgs fields are eliminated, then these bounds are controlled by soft-supersymmetry-breaking gaugino mass parameters, and M_Z and θ_W. Since the latter are known, the former entirely determine the bounds.

We start by recalling the neutralino mass matrix in the minimal supersymmetric standard model [1]. In the basis

$$\psi_j^0 = (-i\lambda' \ , \ -i\lambda^3 \ , \ \psi_{H_1}^1 \ , \ \psi_{H_2}^2) \ , \ j = 1,2,3,4 \ , \quad (1)$$

where λ' and λ^3 are the two-component gaugino states corresponding to the $\mathrm{U}(1)_Y$ and the third component of $\mathrm{SU}(2)_L$ gauge groups, respectively, and $\psi_{H_1}^1$ and $\psi_{H_2}^2$ are the two-component Higgsino states this mass matrix can be written in a form which is well known [1]. We shall denote the neutralino eigenstates of this mass matrix by χ_1^0, χ_2^0, χ_3^0, and χ_4^0 labeled in order of increasing mass [9]. Since some of the neutralino masses resulting from diagonalization of the mass matrix can be negative, we shall consider the squared mass matrix $M^\dagger M$. An upper bound on the squared mass of the lightest neutralino χ_1^0 can be obtained by using the fact that the smallest eigenvalue of $M^\dagger M$ is smaller than the smallest eigenvalue of its upper left 2×2 submatrix,

$$\begin{bmatrix} {M_1}^2 + {M_Z}^2 \sin^2\theta_W & -{M_Z}^2 \sin\theta_W \cos\theta_W \\ -{M_Z}^2 \sin\theta_W \cos\theta_W & {M_2}^2 + {M_Z}^2\cos^2\theta_W \end{bmatrix} \ , \quad (2)$$

thereby resulting in the upper bound [10]

$$M_{\chi_1^0}^2 \le \min({M_1}^2 + {M_Z}^2 \sin^2\theta_W \ , \ \ {M_2}^2 + {M_Z}^2\cos^2\theta_W) . \quad (3)$$

On the other hand, the bigger eigenvalue of submatrix (2) gives a lower bound on the squared mass of the heaviest neutralino:

$$M_{\chi_4^0}^2 \ge \max({M_1}^2 + {M_Z}^2 \sin^2\theta_W \ , \ \ {M_2}^2 + {M_Z}^2\cos^2\theta_W) . \quad (4)$$

Here M_1 and M_2 are supersymmetry-breaking gaugino masses, and g' and g the gauge couplings, associated with the $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$ subgroups of the standard model, respectively. Furthermore, $\tan\beta = v_2/v_1$, where $v_1 = \langle H_1^0\rangle$ and $v_2 = \langle H_2^0\rangle$, and $M_Z^2 = (g^2 + g'^2)(v_1^2 + v_2^2)/2$.

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Since M_Z and θ_W are known, the bounds (3) and (4) are controlled by the soft-supersymmetry- (SUSY-) breaking gaugino masses, in contrast with the bounds on the (tree-level) masses of the lightest and the heaviest scalar Higgs bosons in MSSM, which do not depend on supersymmetry-breaking masses [11].

We now consider a general class of supersymmetric models based on a standard model gauge group with an arbitrary Higgs sector. We shall assume [8], in addition to two Higgs doublets $H_1^{(1)}, H_2^{(1)}$ (with $Y = \mp 1$) which are coupled to the quarks and leptons in the superpotential

$$W^0 = h_U Q_L U_L^c H_2^{(1)} + h_D Q_L D_L^c H_1^{(1)} + h_E L_L E_L^c H_1^{(1)} , \tag{5}$$

an arbitrary number of extra pairs of Higgs doublets $H_1^{(j)}$, $H_2^{(j)}$, $j = 2, ..., d+1$, which are decoupled from quarks and leptons so that there are no dangerous flavor-changing neutral currents [12]. In addition the model contains (i) gauge singlets N^σ, $\sigma = 1, ..., n_s$; (ii) SU(2) triplets $\Sigma^{(\alpha)}$, $a = 1, ..., t_0$, with $Y = 0$; and (iii) SU(2) triplets $\Psi_1^{(k)}, \Psi_2^{(k)}$, $k = 1, ..., t_1$, with $Y = \pm 2$. We note that the above extra Higgs multiplets are the only ones that can provide renormalizable couplings to all possible combinations of $H_1^{(1)}$ and $H_2^{(1)}$ in the superpotential [13].

The most general renormalizable superpotential for the above Higgs supermultiplets can be written as

$$W_1(H_1, H_2, N, \Sigma, \Psi_1, \Psi_2) = f_1^{ij\sigma} H_i^{(i)} H_2^{(j)} N^{(\sigma)} + f_2^{ij\alpha} H_1^{(i)} \Sigma^\alpha H_2^{(j)} + g_1^{ijk} H_1^{(i)} \Psi_1^{(k)} H_1^{(j)} + g_2^{ijk} H_2^{(i)} \Psi_2^{(k)} H_2^{(j)} + h_{\alpha jk} \mathrm{tr}(\Sigma^{(\alpha)} \Psi_1^{(j)} \Psi_1^{(k)}) + \frac{1}{6} \chi_{abc} \mathrm{tr}(\Sigma^{(a)} \Sigma^{(b)} \Sigma^{(c)}) + \frac{1}{6} \lambda_{\mu\nu\sigma} N^{(\mu)} N^{(\nu)} N^{(\sigma)} , \tag{6}$$

where

$$\Sigma = \begin{bmatrix} \zeta^0/\sqrt{2} & \zeta_2^+ \\ \zeta^- & -\zeta^0/\sqrt{2} \end{bmatrix} , \tag{7a}$$

$$\Psi_1 = \begin{bmatrix} \psi_1^+/\sqrt{2} & -\psi_1^{++} \\ \psi_1^0 & -\psi_1^+/\sqrt{2} \end{bmatrix} ,$$

$$\Psi_2 = \begin{bmatrix} \psi_2^+/\sqrt{2} & -\psi_2^{++} \\ \psi_2^0 & -\psi_2^+/\sqrt{2} \end{bmatrix} . \tag{7b}$$

Without loss of generality, we can choose our basis in the space of Higgs doublets $H_1^{(j)}$ and $H_2^{(j)}$ such that only the Higgs doublets $H_1^{(1)}$ and $H_2^{(1)}$ acquire [14] a nonzero vacuum expectation value (VEV). This requires [8] that some of the Yukawa couplings in (6) vanish:

$$f_1^{1j\sigma} = f_2^{1j\alpha} = g_1^{1jk} = g_2^{1jb} = 0 \quad (j \neq 1) . \tag{8}$$

Assuming (8), the superpotential (6) can be written as

$$\begin{aligned} W' = & -f_1^{11\sigma} H_1^{(1)0} H_2^{(1)0} N^{(\sigma)} + \frac{1}{\sqrt{2}} f_2^{11\alpha} H_1^{(1)0} H_2^{(1)0} \xi^{0\alpha} \\ & -g_1^{11k} H_1^{(1)0} H_1^{(1)0} \psi_1^{0k} - g_2^{11k} H_2^{(1)0} H_2^{(1)0} \psi_2^{0k} \\ & +\frac{1}{\sqrt{2}} h_{\alpha jk} \xi^{0\alpha} \psi_1^{0j} \psi_2^{0k} \\ & +\frac{1}{6} \lambda_{\mu\nu\sigma} N^{(\mu)} N^{(\nu)} N^{(\sigma)} + \cdots , \end{aligned} \tag{9}$$

where we have not explicitly written those terms in (9) which involve fields which do not obtain vacuum expectation values. Choosing

$$\begin{aligned} & \psi_I^0 = (-i\lambda', -i\lambda^3, \psi_{H_1}^1, \psi_{H_2}^2, \psi_{N\sigma}, \psi_{\zeta 0\alpha}, \psi_{\psi 0i}, \psi_{\psi 0j}) , \\ & \sigma = 1, ..., n_s , \quad a = 1, ..., t_0 , \quad i, j = 1, ..., t_1 , \\ & I = 1, ..., (4 + n_s + t_0 + 2t_1) \end{aligned} \tag{10}$$

as the basis, it is straightforward to write the neutralino mass matrix M for the general class of supersymmetric models based on standard model gauge group with an arbitrary Higgs sector. We shall not display this mass matrix here, but shall content ourselves with examining the upper left 2×2 submatrix of $M^\dagger M$ corresponding to this mass matrix M. This submatrix can be written as

$$\begin{bmatrix} M_1{}^2 + M_Z{}^2 \sin^2\theta_W + 6g'^2 y^2 & -M_Z{}^2 \sin\theta_W \cos\theta_W - 2gg'y^2 \\ -M_Z{}^2 \sin\theta_W \cos\theta_W - 2gg'y^2 & M_2{}^2 + M_Z{}^2 \cos^2\theta_W + 2g^2 u^2 \end{bmatrix} , \tag{11a}$$

where

$$u^a = \langle \xi^{0a} \rangle , \quad x^\sigma = \langle N^\sigma \rangle , \quad y_1{}^i = \langle \psi_1{}^{0i} \rangle , \quad y_2{}^j = \langle \psi_2{}^{0j} \rangle , \tag{11b}$$

$$y^2 = \sum_i [(y_1{}^i)^2 + (y_2{}^i)^2] , \quad u^2 = \sum_a (u^a)^2 . \tag{11c}$$

In (11a) the expressions for the W and Z masses are

$$M_Z{}^2 = \tfrac{1}{2}(g^2 + g'^2)[v_1{}^2 + v_2{}^2 + 4y^2] , \tag{12a}$$

$$M_W{}^2 = \tfrac{1}{2} g^2 [v_1{}^2 + v_2{}^2 + 4u^2 + 2y^2] . \tag{12b}$$

The vacuum expectation values that enter into the W and Z masses (12a) and (12b) are experimentally constrained by the ρ parameter. From a recent global fit, which includes the Collider Detector at Fermilab (CDF) data, we have [15]

$$\rho = \frac{M_W{}^2}{M_Z{}^2 \cos^2\theta_W} = 1.0004 \pm 0.0022 \pm 0.002 , \tag{13}$$

where the second error is due to the Higgs boson mass. This result is remarkably close to the expected standard model value of $\rho = 1$. Taking a value of $\rho = 1$ implies, through (12a) and (12b), the following constraint on the triplet vacuum expectation values:

$$4u^2 = 2y^2 \ . \tag{14}$$

We note that (14) implies a fine-tuning of the parameters of the triplet vacuum expectation values in order to maintain the experimental result (13) in the general supersymmetric theory that we are considering. However, we shall assume the constraint (14) to be true, although our final results do not depend on this constraint. With this constraint the W and Z masses can be written as [16]

$$M_W{}^2 = M_Z{}^2 \cos^2\theta_W = \tfrac{1}{2} g^2 (v_1{}^2 + v_2{}^2 + 4y^2) \ . \tag{15}$$

The combination $(v_1^2 + v_2^2 + 4y^2)$ is constrained to be $\approx$ (174 GeV)^2, but the ratio $y/(v_1{}^2 + v_2{}^2)$ is unconstrained. In the general supersymmetric standard model the smallest and the largest eigenvalue of (11a) serve as the upper bound on the mass of the lightest neutralino (χ_1^0) and the lower bound on the mass of the heaviest neutralino (χ_n^0), respectively. Using (14) in (11a), we can write these bounds as

$$M^2_{\chi_1^0} \le \min(M_1{}^2 + M_Z{}^2 \sin^2\theta_W + 6g'^2y^2, M_2{}^2 + M_Z{}^2\cos^2\theta_W + g^2y^2) \ , \tag{16}$$

$$M^2_{\chi_n^0} \ge \max(M_1{}^2 + M_Z{}^2 \sin^2\theta_W + 6g'^2y^2, M_2{}^2 + M_Z{}^2\cos^2\theta_W + g^2y^2) \ . \tag{17}$$

The bounds (16) and (17) depend on *a priori* unknown vacuum expectation values of the triplet Higgs fields and gaugino mass parameters. Nevertheless, as we shall see, these bounds can become meaningful in theories with gauge coupling unification [17]. It is important to note that the singlet vacuum expectation values decouple from these bounds.

In the general supersymmetric theory that we are considering, the renormalization-group equations (RGE's) [8] for the standard SU(3)×SU(2)×U(1) gauge couplings can be written as [$g_1^2 = \frac{5}{3}g'^2$, $g_2^2 = g^2$, $\tan\theta_W = g'/g$, g_3 is the SU(3) gauge coupling constant]

$$16\pi^2 \frac{dg_1}{dt} = [\tfrac{33}{5} + \tfrac{3}{5}(6t_1 + d)]g_1^3 \ ,$$
$$16\pi^2 \frac{dg_2}{dt} = (1 + 2t_0 + 4t_1 + d)g_2^3 \ , \tag{18}$$
$$16\pi^2 \frac{dg_3}{dt} = -3g_3^3 \ .$$

The RGE's depend on the number and the type of the Higgs representations (d, t_0, t_1). We note that additional doublets increase the β functions of the gauge couplings, even though the VEV's of these doublets have been rotated away. If we assume that the gauge couplings unify at some grand-unification scale M_U, i.e., $g_1(M_U) = g_2(M_U) = g_3(M_U) = g_U$, then the simplest choice is

$$d = t_0 = t_1 = 0, \tag{19}$$

with n_s arbitrary, i.e., the MSSM with an arbitrary number of singlet superfields. In other words only Higgs singlets, in addition to the two Higgs doublets of the MSSM, are consistent with unification [18] without fine-tuned cancellations [19]. The bounds (16) and (17) now reduce to

$$M^2_{\chi_1^0} \le \min(M_1{}^2 + M_Z{}^2 \sin^2\theta_W \ , \ M_2{}^2 + M_Z{}^2\cos^2\theta_W), \tag{20}$$

$$M^2_{\chi_n^0} \ge \max(M_1{}^2 + M_Z{}^2 \sin^2\theta_W \ , \ M_2{}^2 + M_Z{}^2\cos^2\theta_W), \tag{21}$$

which are the same as the bounds (3) and (4) in MSSM. With condition (19), the gaugino mass parameters satisfy the one-loop RGE's ($|M_3| \equiv m_{\tilde{g}}$, the gluino mass) [20]

$$16\pi^2 \frac{dM_i}{dt} = b_i M_i g_i^2 \ , \ \ b_i = (\tfrac{33}{5}, 1, -3) \ . \tag{22}$$

Equations (18), (19), and (22) imply ($\alpha_i = g_i^2/4\pi, \alpha_U = g_U/4\pi$)

$$M_1(M_Z)/\alpha_1(M_Z) = M_2(M_Z)/\alpha_2(M_Z) = M_3(M_Z)/\alpha_3(M_Z) = m_{1/2}/\alpha_U \ , \tag{23}$$

where $m_{1/2}$ is the common gaugino mass at the grand unification ion sale, and α_U is the unified coupling constant [21–23]. Relation (23), which is the same as occurs in the MSSM with grand unification [24], reduces the three gaugino mass parameters to one, which we take to be $m_{\tilde{g}}$. The other two gaugino mass parameters are then determined through

$$M_2(M_Z) = (\alpha/\alpha_3 \sin^2\theta_W)m_{\tilde{g}} \simeq 0.27 m_{\tilde{g}} \ ,$$
$$M_1(M_Z) = (5\alpha/3\alpha_3 \cos^2\theta_W)m_{\tilde{g}} \simeq 0.14 m_{\tilde{g}} \ , \tag{24}$$

where we have used [25–27,15] the value of couplings at the Z^0 mass:

$$\alpha^{-1}(M_Z) = 127.9 \ , \ \ \sin^2\theta_W = 0.2324 \ , \ \ \alpha_3 = 0.123 \ . \tag{25}$$

Results (20) and (21), together with (24), imply

$$M^2_{\chi_1^0} \le M_1{}^2 + M_Z{}^2 \sin^2\theta_W \sim (0.02 m_{\tilde{g}}^2 + 1924.5) \text{ GeV}^2 \ , \tag{26}$$

$$M^2_{\chi_n^0} \ge M_2{}^2 + M_Z{}^2 \cos^2\theta_W \sim (0.07 m_{\tilde{g}}^2 + 6356.5) \text{ GeV}^2 \ . \tag{27}$$

For a gluino mass of 200 GeV the upper bound (26) for the lightest neutralino is 52 GeV, and the lower bound (27) for the heaviest neutralino is 96 GeV. Similarly, for a gluino mass of 1 TeV, these upper and lower bounds become 148 and 276 GeV, respectively.

The bounds (16), (17) and (26), (27) constitute the main results of this paper. While the former are valid in a supersymmetric theory based on standard model gauge group and an arbitrary Higgs sector, the latter are valid only when we impose the constraint of grand unification on gauge couplings in the simplest form on such a theory. In the latter case the general model is reduced to MSSM with an arbitrary number of singlet Higgs fields. Thus, although a bound on the lightest neutralino mass exists in an arbitrary supersymmetric theory, it is calculable in terms of gluino mass only under the assumption of gauge coupling unification. We further note that since the soft-supersymmetry-breaking gaugino masses satisfy the GUT relation (23) in any grand-unified theory based on a simple group independently of the breaking pattern to the standard model gauge group (see [24]), our results (26) and (27) are valid in any such supersymmetric grand-unified theory.

Finally, we discuss the effect of radiative corrections to the upper and lower bounds on the lightest and heaviest neutralino masses derived above, which arise from the dominant top-quark (t-)-top-squark ($\tilde{t}$) loops. For Higgsino-like neutralino, a simple estimate of the radiative corrections to its mass arising from top-quark–top-squark loops is given by

$$\frac{\Delta M_{\chi^0}}{M_{\chi^0}} \approx 3\frac{h_t^2}{16\pi^2}\ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right) \approx 5\% \;, \qquad (28)$$

where we have taken the stop mass to be equal to 1 TeV. A similar estimate holds for a gauginolike neutralino. These estimates are very close to the generic results which emerge from detailed calculations [28] carried out in the context of the MSSM, although, for some extreme values of parameters, the radiative corrections to the lightest neutralino mass can be as large as 20%. On the other hand, the radiative corrections to the heaviest neutralino mass in the minimal supersymmetric standard model do not exceed 5%. Taking these results as indicative of the radiative corrections in the general model we are considering, we estimate a conservative radiatively corrected upper bound on the mass of the lightest neutralino to be about 62 GeV for a 200 GeV gluino, which increases to 178 GeV for a gluino mass of 1 TeV [29]. Similarly, a conservative estimate for the lower bound on the heaviest neutralino mass becomes 101 GeV for a gluino mass of 200 GeV, the bound increasing to 290 GeV as the gluino mass increases to 1 TeV.

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