Pramāṇa, Vol. 22, No. 2, February 1984, pp. 97–106, © Printed in India.

Colour dielectric model of the proton

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MS received 18 March 1983; revised 23 August 1983

Abstract. A model of the proton with its constituent quarks bound in a colour polarizable medium with dielectric constant varying as $(a/r - b^2)$ from a fixed centre, is presented. The Dirac equation modified by the colour polarization is solved and the analytic expression for the wavefunction of the quarks obtained shows that quarks with higher energy lie closer to the fixed centre. The energy spectrum is equispaced without any continuum. A semiclassical approximation scheme yields closed orbits for quarks which have smaller size for higher energies and no orbits with size bigger than a certain maximum, thereby rendering the quarks permanently confined. The wavefunctions of the three quarks constituting the proton are used to calculate physical parameters of the proton such as its mass, charge radius and weak coupling constant which with suitable choice of the constants a and b appearing in the dielectric constant agree fairly well with experimental results.

Keywords. Colour dielectric; confined quarks; proton; elliptic orbits; equispaced energy spectrum.

1. Introduction

The quark model of hadrons is now highly successful in explaining their symmetries. Quantum chromodynamics (QCD), the gauge field theory based on exact SU(3) colour symmetry of quarks has emerged as a reasonably respectable theory for strong interaction of hadrons. Although there have been several models such as the bag model (Chodos *et al* 1974; Hasenfratz and Kuti 1978) and various potential models to describe some properties of hadrons in the above framework at phenomenological levels, the mechanism of quark confinement is not yet fully understood.

Recently the concept of introducing colour dielectric constant in Dirac equation has been suggested by Lee (1980) to explain confinement of quarks. In the present paper, we implement this idea in a different way analogous to our work (Jena and Pradhan 1981) of confinement of photon where Maxwell equation in a suitably chosen dielectric medium has been used. Confinement of gluons can be described in a similar manner by solving the Maxwell-like equations for chromoelectric and chromomagnetic fields. Here, the nonlinear terms are considered to simulate the effect of a medium (in colour space) which is taken to be nonmagnetic but dielectric. In the present paper we extend this idea to coloured quarks which are capable of producing colour polarization. They are assumed to be massless and their interaction is taken care of by considering them to move in a colour-polarized medium characterized by a colour dielectric constant. These massless coloured quarks are described by Dirac equation modified by the colour dielectric constant. Since the massless Dirac equation has the same form as the Maxwell equation, we incorporate the colour dielectric constant into massless Dirac equation in the same way as the conventional dielectric constant enters the Maxwell equations. The colour dielectric constant is chosen to be spherically symmetric and of the form

$$\varepsilon_c(\mathbf{r}) \equiv \varepsilon_c(r) = (a/r) - b^2, \tag{1}$$

where a and b are constants. This choice is purely phenomenological and is motivated partly by the form of dielectric constant in our discussion of confinement of photon (Jena and Pradhan 1981). An *a posteriori* justification comes from those features of the solution *i.e.* (i) equispaced energy spectrum without continuum (ii) wavefunctions with higher energy states lying closer to the centre than the lower energy ones (iii) closed elliptic orbits with their size decreasing with increase of energy, which are characteristic of confinement.

Since the colour dielectric medium takes care of all the interaction among the quarks, they are assumed to move as independent particles inside a hadron. We consider the lightest baryon: the proton, and calculate some static properties such as its mass, charge radius and weak coupling constant in terms of the two parameters a and b whose values are obtained by fitting with experimental results. In §2, we set up and solve the Dirac equation for massless quarks in a colour dielectric medium characterized by the dielectric constant ε_c of (1). The energy spectrum of the confined quarks is obtained in §3 and in §4, we obtain the quark orbits by solving the semiclassical eikonal equation. In §5, the proton wavefunction is written from those of its constituent quarks and its various static properties are calculated in terms of the two parameters appearing in the colour dielectric constant.

2. Equation of motion

Since quarks in this model are massless, coloured, spin-$\frac{1}{2}$ objects moving in a colour-dielectric medium, they are described by massless Dirac equation into which the colour dielectric constant is incorporated. This is done in the same manner as the conventional dielectric constant enters the Maxwell equation,

$$\nabla \times \mathbf{E} = -(\partial \mathbf{B}/\partial t), \tag{2}$$

$$\nabla \times \mathbf{B} = \varepsilon(\partial \mathbf{E}/\partial t), \tag{3}$$

in a non-magnetic, dielectric medium with dielectric constant ε. With the choice of time dependence of $\mathbf{E}$ and $\mathbf{B}$ as

$$(\mathbf{E}, \mathbf{B}) \sim \exp(-i\omega t),$$

(2) and (3) take the form, (Akhiezer and Berestetskii 1965)

$$(\mathbf{S}\cdot\mathbf{p})_{jk} E_k = i\omega B_j, \quad (\mathbf{S}\cdot\mathbf{p})_{jk} B_k = -i\omega\varepsilon E_k, \tag{4}$$

where ω is the frequency, $p_k = -i\partial_k$ and $(S_i)_{jk} = -i\varepsilon_{ijk}$. This has the same form as the massless Dirac equations

$$\boldsymbol{\sigma}\cdot\mathbf{p}\varphi = \omega\chi, \quad \boldsymbol{\sigma}\cdot\mathbf{p}\chi = \omega\varphi, \tag{5}$$

where $\varphi \equiv \mathbf{E}$, $\chi = i\mathbf{B}$ and $\boldsymbol{\sigma} \equiv \mathbf{S}$ except that the analogue of dielectric constant in the latter is unity. This equation can be modified and cast into the form of (4) as

$$\boldsymbol{\sigma}\cdot\mathbf{p}\varphi = \omega\chi, \quad \boldsymbol{\sigma}\cdot\mathbf{p}\chi = \omega\varepsilon_c\varphi, \tag{6}$$

and be taken to represent massless quarks moving in a colour dielectric medium with colour dielectric constant $\varepsilon_c(r)$ which is assumed to be diagonal in colour space and same for all colours. It is worth mentioning that (6) can also be obtained from massless Dirac equation

$$[\gamma_\mu p_\mu + \gamma_\mu V_\mu(\omega, r) + S(\omega, r)]\psi = 0,$$

with energy-dependent scalar (S) and vector $V_\mu = (0,0,0,V_0)$ potentials with

$$S = V_0 = \frac{\omega}{2}\left(\frac{a}{r} - b^2 - 1\right).$$

However, the corresponding Hamiltonian is not hermitian. To take care of this a space-dependent metric has to be introduced in calculation of expectation values and fixation of normalisation constant. This will be dealt with at the end of §2.

With the choice $\varepsilon_c(r) = (a/r - b^2)$, the solution of (6) can be put in the standard form:

$$\begin{pmatrix} \varphi \\ \chi \end{pmatrix} = \begin{pmatrix} f(r)\,\Omega_{jlm}(\theta,\phi) \\ i\,g(r)\,\Omega_{jl'm}(\theta,\phi) \end{pmatrix}, \tag{7}$$

with $l = j \pm \frac{1}{2}$ and $l' = 2j - l$. The Ω_{jlm} are the spinor spherical harmonics which are related to the spherical harmonics $Y_{lm}(\theta, \varphi)$ by,

$$\Omega_j(= l \pm \tfrac{1}{2}), l, m = 1/(2l+1)^{\frac{1}{2}} \begin{pmatrix} \pm\sqrt{l \pm m + \frac{1}{2}} & Y_{l,m-\frac{1}{2}} \\ {}_s\sqrt{l \mp m + \frac{1}{2}} & Y_{l,m+\frac{1}{2}} \end{pmatrix}. \tag{8}$$

Using the relation $\Omega_{jl'm} = (i)^{l-l'}\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\Omega_{jlm}$, in (6) we get the radial equations

$$f' + \frac{1+\kappa}{r} f - \omega g = 0, \tag{9a}$$

$$g' + \frac{1-\kappa}{r} g + \omega\varepsilon_c(r) f = 0, \tag{9b}$$

where

$$\kappa = \begin{cases} (j+\frac{1}{2}) = l, & \text{for } j = l - \frac{1}{2} \\ -(j+\frac{1}{2}) = -(l+1), & \text{for } j = l + \frac{1}{2}, \end{cases}$$

is either a (+)ve or (−)ve integer and can never take the value zero. From (1) and (9) we get

$$f'' + \frac{2}{r} f' + \left\{\frac{a\omega^2}{r} - b^2\omega^2 - \frac{\kappa(\kappa+1)}{r^2}\right\} f = 0, \tag{10}$$

which on changing to the dimensionless variable $\rho = 2bwr$ and taking

$$f \sim \rho^\kappa \exp(-\rho/2)\xi(\rho), \tag{11}$$

yields

$$\rho\xi'' + (2\kappa + 2 - \rho)\xi' + \left(\frac{a\omega}{2b} - \kappa - 1\right)\xi = 0 \tag{12}$$

This is a confluent hypergeometric equation whose general solution is

$$\xi_\kappa(\rho) = AF\left(\kappa + 1 - \frac{a\omega}{2b}, 2\kappa + 2; \rho\right) + B(\rho)^{-1-2\kappa} F\left(-\kappa - \frac{a\omega}{2b}, -2\kappa; \rho\right), \tag{13}$$

where A and B are constants and $F(\ldots,\ldots;\ldots)$ is confluent hypergeometric function. There are two sets of solutions for $(+)$ve and $(-)$ve values of κ

(i) For $\kappa > 0$, $j = l - 1/2$ and $\kappa = (j + 1/2) = l = 1, 2, \ldots$ etc. Here $l = 0$ is excluded as $\kappa \neq 0$. In this case $(\rho)^{-1-2\kappa} \doteq (\rho)^{-1-2l}$ blows up at the origin for all values of l. Further, since $-2\kappa < 0$, the function

$$F\left(-\kappa - \frac{a\omega}{2b}, -2\kappa; \rho\right)$$

cannot be defined. So, we take $B = 0$ in which case

$$\xi_l(\rho) = AF\left(l + 1 - \frac{a\omega}{2b}, 2l + 2; \rho\right). \tag{14a}$$

(ii) For $\kappa < 0$, $j = l + 1/2$ and $\kappa = -(l+1) = -1, -2, \ldots$ etc. with $l = 0, 1, 2, \ldots$ etc. Here $(2\kappa + 2) = -2l \leqslant 0$ and hence the function $F(\kappa + 1 - (a\omega/2b), 2\kappa + 2; \rho)$ cannot be defined. So, we set $A = 0$ in this case and the solution reduces to

$$\xi_l(\rho) = B(\rho)^{2l+1} F\left(l + 1 - \frac{a\omega}{2b}, 2l + 2; \rho\right). \tag{14b}$$

Thus from (11) and (14), the two sets of solutions for $(+)$ve and $(-)$ve values of κ are,

$$f_l(r) = \begin{cases} A(2b\omega r)^l \exp(-b\omega r) F\left(l + 1 - \frac{a\omega}{2b}, 2l + 2; 2b\omega r\right) \\ \quad \text{for } j = l - \frac{1}{2}, \text{ with } l = 1, 2, 3, \ldots \text{ etc.,} \\ B(2b\omega r)^l \exp(-b\omega r) F\left(l + 1 - \frac{a\omega}{2b}, 2l + 2; 2b\omega r\right) \\ \quad \text{for } j = l + \frac{1}{2}, \text{ with } l = 0, 1, 2, \ldots \text{ etc.} \end{cases} \tag{15}$$

To ensure good asymptotic behaviour of $f_l(r)$ the confluent hypergeometric series must terminate and reduce to a polynomial. This imposes a condition,

$$l + 1 - (a\omega/2b) = -n_r,$$

with $n_r = 0, 1, 2, \ldots$ etc. which restricts the ω value to

$$\omega_n = \frac{2b}{a} n, \tag{16}$$

where $$n = l + 1 + n_r = \begin{cases} 1, 2, 3, \ldots \text{ etc., for } j = l + \frac{1}{2}, \\ 2, 3, 4, \ldots \text{ etc., for } j = l - \frac{1}{2}. \end{cases} \tag{17}$$

Putting the allowed values of ω from (16) in (15) we get,

$$f_{nl}(r) = \begin{cases} A(4b^2nr/a)^l \exp(-2b^2nr/a) F(l + 1 - n, 2l + 2; 4b^2nr/a), \\ \quad \text{for } j = l + \frac{1}{2}, \text{ with } l = 0, 1, 2, \ldots \text{ etc., and } n = 1, 2, 3, \ldots \text{ etc.} \\ B(4b^2nr/a)^l \exp(-2b^2nr/a) F(l + 1 - n, 2l + 2; 4b^2nr/a) \\ \quad \text{for } j = l - \frac{1}{2}, \text{ with } l = 1, 2, 3, \ldots \text{ etc., and } n = 2, 3, \ldots \text{ etc.} \end{cases} \tag{18}$$

In a similar manner we obtain the solution $g_{nl}(r)$ of (9a) in the form

$$g_{nl}(r)=\begin{cases} A(4b^2nr/a)^l\exp(-2b^2nr/a) \\ [\{b(l+1-n)/(l+1)\}F(l+2-n,\,2l+3;\,4b^2nr/a) \\ -bF(l+1-n,\,2l+2;\,4b^2nr/a)], \\ \quad\text{for } j=l+\tfrac{1}{2},\text{ with } l=0,1,2,\ldots\text{ etc., and } n=1,2,3,\ldots\text{ etc.,} \\ B(4b^2nr/a)^l\exp(-2b^2nr/a) \\ [\{b(l+1-n)/(l+1)\}F(l+2-n,\,2l+3;\,4b^2nr/a) \\ \quad+\{a(2l+2)/2bnr-b\}F(l+1-n,\,2l+2;\,4b^2nr/a)], \\ \text{for } j=l-\tfrac{1}{2},\text{ with } l=1,2,3,\ldots\text{ etc., and } n=2,3,\ldots\text{ etc.} \end{cases} \tag{19}$$

The quark in the $|njl\rangle$ state is thus described by,

$$\psi_{njl}=\begin{pmatrix}\phi_{njl}\\ \chi_{njl}\end{pmatrix}=N_{nl}\begin{pmatrix}f_{nl}(r)\Omega_{jlm}(\theta,\phi)\\ ig_{nl}(r)\Omega_{jl'm}(\theta,\phi)\end{pmatrix}. \tag{20}$$

To determine the normalisation constant N_{nl}, we write the equation of continuity using (6) from which the probability density turns out to be

$$\rho=\varphi^{\dagger}\varepsilon_c\varphi+\chi^{\dagger}\chi. \tag{21}$$

Using the property,

$$\int d\Omega\,\Omega^{*}_{jlm}(\theta,\phi)\Omega_{j'l'm'}(\theta,\varphi)=\delta_{jj'}\delta_{ll'}\delta_{mm'},$$

of the spinor spherical harmonics, the normalization condition $\int\rho\, d^3x=1$, reduces to

$$|N_{nl}|^2\int_0^{\infty}dr\,r^2\{|f_{nl}(r)|^2\varepsilon_c(r)+|g_{nl}(r)|^2\}=1,$$

which on simplification gives

$$N_{nl}=[(4b^2n/a)^3/4b^2n(n-l-1)!\{(2l+1)!\}^2]^{1/2}. \tag{22}$$

The probability density given by (21) can be put in the form

$$\rho=\psi^{\dagger}\tilde{\varepsilon}\psi,\ \text{where } \tilde{\varepsilon}\equiv\begin{pmatrix}\varepsilon_c(r) & 0\\ 0 & 1\end{pmatrix}.$$

Here $\tilde{\varepsilon}$ can be considered as a space-dependent metric for the massless coloured quarks in the colour-dielectric medium.

3. Energy spectrum of the quarks

From (16), the allowed values of ω for the quarks in the state $|njl\rangle$ are seen to depend only on n and are independent of j and l. So,

$$E_{njl}=E_n=\omega_n=\frac{2b}{a}n. \tag{23}$$

The energy levels of the quarks are thus equispaced like the spectrum of a simple harmonic oscillator. It will be noticed that the spectrum has no continuum. Therefore, it would not be possible to liberate a quark by supplying energy from outside.

Further, the radial parts of the quark wavefunction given by (18) and (19) look very much like that of hydrogen-like atoms with one important difference. From (23) it is seen that in the radial wavefunction the quantum number n appears in the numerator of the exponent rather than in the denominator as is in the hydrogen atom. As a consequence, the higher energy states lie closer to the centre than those with lower energy which is quite the opposite of hydrogenic states. In other words, increasing energy of a quark brings it closer to the centre rather than taking it away. So it is not possible to make a quark free by increasing its energy. The hadron which in the present model is a colour dielectric medium characterized by the colour dielectric constant given by (1), thus privides a trap for confining coloured quarks.

We have obtained bound states of massless quarks in a colour dielectric with $\varepsilon_c = a/r - b^2$, about a fixed centre and those of photons (Jena and Pradhan 1981) in an ordinary dielectric with the same variation of dielectric constant. It may be noted that a/r variation of the static dielectric constant in translationally invariant quantum electrodynamics can make the photon massive (Schwinger 1962) because this would follow (Källen 1972) from the vacuum polarization tensor

$$\pi_{\mu\nu}(k) = m^2 (k_\mu k_\nu - k^2 g_{\mu\nu}) \frac{1}{k^2},$$

which in turn would lead to the photon propagator

$$D_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{1}{k^2 + m^2}.$$

This seems to be the case in our theory since the bound photons in a sense can be considered massive.

4. Quark orbits

To get a simple picture of quark motion inside the hadron, we take recourse to a semiclassical approximation of (6) which is valid in the limit $\omega \to \infty$. In this approximation, quarks are found to move in elliptic and circular orbits. For this we first eliminate χ from (6) and obtain

$$\nabla^2 \varphi + \omega^2 \varepsilon_c(r) \varphi = 0. \tag{24}$$

Next, we set

$$\varphi \sim \varphi_0 \exp(iS\omega),$$

and finally obtain in the limit $\omega \to \infty$,

$$(\nabla S)^2 = \varepsilon_c(r) = a/r - b^2. \tag{25}$$

Here, S is the so-called eikonal and (∇S) gives the direction of propagation of the quarks. Since the colour dielectric constant is spherically symmetric, the path of the quarks will be confined to a single plane. We, therefore, use plane polar co-ordinates in which (25) takes the form,

$$\left(\frac{\mathrm{d}S_r}{\mathrm{d}r}\right)^2+\frac{1}{r^2}\left(\frac{\mathrm{d}S_\theta}{\mathrm{d}\theta}\right)^2=\frac{a}{r}-b^2, \tag{26}$$

where we have used

$$S(\bar{\mathbf{r}})\equiv S(r,\theta)=S_r(r)+S_\theta(\theta). \tag{27}$$

It is clear from (26) that

$$\frac{\mathrm{d}S_\theta}{\mathrm{d}\theta}=\text{constant}=\alpha_\theta \text{ (say)}, \tag{28a}$$

and hence

$$\frac{\mathrm{d}S_r}{\mathrm{d}r}=\left(\frac{a}{r}-b^2-\frac{\alpha_\theta^2}{r^2}\right)^{\frac{1}{2}}. \tag{28b}$$

For convenience we introduce two quantities J_r and J_θ:

$$J_\theta=\oint \mathrm{d}\theta\frac{\mathrm{d}S_\theta}{\mathrm{d}\theta}=2\pi\alpha_\theta, \tag{29}$$

and

$$J_r=\oint \mathrm{d}r\frac{\mathrm{d}S_r}{\mathrm{d}r}=\oint \mathrm{d}r\left(\frac{a}{r}-b^2-\frac{\alpha_\theta^2}{r^2}\right)^{\frac{1}{2}}. \tag{30}$$

Integrating (30) by standard methods and using (29) we get

$$J_r=-J_\theta+\pi a/b,$$

so that

$$J_r+J_\theta=\pi a/b. \tag{31}$$

This looks very much like a relation that one comes across in the solution of Hamilton-Jacobi equation for motion of a massive charged particle in a Coulomb potential. It is therefore natural to impose the "Bohr-Sommerfeld conditions",

(a) $$\omega J_r=2\pi n_r, \tag{32}$$

with $n_r=0, 1, 2, \ldots$ etc. and

(b) $$\omega J_\theta=2\pi(l+1), \tag{33}$$

with $l=0, 1, 2, 3, \ldots$ etc.

Combining these two conditions, we get

$$\omega(J_r+J_\theta)=2\pi(n_r+l+1)=2\pi n, \tag{34}$$

where, $n=(n_r+l+1)=1, 2, 3, \ldots$ etc. which along with (31) gives

$$\omega_n=2bn/a. \tag{35}$$

This is precisely the same as (16) obtained from solution of Dirac equation (6).

Now, using (27), (28) and (33) we get

$$\theta=\theta_0+\frac{(l+1)}{\omega}\int\frac{\mathrm{d}r}{r^2}\left\{\frac{a}{r}-b^2-(l+1)^2/\omega^2r^2\right\}^{-\frac{1}{2}}, \tag{36}$$

where θ_0 is constant of integration. This leads to the orbit equation

$$\frac{1}{r}=\frac{a\omega^2}{2(l+1)^2}\left[1+\left\{1-\frac{4b^2(l+1)^2}{a^2\omega^2}\right\}\cos(\theta-\theta_0)\right] \tag{37}$$

which represents a conic section with one of the focii at the origin and eccentricity

$$e=\{1-4b^2(l+1)^2/a^2\omega^2\}^{\frac{1}{2}}. \tag{38}$$

Substituting the allowed values of ω from (35), we get the orbit equation as

$$\frac{1}{r}=\left\{2b^2n^2/a(l+1)^2\right\}\left[1+\left\{1-\frac{(l+1)^2}{n^2}\right\}^{\frac{1}{2}}\cos(\theta-\theta_0)\right], \tag{39}$$

and

$$e=\{1-(l+1)^2/n^2\}^{1/2}. \tag{40}$$

This shows that in general the path of the massless quarks form elliptic orbits with the semimajor and semiminor axes given by

$$r_{\text{semimajor}}=a/2b^2, \tag{41}$$

and

$$r_{\text{semiminor}}=a(l+1)/2b^2n. \tag{42}$$

They are circular with radius $(a/2b^2)$ when $n=l+1$, *i.e.* when $n_r=0$. Thus we see that no orbit can have size greater than $(a/2b^2)$. Further, (42) shows that for a given value of l, the semiminor axis decreases with increase of n (*i.e.* with increase of energy). So, higher energy orbits are smaller in size. It is thus impossible to liberate quarks by supplying energy from outside.

5. Structure of proton

We can now write the wavefunction of the proton in terms of those of its constituent quarks. Since the quarks are taken as independent particles in this model, the proton wavefunction would be the properly symmetrized products of the quark wavefunctions.

The normalized proton wavefunction can be written as

$$\begin{aligned}|p\rangle=(2b^2/1+b^2)^{3/2}(18)^{-1/2}\,&(2\overset{\uparrow}{u}\overset{\uparrow}{u}\overset{\downarrow}{d}+2\overset{\downarrow}{d}\overset{\uparrow}{u}\overset{\uparrow}{u}+2\overset{\uparrow}{u}\overset{\downarrow}{d}\overset{\uparrow}{u}\\&-\overset{\uparrow}{u}\overset{\downarrow}{u}\overset{\uparrow}{d}-\overset{\downarrow}{u}\overset{\uparrow}{u}\overset{\uparrow}{d}-\overset{\uparrow}{d}\overset{\uparrow}{u}\overset{\downarrow}{u}-\overset{\uparrow}{d}\overset{\downarrow}{u}\overset{\uparrow}{u}-\overset{\uparrow}{u}\overset{\uparrow}{d}\overset{\downarrow}{u}-\overset{\downarrow}{u}\overset{\uparrow}{d}\overset{\uparrow}{u})\end{aligned} \tag{43}$$

Here each of the u and d quarks is in the ground state and the $\uparrow$, $\downarrow$ arrows represent spin up and spin down configuration respectively. The u and d appearing in the above wavefunction can be obtained from (20):

$$\overset{\uparrow}{u}=\overset{\uparrow}{d}=\psi^{\uparrow}_{1\frac{1}{2}0}=N_{10}\begin{pmatrix}f_{10}(r)\Omega_{\frac{1}{2}0\frac{1}{2}}\\ ig_{10}(r)\Omega_{\frac{1}{2}1\frac{1}{2}}\end{pmatrix},$$

and

$$\overset{\downarrow}{u}=\overset{\downarrow}{d}=\psi^{\downarrow}_{1\frac{1}{2}0}=N_{10}\begin{pmatrix}f_{10}(r)\Omega_{\frac{1}{2}0-\frac{1}{2}}\\ ig_{10}(r)\Omega_{\frac{1}{2}1-\frac{1}{2}}\end{pmatrix}.$$

We can now calculate various static properties of the proton in terms of the two parameters a and b appearing in the expression for colour-dielectric constant.

5.1 *Proton mass*

Since the quarks are massless in this model, the mass of the proton is equal to the sum of the energies of the quarks. From (23), the ground state energy of a single quark is

$$E_1 = 2b/a,$$

and hence the proton mass is given by

$$m_p = 3E_1 = 6b/a \tag{44}$$

5.2 *Charge radius*

The contribution of a quark to the charge radius of proton is given by

$$\langle r^2 \rangle_q = e_q \int d^3r (\phi^\dagger \varepsilon_c \phi + \chi^\dagger \chi) r^2, \tag{45}$$

where e_q is the charge of the quark q. By use of the wavefunction given by (43), the proton charge radius comes out to be

$$\langle r^2 \rangle_p^{\frac{1}{2}} = [3a^2/4b^2(1+b^2)]^{\frac{1}{2}}. \tag{46}$$

5.3 g_A/g_V *for* $n \to pe^- \bar{\nu}_e$

For this process

$$g_A/g_V = \langle p | \sum_{i=1}^{3} (\Sigma_3^i \tau_3^i) | p \rangle, \tag{47}$$

where $i = 1, 2, 3$, is the quark index, τ_3^i is the third component of isospin in the (u, d) space acting on the ith quark and

$$\Sigma_3^i = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix},$$

is the third component of the Dirac spin matrix operating on the ith quark. Direct calculation now yields

$$\langle \psi^{\uparrow}_{1\frac{1}{2}0} | \Sigma_3 | \psi^{\uparrow}_{1\frac{1}{2}0} \rangle = (3 - b^2)/3(1+b^2),$$

and

$$\langle \psi^{\downarrow}_{1\frac{1}{2}0} | \Sigma_3 | \psi^{\downarrow}_{1\frac{1}{2}0} \rangle = (b^2 - 3)/3(1+b^2).$$

Finally using (43) we get

$$g_A/g_V = (5/3) \cdot (3 - b^2)/3(1+b^2). \tag{48}$$

If we choose the parameters a and b as $a = 0{\cdot}63$ fm and $b = 0{\cdot}50$, the various static properties of the proton calculated above take on the values given below

	Calculated	*Expt*
Proton mass (m_p)	939·3 MeV	939 MeV
g_A/g_V	1·22	1·25
Proton charge radius	0·95 fm	0·89 fm

Although we have considered only the proton here, these ideas can hopefully be extended to other light hadrons with suitable choice and addition of extra parameters.

Acknowledgements

The authors would like to thank Dr P K Patnaik of Sambalpur University for stimulating discussions and correspondence at the early state of this work. They also thank Professors S P Misra, A Khare and J Maharana for fruitful discussions.

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