

The fifth interaction: universal long range force between spins

T PRADHAN, R P MALIK and P C NAIK*

Institute of Physics, Sachivalaya Marg, Bhubaneswar 751 005, India

* Department of Physics, S.C.S. College, Puri 752 001, India

Abstract. In this paper we present a review of our investigations on universal long range force between spins mediated by a massless axial vector gauge field which we name as "axial photon". The invariance of the Lagrangian field theory of particles, possessing spin degrees of freedom, under local Lorentz transformations, necessitates the introduction of such an axial vector gauge field which interacts with spin current of the particles. Classical as well as quantum dynamics of electrons interacting with photon and axial photon are worked out. The new interaction is found to be asymptotically free. It is shown that QED can be made finite if the coupling strengths of electron to photon and axial photon can be made equal. Experimental consequences of the existence of axial photon are discussed and the strength of the interaction is estimated by comparing predictions of the theory with experiments.

Keywords. Fifth interaction; axial photon; local Lorentz group; divergences; stability of the electron; asymptotic freedom; hyperfine anomaly; phase of the wavefunction.

PACS No. 12-90

1. Introduction

There are at present four kinds of known fundamental interactions in nature namely: strong, electromagnetic, weak and gravitational. It is the present-day conviction that all these interactions stem from certain symmetries and are mediated by gauge fields that become necessary to restore symmetry when the parameters of global transformation are made space-time dependent. For instance the electromagnetic field is required to restore phase invariance of the Lagrangian when the parameter of the phase transformation is made space-time dependent. The coloured gluons (Fritzsch *et al* 1973), the W and Z bosons (Glashow 1980; Weinberg 1980; Salam 1980) that mediate strong and weak interactions are gauge particles associated with space-time dependent $SU(3)_c$ and $SU(2) \otimes U(1)$ transformations respectively. The gravitational interaction is thought to be mediated by spin-2 massless gauge particle required in space-time dependent translation group, (Cho 1976; Hayward 1979). It is natural to speculate about gauge particle associated with the Lorentz group. Although the latter has been considered by various authors (Utiyama 1956; Kibble 1961; Sciama 1962) who obtain the gravitational interaction by gauging the Lorentz group it is somewhat artificial. In all gauge theories the gauge particle usually couples to the conserved current of the global group which is gauged. In Einstein's theory the gravitational field couples to energy-momentum tensor (Carmeli 1982) which is the conserved current for the space-time translation group. It is therefore natural to associate gravitational interaction with the local translation group. In the Lorentz group the conserved current is the angular momentum tensor density and one would therefore expect that the gauge particle in the local Lorentz group should couple to this density. In other words, a local Lorentz group

should lead to interaction between spins mediated by an axial vector gauge boson. We have shown that this is indeed the case (Naik and Pradhan 1981). Our approach to gauging the Lorentz group is different from those of Kibble, Sciama and Utiyama. We name the gauge particle as "axial photon". When the gauge theory is worked out in detail one finds that axial photon couples to the axial vector currents of the Dirac particle as well as that of the photon and the space integrals of the space components of these currents are the spin vectors of the respective particles. We thus have a fifth interaction which operates universally between spins of particles. Further we have a mechanism of dynamically measuring spin through axial photon probe.

That such an interaction should exist, can be inferred from physical considerations without going to gauge theory. This had been done when axial photon was first introduced (Pradhan and Lahiri 1974). Their motivation for introducing the axial photon was to give stability to the classical electron and also to make quantum electrodynamics divergence free. It was argued that if an axial vector boson is coupled to the axial vector current of the electron it would result in spin-spin interaction which is attractive for parallel spins. The parts of classical spinning electron would therefore attract each other and compensate the force of Coulomb repulsion. It turns out, as we shall see in § 3 that this is indeed the case but only when the spin-spin coupling strength is equal to the strength of the Coulomb interaction. It also turns out that in quantum theory the divergence associated with electron-self-energy due to exchange of virtual photon cancels with that arising from exchange of virtual axial photon under exactly the same condition (Pradhan and Lahiri 1974). However, the axial photon coupling to electron and photons with such large strength would destroy the excellent agreement of present quantum electrodynamics with experiment unless of course the axial photon acquires a large mass through spontaneous breaking of the gauge symmetry. This possibility was considered by Pradhan and Lahiri. The large mass suppresses physical processes involving axial photon exchanges, but do not affect divergent diagrams.

As mentioned earlier, the force arising out of exchange of axial photon between particles with spin is attractive when the spins are parallel and repulsive when they are antiparallel. Strikingly enough, this is exactly the situation with the experiments carried out with polarized laser beams by Tam and Happer (1977) who found that two right circularly polarized laser beams passing through dense sodium vapour attract each other while those with opposite circular polarization repel. They also found that the force between the beams was of long range. The sodium vapour acts as a spin polarizable medium which on account of asymptotic freedom of axial photon coupling enhances the spin-spin force between laser beams. By analysing the experimental results of Tam and Happer and considering the enhancement by spin-polarizability of the medium, Naik and Pradhan found that good agreement between theory and the experimental results, over the range of densities of sodium vapour used in these experiments, can be achieved if the coupling strength is taken to be $\alpha_g = g^2/4\pi = 0.7 \times 10^{-9}$.

An independent estimate of the coupling strength can be made from the contribution of the long range force between nuclear and electronic spin to hyperfine splitting in the hydrogen and the deuterium atoms for which only a small discrepancy exists between theory and experiment. This discrepancy can be accounted for by the long range spin-spin interaction if the axial photon coupling strength is taken to be $\alpha_g = g^2/4\pi \simeq 10^{-13}$ which is several orders of magnitude smaller than that estimated from the force between circularly polarized laser beams in sodium vapour. We take the latter value to

be more reliable since precision of measurement of hyperfine splitting is much higher than that of the laser beam experiment.

In view of these two experimental evidences for the existence of the fifth interaction we think it worthwhile to review the classical and quantum aspects of this interaction. In this paper we begin with a presentation of the gauging of the Lorentz group in § 2. The classical dynamics of the theory is worked out in § 3. Equations of motion for the axial photon field and its spin source are worked out and their static solution for point sources obtained. Stability of the classical electron and radiation of axial photon by spin sources are discussed in the classical framework. Section 4 is devoted to the quantum dynamics of the theory where questions such as the removal of divergences, asymptotic freedom of the interactions, its contribution to hyperfine splitting and two-body bound states resulting from long range spin-spin forces are discussed. In this section we also discuss a Bohm-Aharonov type of experiment which can establish the existence of the fifth interaction as well as measure its strength.

2. Local Lorentz group

Invariance of Lagrangian density under global Lorentz group of transformations:

$$\left. \begin{aligned} x_\mu \rightarrow x'_\mu &= x_\mu + \alpha_{\mu\nu} x_\nu \\ \alpha_{\mu\nu} &= -\alpha_{\nu\mu} \end{aligned} \right\} \quad (1)$$

leads to conservation of angular momentum $J_{\mu\nu}$:

$$\left. \begin{aligned} \frac{\partial J_{\mu\nu}}{\partial t} &= 0; \quad J_{\mu\nu} = \int d\sigma_\lambda M_{\mu,\nu\lambda} \\ \partial_\lambda M_{\mu,\nu\lambda} &= 0 \end{aligned} \right\} \quad (2)$$

where $M_{\mu,\nu\lambda}$ is the angular momentum density. As mentioned in § 1 one expects that in the local Lorentz invariant theory the gauge-field would couple to the conserved current $M_{\mu,\nu\lambda}$. This implies that the gauge field would be a third rank tensor $B_{\mu,\nu\lambda}$. That this is actually the case can be seen as follows:

$$\mathcal{L}_D^{(0)} = \frac{1}{2} [\bar{\psi} \gamma_\mu \partial_\mu \psi - (\partial_\mu \bar{\psi}) \gamma_\mu \psi] - m \bar{\psi} \psi. \quad (3)$$

The above Dirac Lagrangian density which is invariant under global transformation (1) under which,

$$\left. \begin{aligned} \psi(x) \rightarrow \psi'(x) &= \exp \left[\frac{i}{2} \sum_{\nu\lambda} \alpha_{\nu\lambda} \right] \psi(x) \equiv \Omega \psi(x) \\ \sum_{\nu\lambda} &= \frac{1}{4} (\gamma_\nu \gamma_\lambda - \gamma_\lambda \gamma_\nu) \end{aligned} \right\} \quad (4)$$

ceases to be so when the parameters $\alpha_{\mu\nu}$ are made space-time dependent. However, the Lagrangian

$$\mathcal{L}_D = \frac{1}{2} [\bar{\psi} \gamma_\mu D_\mu \psi - (D_\mu \bar{\psi}) \gamma_\mu \psi] - m \bar{\psi} \psi, \quad (5)$$

where $D_\mu \psi = \partial_\mu \psi + ig B_\mu \psi,$ (6)

and $B_\mu = B_{\mu,\nu\lambda} \sum_{\nu\lambda},$

remains invariant when the massless gauge fields B_μ transform as:

$$B_\mu(x) \rightarrow B'_\mu(x') = \Omega(x) B_\mu(x) \Omega^{-1}(x) - \frac{1}{ig} (\partial_\mu \Omega(x)) \Omega^{-1}(x). \quad (7)$$

At this stage it is necessary to point out that in the above, we have considered the invariance of the Lagrangian density which for global transformation ensures the invariance of the action $S = \int d^4x \mathcal{L}(x)$. For local transformation this is not the case; one needs extra conditions because,

$$S = \int d^4x \mathcal{L}(x) \rightarrow S' = \int d^4x' \mathcal{L}'(x') = \int d^4x J \mathcal{L}'(x'), \quad (8)$$

where J is the Jacobian for the transformation of the four-volume element. Since $J = 1 + \partial_\mu(\delta x_\mu)$ one requires both

$$\left. \begin{aligned} \mathcal{L}'(x') &= \mathcal{L}(x) \\ \partial_\mu(\delta x_\mu) &= 0. \end{aligned} \right\} \quad (9)$$

The latter is automatically satisfied for global transformation. For our local Lorentz group this is equivalent to,

$$\partial_\mu \alpha_{\mu\lambda}^{(x)} = 0. \quad (10)$$

In the Kibble-Sciama (Kibble 1961; Sciama 1962) approach to local Lorentz group this restriction is not put, they use vierbeines to define covariant derivatives and their theory leads to Einstein-Cartan theory of gravitation.

The tensor gauge field $B_{\mu,\nu\lambda}$ is antisymmetric in the last two indices. We can decompose it into symmetric and anti-symmetric parts in the first and last two indices as:

$$B_{\mu,\nu\lambda} = \frac{1}{2} \varepsilon_{\mu\nu\lambda\zeta} a_\zeta + S_{\mu,\nu\lambda}, \quad (11)$$

where a_μ is an axial vector field and,

$$S_{\mu,\nu\lambda} = \frac{1}{2} (B_{\mu,\nu\lambda} + B_{\nu,\mu\lambda} + B_{\lambda,\nu\mu})$$

is a symmetric tensor field which on account of (10) transform as,

$$a_\mu(x) \rightarrow a'_\mu(x') = a_\mu(x) + 2/3 \alpha_{\mu\nu}(x) a_\nu(x) - \frac{1}{g} \partial_\mu \Lambda$$

where

$$\partial_\mu \alpha_{\lambda\rho} = \varepsilon_{\mu\lambda\rho\alpha} \partial_\alpha \Lambda$$

and

$$S_{\mu,\nu\lambda}^{(x)} \rightarrow S'_{\mu,\nu\lambda}(x') = \Omega(x) S_{\mu,\nu\lambda}^{(x)} \Omega^{-1}(x).$$

We shall name the axial vector field as "axial photon" field. Substituting (6) and (11) into (5) we obtain

$$\mathcal{L}_D = \mathcal{L}_D^{(0)} + \mathcal{L}_D^{(1)}$$

where

$$\mathcal{L}_D^{(1)} = g M_{\mu,\nu\lambda} B_{\mu,\nu\lambda}$$

Or

$$\mathcal{L}_D^{(1)} = \frac{ig}{4} \varepsilon_{\mu\nu\lambda\zeta} \bar{\psi} (\gamma_\mu \sum_{\nu\lambda} + \sum_{\nu\lambda} \gamma_\mu) \psi a_\zeta$$

$$= 3/2 g \bar{\psi} \gamma_5 \gamma_\mu \psi a_\mu$$

(13)

which is equivalent to taking

$$D_\mu \psi = \partial_\mu \psi - 3/2 g \gamma_5 \psi a_\mu. \quad (14)$$

It will be noticed that the tensor field $S_{\mu,\nu\lambda}$ does not couple to the Dirac field; only the axial vector field a_μ couples to the axial vector current $\bar{\psi} \gamma_5 \gamma_\mu \psi$ of the Dirac field. The space integral of the space components of this current is the spin of the Dirac particle and that of the time component its helicity. It thus follows that the axial vector field couples to the spin and helicity of the Dirac particle just as the Maxwell field couples to its electric charge and current. Thus axial photon provides a means of dynamical measurement of spin in the same sense as photon measures the charge of the electron.

The kinetic energy term of the gauge field must now be included in the Lagrangian. It is given by

$$\mathcal{L}_G = -\frac{1}{4} B_{\mu\nu} B_{\mu\nu} \quad (15)$$

where

$$\left. \begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu + ig [B_\mu, B_\nu] \\ B_\mu &= B_{\mu,\nu\lambda} \sum_{\nu\lambda} \end{aligned} \right\} \quad (16)$$

The axial photon part of \mathcal{L}_G is:

$$\mathcal{L}_G^{\text{axial}} = -\frac{1}{2} (\partial_\mu a_\nu) (\partial_\mu a_\nu) - \frac{3g^2}{16} (a_\mu a_\mu)^2. \quad (17)$$

Since the photon has spin it will also couple to axial photon. To obtain the interaction Lagrangian we start with

$$\mathcal{L}_M = -\frac{1}{4} (D_\mu A_\nu - D_\nu A_\mu) (D_\mu A_\nu - D_\nu A_\mu), \quad (18)$$

$$\text{where } D_\mu A_\nu = \partial_\mu A_\nu + ig B_{\mu;\lambda\eta} \sum_{\nu\zeta, \lambda\eta} A_\zeta, \quad (19)$$

$$\text{with } \sum_{\nu\zeta, \lambda\eta} = -i(\delta_{\nu\lambda} \delta_{\zeta\eta} - \delta_{\nu\eta} \delta_{\zeta\lambda}). \quad (20)$$

Substituting (11) and (20) in (19) and evaluating (18) gives the following interaction Lagrangian between photon and axial photon

$$\mathcal{L}_{MG}^{(1)} = -g \epsilon_{\mu\nu\lambda\zeta} (\partial_\mu A_\nu) A_\lambda a_\zeta - \frac{1}{2} g^2 [a^2 A^2 - (a \cdot A)^2]. \quad (21)$$

It will be noticed that this Lagrangian is not invariant under the electromagnetic gauge transformation

$$A_\mu \rightarrow A'_\mu = A_\mu + \frac{1}{e} \partial_\mu \alpha. \quad (22)$$

This can be rectified by replacing A_μ occurring in the Lagrangian by $(A_\mu + \lambda \partial_\mu \Phi)$ where $\Phi(x)$ is a massless scalar field which transforms as

$$\phi(x) \rightarrow \phi'(x) = \phi(x) - \frac{1}{\lambda e} \alpha(x). \quad (23)$$

In order to ensure the invariance of the kinetic energy term $-\frac{1}{2} (\partial_\mu \Phi) (\partial_\mu \Phi)$ of the scalar field we add a counter term $-\frac{1}{\lambda} (\partial_\mu \Phi) A_\mu$ to the Lagrangian. In order to decouple the scalar field from the rest, one goes to the limit $\lambda \rightarrow 0$. This has been discussed in detail (Naik and Pradhan 1981) and is a slightly modified version of Stueckelberg's (1938) compensating field method.

3. Classical dynamics of axial vector gauge field

3.1 Equations of motion for the field and its source

The Lagrangian density for the field interacting with its spin source as worked out in the preceding section can be written as

$$\mathcal{L} = \mathcal{L}_G^{(0)} + \mathcal{L}_D^{(0)} + \mathcal{L}_M^{(0)} + \mathcal{L}^{(1)} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)}, \quad (24a)$$

where

$$\mathcal{L}^{(0)} = -\frac{1}{2}(\partial_\mu a_\nu)^2 - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}[\bar{\psi}\gamma_\mu\partial_\mu\psi - (\partial_\mu\bar{\psi})\gamma_\mu\psi] - m\bar{\psi}\psi, \quad (24b)$$

$$\mathcal{L}^{(1)} = -\frac{3g^2}{16}(a_\mu a_\mu)^2 - \frac{1}{2}g^2[a^2 A^2 - (a \cdot A)^2] + s_\mu(x)a_\mu(x), \quad (24c)$$

where

$$s_\mu(x) = 3/2g\bar{\psi}\gamma_5\gamma_\mu\psi - g\varepsilon_{\mu\nu\lambda\zeta}(\partial_\nu A_\lambda)A_\zeta, \quad (24d)$$

is the spin current density of the electron and the photon. If we add a four divergence $\frac{1}{2}\partial_\mu(a_\nu\partial_\nu a_\mu)$ to the $\mathcal{L}^{(0)}$ in (24b), the kinetic energy of the axial photon can be written as

$$\mathcal{L}^{(\text{axial})} = -\frac{1}{4}f_{\mu\nu}f_{\mu\nu}, \quad (25a)$$

where

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu, \quad (25b)$$

and the equations of motion that follow from the Lagrangian are,

$$\left. \begin{aligned} \partial_\nu f_{\mu\nu} &= s_\mu(x) \\ \varepsilon_{\mu\nu\lambda\zeta}\partial_\nu f_{\lambda\zeta} &= 0 \end{aligned} \right\} \quad (26)$$

if we neglect nonlinear terms of order g^2 in (24c). In terms of $\mathcal{E}_i = f_{0i}$ and $\mathcal{B}_i = \frac{1}{2}\varepsilon_{ijk}f_{jk}$ these equations read

$$\left. \begin{aligned} \nabla \times \mathcal{E} - \frac{1}{c} \frac{\partial \mathcal{B}}{\partial t} &= 0, \quad \nabla \cdot \mathcal{B} = 0 \\ \nabla \times \mathcal{B} - \frac{1}{c} \frac{\partial \mathcal{E}}{\partial t} &= \mathbf{s}(\mathbf{x}), \quad \nabla \cdot \mathcal{E} = h(\mathbf{x}) \end{aligned} \right\} \quad (27)$$

where $h(\mathbf{x}) = s_0(\mathbf{x})$ is the helicity density and together with spin density $\mathbf{s}(\mathbf{x})$ obeys the continuity equation:

$$\partial_\mu s_\mu(\mathbf{x}) = 0. \quad (28)$$

The equation for the axial vector potential a_μ that follows from (24b) when terms of order g^2 are neglected, are:

$$\square a_\mu = s_\mu(\mathbf{x}). \quad (29)$$

This equation can also be regarded as Hamilton's equations:

$$\frac{\partial \pi_i}{\partial t} = \partial_j \frac{\delta H}{\delta(\partial_j a_i)} - \frac{\delta H}{\delta a_i}, \quad (30)$$

where

$$\pi_i = -\frac{\delta H}{\delta\left(\frac{\partial a_i}{\partial t}\right)} \quad (31)$$

is the field canonically conjugate to a_i and

$$H = \int d^3 x \left\{ \frac{1}{2} (\nabla \times \mathbf{a})^2 + \frac{1}{2} \left(\frac{\partial \mathbf{a}}{\partial t} \right)^2 - \mathbf{s} \cdot \mathbf{a} \right\} + H_0, \quad (32)$$

H_0 being the energy of the bare source. It will be seen that the axial vector field couples to the spin of the source.

The equation of motion for the spin of the source can be obtained by using the Poisson Bracket equation:

$$\partial S_i / \partial t = i [H, S_i]. \quad (33)$$

Using the definition,

$$S_i = \int d^3 x s_i(x), \quad (34)$$

and the ansatz

$$[s_i(x), s_j(x')]_{t'=t} = i \varepsilon_{ijk} \delta(\mathbf{x} - \mathbf{x}') s_k(\mathbf{x}),$$

yields

$$\partial S_i / \partial t = -\varepsilon_{ijk} \int d^3 x s_j(\mathbf{x}t) a_k(\mathbf{x}t). \quad (35)$$

This equation shows that the axial photon field exerts a torque on the spin the direction of which is perpendicular to the field potential \mathbf{a} as well as to the spin of the source so that $S^2 = \text{constant}$. If \mathbf{a} were space-time independent, the solution of (35) would give a gyration of the spin like that of a spinning top in a gravitational field. This situation is to be contrasted with the interaction of the photon field on a charged particle which gets accelerated by it.

3.2 Static field of a spin-source

Solution of (29) can be written as

$$a_i(\mathbf{x}) = \frac{1}{4\pi} \int d^3 x' \frac{s_i(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \quad (36)$$

For the spin distribution we can take

$$s_i(\mathbf{x}) = g\rho(\mathbf{x})S_i, \quad (37)$$

where $\rho(\mathbf{x})$ is the spin distribution function. Substitution of (37) in (36) leads to

$$a_i(\mathbf{x}) = \frac{S_i}{4\pi} \int d^3 x' \frac{g\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \quad (38)$$

For a point spin source $\rho(\mathbf{x}') = 3/2\delta(\mathbf{x}')$

$$a_i(\mathbf{x}) = \frac{3gS_i}{8\pi|\mathbf{x}|}, \quad (39)$$

and

$$\mathcal{E} = -\nabla \times \mathbf{a} = \frac{3g}{8\pi} \left(\frac{\mathbf{r} \times \mathbf{s}}{r^3} \right) \quad (40)$$

$$\mathcal{H} = -\partial \mathbf{a} / \partial t = 0. \quad (41)$$

The situation here is to be compared with static photon field of a steady electric current which produces a steady magnetic field but no electric field. It will be seen from (40) that for a right circularly polarized light beam travelling in a straight line for which the spin is in the direction of motion, the \mathcal{E} lines of force will be circles around the beam axis just like magnetic lines of force (\mathbf{B}) around a straight wire carrying current. On the other hand for circularly polarized light beam travelling in a circular path, the \mathcal{E} lines of force would be on the surface of a torous around the path like those of \mathbf{B} lines around a circular current. It then follows that the \mathcal{E} lines for an infinitely long helical light path as in a selfoc fibre would be exactly like the \mathbf{B} lines produced by a solenoid, i.e. the \mathcal{E} lines would be parallel to the fibre and confined to the inside of the fibre.

3.3 Stability of classical electron

It is well known that classical finite size electron is not stable; its parts would fly apart due to repulsive Coulomb interaction which cannot be prevented unless forces of non-electromagnetic origin are invoked. It will readily be seen that such forces would arise from the new interaction. The static fields produced by electrons' spin and electric charge would be

$$\mathbf{E} = \frac{e\mathbf{r}}{4\pi r^3}, \quad \mathbf{B} = 0; \quad \mathcal{E} = \frac{3g}{8\pi} \frac{\mathbf{r} \times \mathbf{s}}{r^3}; \quad \mathcal{D} = 0. \quad (42)$$

The \mathbf{E} and \mathcal{E} lines of force are shown in figures 1a, b. These fields would give rise to a self-force:

$$\mathbf{F} = e\mathbf{E} - \frac{3g}{2} \mathbf{s} \times \mathcal{E} = \frac{e^2 \mathbf{r}}{4\pi r^3} - \frac{9g^2 S^2 \mathbf{r}}{16\pi r^3}, \quad (43)$$

which can be made to vanish by suitably choosing 'g'

$$(3/2g)^2 = e^2/S^2 \quad \text{or} \quad \frac{9}{4}g^2 = \frac{e^2}{S^2}. \quad (44)$$

This would ensure stability of the electron. For an extended electron calculations will be somewhat more complex. We thus see that the fifth interaction can provide a "Poincare stress" for ensuring stability of the classical electron.

3.4 Static interaction between two spins

The potential energy between two particles possessing spins, one located at the origin and another at \mathbf{r} is given by

$$v_{AB}(\mathbf{r}) = - \int d^3r' s_i^{(B)}(\mathbf{r} - \mathbf{r}') a_i^{(A)}(\mathbf{r}'). \quad (45)$$

Using expression (39) for $a_i(\mathbf{r})$ in (45) we obtain after some manipulations,

$$v_{AB}(\mathbf{r}) = - \frac{(3g/2)^2}{2\pi |\mathbf{r}|} [\mathbf{S}_A \cdot \mathbf{S}_B], \quad (46)$$

which shows that the force between two spins is attractive when the two are parallel while it is repulsive when they are anti-parallel. This can be considered as the Coulomb law for the fifth interaction. It is important to note that identical (parallel) spins attract

With $k = \omega/c$ and for the limit $kr \gg 1$ this reduces to

$$\lim_{kr \rightarrow \infty} \mathbf{a}(\mathbf{r}, 0) = \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{|\mathbf{r}|} \int d^3r' \mathbf{s}(\mathbf{r}') \exp(-ik\hat{\mathbf{n}} \cdot \mathbf{r}'), \quad (50)$$

where $\hat{\mathbf{n}}$ is a unit vector along \mathbf{r} . For the case where the dimensions of the source are small compared to wavelength we can expand the exponential in (50) in powers of k and the first term in the expansion namely

$$\mathbf{a}(\mathbf{r}) = \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{|\mathbf{r}|} \int d^3r' \mathbf{s}(\mathbf{r}') = \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{|\mathbf{r}|} \mathbf{S} \quad (51)$$

gives the dominant contribution to the radiation of axial photon. Since $e\mathbf{S}/mc = \boldsymbol{\mu}$ is the Dirac magnetic dipole moment, this radiation can be termed as the magnetic dipole radiation of axial photon. The fields \mathcal{E} and \mathcal{B} outside the source derived from (51) work out to be

$$\mathcal{E} = -\nabla \times \mathbf{a} = ik \left(\frac{3mc^2g}{2e} \right) (\mathbf{n} \times \boldsymbol{\mu}) \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{|\mathbf{r}|} \left(1 - \frac{1}{ikr} \right), \quad (52a)$$

and
$$\mathcal{B} = -\frac{i}{k} \nabla \times \boldsymbol{\varepsilon}$$

$$\mathcal{B} = -\left(\frac{3gmc^2}{2e} \right) \left\{ (\mathbf{n} \times \boldsymbol{\mu}) \times \mathbf{n} k^2 + [3\mathbf{n}(\mathbf{n} \cdot \boldsymbol{\mu})k^2 - \boldsymbol{\mu}] \right. \\ \left. \times \left(\frac{1}{r^2} - \frac{ik}{r} \right) \right\} \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{ikr}. \quad (52b)$$

In the radiation zone, they take the limiting values:

$$\mathcal{E} = ik \left(\frac{3mc^2g}{2e} \right) \left\{ (\mathbf{n} \times \boldsymbol{\mu}) \times \mathbf{n} \right\} \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{|\mathbf{r}|} \quad (53a)$$

$$\mathcal{B} = -ik \left(\frac{3mc^2g}{2e} \right) \left\{ (\mathbf{n} \times \boldsymbol{\mu}) \times \mathbf{n} \right\} \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{|\mathbf{r}|}. \quad (53b)$$

Thus the time-averaged power radiated works out to be

$$\frac{d\mathcal{P}}{d\Omega} = \frac{c}{8\pi} R_e [r^2 \hat{\mathbf{n}} \cdot (\boldsymbol{\varepsilon} \times \mathcal{B}^*)] = \frac{ck^2}{8\pi} \left(\frac{3mc^2g}{2e} \right) |\boldsymbol{\mu}|^2 \sin^2 \theta, \quad (54)$$

where θ is the angle between the direction of radiation with respect to $\boldsymbol{\mu}$. The total power radiated is:

$$\mathcal{P} = \frac{ck^2}{3} \left(\frac{3mc^2g}{2e} \right)^2 |\boldsymbol{\mu}|^2. \quad (55)$$

The formula for the counter part in magnetic dipole radiation of photons reads,

$$\mathcal{P} = \frac{ck^4}{3} |\boldsymbol{\mu}|^2. \quad (56)$$

4. Quantum dynamics of axial vector gauge field

4.1 Two-body force and asymptotic freedom

The force between two particles with spins \mathbf{S}_A and \mathbf{S}_B is mediated by axial photon exchanged between them. The two-body potential has been worked out (Naik and Pradhan 1981) using Coulomb gauge axial vector propagator. It is more convenient to use the Feynman gauge propagator

$$D_{\mu\nu} = g_{\mu\nu}/(q^2 - \omega^2/c^2). \quad (57)$$

The two-body potential would then have the form

$$v_{AB} = -\frac{(\beta g)^2}{8\pi r} (\mathbf{S}_A \cdot \mathbf{S}_B) \quad (58)$$

where

$$\beta = \begin{cases} 3, & \text{A and B fermions,} \\ \sqrt{3}, & \text{A is boson, B fermion.} \end{cases} \quad (59)$$

For $\beta = 3$ it is identical to the classical expression (46). The two-body force being attractive for parallel spins has far-reaching consequences. A test body with spin, in a spin-polarizable medium would be antiscreened by the medium rather than screened like a test charge in a charge-polarizable medium. This is because the test spin would attract other spins parallel to it and repel those antiparallel to it. Thus the effective spin-spin force in a spin-polarizable medium would be stronger than in vacuum. In other words, the force will decrease as one gets closer to the test-body. Thus, the effective—axial—photon coupling constant would decrease as momentum transfer is increased. Such forces are called asymptotically free forces. This conclusion will be confirmed from a calculation of the Callan-Symanzik β -function in a later section.

4.2 Cancellation of divergence in QED

We shall first consider the problem in the non-relativistic limit. The ground state self-energy of an electron in the second order of perturbation theory is given by (Bethe and Salpeter 1957)

$$\Delta E_0 = \sum_n \frac{\langle 0|H_I|n\rangle \langle n|H_I|0\rangle}{E_n - E_0 + \omega} \quad (60)$$

$$\text{where } H_I = \frac{e\mathbf{p} \cdot \mathbf{A}}{m} + \frac{3}{2}g\boldsymbol{\sigma} \cdot \mathbf{a}. \quad (61)$$

Inserting (61) in (60) gives

$$\Delta E = \frac{1}{16\pi^2} \sum_n \int_0^\infty d\omega \omega \frac{\frac{-e^2}{m^2} \langle 0|p_i|n\rangle \langle n|p_i|0\rangle + \left(\frac{3}{2}g\right)^2 \langle 0|\sigma_i|n\rangle \langle n|\sigma_i|0\rangle}{E_n - E_0 + \omega} \quad (62)$$

which can be written as

$$\Delta E = \frac{1}{6\pi^2} \sum_n \int_0^\infty d\omega \omega \frac{\frac{-e^2}{m^2} (E_0 - E_n) \langle 0|P_i|n \rangle \langle n|p_i|0 \rangle}{\omega(\omega + E_n - E_0)} - \frac{e^2/m^2 \langle 0|p_i|n \rangle \langle n|p_i|0 \rangle}{\omega} + \frac{(3g/2)^2}{6\pi^2} \sum_m \int_0^\infty d\omega \langle 0|\sigma_i|m \rangle \langle m|\sigma_i|0 \rangle, \quad (63)$$

where m stands for the spin sublevels of the ground state. It will be noticed that the first term in the above expression is convergent while the last two terms are linearly divergent and would cancel with each other if $\left(\frac{3}{2}g\right)^2 = e^2$.

We shall next consider the relativistic case. The free electron self-energy contributed by the photon and axial photon shown in figures 2a, b,

$$\Sigma(p) = \frac{ie^2}{(2\pi)^4} \int d^4k \gamma_\mu \frac{i\gamma \cdot (p-k) - m}{(p-k)^2 + m^2} \gamma_\nu D_{\mu\nu}(k) - \frac{i(3/2g)^2}{(2\pi)^4} \int d^4k \gamma_\mu \gamma_5 \frac{i\gamma \cdot (p-k) - m}{(p-k)^2 + m^2} \gamma_\nu \gamma_5 D_{\mu\nu}(k) \quad (64)$$

from which after a certain amount of calculation one obtains the self-mass,

$$\left. \begin{aligned} \delta m &= \frac{\alpha_e m}{2\pi} \left(\frac{3D}{2} + \frac{9}{4} \right) - \frac{\alpha_g^* m}{(2\pi)} \left(\frac{3D}{2} - 5/4 \right) \\ \text{where } \alpha_e &= \frac{e^2}{4\pi}, \quad \alpha_g^* = \frac{(3/2g)^2}{4\pi} = \frac{9}{4} \alpha_g \end{aligned} \right\} \quad (65)$$

It will be noticed that the divergent parts in δm cancel if

$$e^2 = (3g/2)^2.$$

4.3 Callan-Symanzik beta function

The asymptotic freedom of the axial photon interaction can be confirmed by calculating the Callan-Symanzik β -function (Callan 1970; Symanzik 1970). For the electron axial photon interaction

$$\beta(g) = \frac{-3g}{2} \frac{\partial}{\partial \ln \Lambda} (\ln Z_3^{1/2}), \quad (66)$$

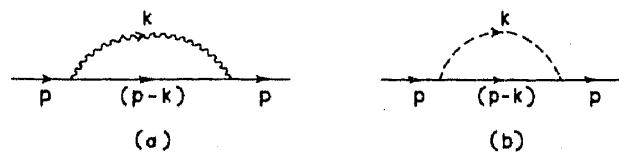


Figure 2. (a) Photon contribution and (b) axial-photon contribution to the self-energy of the electron.

where \tilde{Z}_3 is the renormalization constant associated with axial photon self-energy. Explicit calculations of the relevant diagrams (Naik 1980) gives

$$\tilde{Z}_3 = 1 + \frac{2\alpha_g}{3\pi} \ln \frac{\Lambda^2}{m^2} \quad (67)$$

from which we get

$$\beta(g) = -\frac{(3/2g)^2}{6\pi^2} + O(g^5). \quad (68)$$

The negative sign confirms the asymptotic freedom of the electron-axial-photon interaction.

4.4 Contribution of axial-photon interaction to hyperfine anomaly

The ratio of frequencies of hyperfine splitting in deuterium and hydrogen atoms is given by (Bethe and Salpeter 1957)

$$\frac{\nu_D}{\nu_H} = \frac{3}{4} \left(\frac{\mathcal{M}_D}{\mathcal{M}_H} \right)^3 \frac{2g_D}{g_H} (1 + \Delta), \quad (69)$$

where \mathcal{M}_D and \mathcal{M}_H are reduced masses and g_D and g_H are Lande g -factors for deuterium and hydrogen respectively. Δ represents nuclear structure and relativistic recoil-effects, which have been calculated from theory. It is found that there exists a very minute discrepancy between the experimental and theoretical values of Δ ;

$$\begin{aligned} \Delta_{\text{theory}} &= 28 \times 10^{-5}, \\ \Delta_{\text{expt}} &= 17 \times 10^{-5}. \end{aligned} \quad (70)$$

The hyperfine splitting discussed above is taken to be due to interaction between the electron's spin magnetic moment and the nuclear magnetic moment. It is a short range interaction between their spins. The discrepancy could be ascribed to the existence of long range interaction between spins of the electron and the nucleus caused by axial-photon exchange. We therefore take this extra interaction into account in the calculation of hyperfine splitting.

Taking the nuclear spin as \mathbf{I} and the electron spin magnetic moment as $\boldsymbol{\mu}$, the standard hyperfine interaction Hamiltonian can be written as (Bethe and Salpeter 1957)

$$\mathcal{H}_c = -2\mu_0 \left[\frac{-8\pi}{3} (\mathbf{I} \cdot \boldsymbol{\mu}) \delta(r) + \frac{1}{r^3} \left\{ \mathbf{I} \cdot \boldsymbol{\mu} - \frac{3(\mathbf{I} \cdot \mathbf{r})(\boldsymbol{\mu} \cdot \mathbf{r})}{r^2} \right\} - \frac{1}{r^3} (\mathbf{k} \cdot \boldsymbol{\mu}) \right], \quad (71)$$

where \mathbf{k} is the orbital angular momentum of the electron and $\mu_0 = e/2m$, a Bohr-magneton. The expectation value of \mathcal{H}_c gives the lowest order contribution to the hyperfine splitting.

Now to \mathcal{H}_c , we add the long range spin-spin part of the interaction so that the total interaction is given by

$$\mathcal{H}_{\text{total}} = \mathcal{H}_c - \frac{(\beta g)^2}{8\pi r} \mathbf{S} \cdot \mathbf{I}, \quad (72)$$

where \mathbf{S} is the electron spin; \mathbf{I} is nuclear spin.

The expectation value of the additional term is easily seen to be

$$-\frac{(\beta g)^2 \left\langle \frac{1}{r} \right\rangle \left\langle \frac{F^2 - S^2 - I^2}{2} \right\rangle}{8\pi}; \mathbf{F} = \mathbf{I} + \mathbf{S}. \quad (73)$$

For hydrogen-like atoms

$$\left\langle \frac{1}{r} \right\rangle = \frac{2z}{\alpha_e n^2} R_y; \quad \alpha_e = \frac{e^2}{4\pi}. \quad (74)$$

Thus the final result of the expectation value is

$$-\beta^2 \left(\frac{\alpha_g}{\alpha_e} \right) \frac{z}{n^2} \left(\frac{F(F+1) - I(I+1) - S(S+1)}{2} \right) R_y; \quad \alpha_g = \frac{g^2}{4\pi}. \quad (75)$$

For hydrogen atom,

$$\mathbf{S} = 1/2, \mathbf{I} = 1/2; \mathbf{F} = 1, 0; \beta = 3$$

$$\Delta E_H^{(g)} = -9 \left(\frac{\alpha_g}{\alpha_e} \right) R_y. \quad (76)$$

For deuterium atom,

$$\mathbf{S} = 1/2; \mathbf{I} = 1; \mathbf{F} = 1/2, 3/2; \beta = \sqrt{3}$$

$$\Delta E_D^{(g)} = -9/2 \left(\frac{\alpha_g}{\alpha_e} \right) R_y. \quad (77)$$

These have to be added to the existing Fermi formula (Bethe and Salpeter 1957, p. 196)

$$\Delta E_H^{(e)} = \frac{16}{3} \alpha_e^2 \left(\frac{g_p \mu_N}{2\mu_0} \right) R_y, \quad (78)$$

$$\Delta E_D^{(e)} = \frac{8\alpha_e^2}{1} \left(\frac{g_D \mu_D}{2\mu_0} \right) R_y, \quad (79)$$

leading to

$$\begin{aligned} \Delta E_H^{(\text{total})} &= \Delta E_H^{(e)} + \Delta E_H^{(g)}, \\ \Delta E_D^{(\text{total})} &= \Delta E_D^{(g)} + \Delta E_D^{(e)}. \end{aligned} \quad (80)$$

Taking effects of nuclear motion and structure one gets for the ratio of splitting in deuterium and hydrogen

$$\frac{\nu_D}{\nu_H} = \frac{3}{4} \left(\frac{\mathcal{M}_D}{\mathcal{M}_H} \right)^3 \left[\frac{\alpha_e^2 \left(\frac{g_D \mu_D}{\mu_0} \right) - 9/8 (\alpha_g/\alpha_e)}{\alpha_e^2 \left(\frac{g_p \mu_N}{2\mu_0} \right) - \frac{9 \times 3}{16} (\alpha_g/\alpha_e)} \right] \quad (81)$$

or

$$\frac{\nu_D}{\nu_H} = \frac{3}{4} \left(\frac{\mathcal{M}_D}{\mathcal{M}_H} \right)^3 \frac{2g_D}{g_p} (1 + \Delta - \Delta'), \quad (82)$$

where,

$$\Delta' = \frac{9}{8} \frac{\alpha_g}{\alpha_e^3} \mu_0 \left[\frac{1}{g_D \mu_D} - \frac{3}{g_p \mu_p} \right]. \quad (83)$$

It will be seen that the theoretical value of the ratio is reduced as a result of taking long range spin-spin force into account. This is what is required to obtain agreement with experimental result which requires $\Delta' = 10^{-4}$. For this to be satisfied we need $\alpha_g \simeq 10^{-13}$ which is several orders of magnitude smaller than obtained from laser beam experiment.

4.5 Electron-neutron and two-neutron bound states

Neutron and electron as well as two neutrons can form bound states when their spins are parallel so that the force between them is attractive. The Dirac equation for an electron whose spin interacts with that of a neutron can be written as,

$$\left(\alpha \cdot \mathbf{p} + \beta m - \left(\frac{3g}{2} \right)^2 \frac{\Sigma \cdot \mathbf{S}}{r} \right) \psi = 0, \quad (84)$$

where

$$\Sigma = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} \quad \text{and} \quad \mathbf{S} = \begin{pmatrix} \mathbf{s} & 0 \\ 0 & \mathbf{s} \end{pmatrix}$$

are spins of electron and neutron respectively. The latter has been taken to be infinitely heavy compared to the electron. After separating the angular and radial part of ψ in the conventional manner (Akhiezer and Berestetskii 1965)

$$\psi_{jIM} = \begin{pmatrix} iG(r)\Omega_{jIM}(\mathbf{n}) \\ -F(r)\Omega_{j'M}(\mathbf{n}) \end{pmatrix}; \quad l' = 2j - l; \quad \mathbf{n} = \frac{\mathbf{r}}{|\mathbf{r}|} \quad (85)$$

One gets the radial equations:

$$\frac{dG}{dr} + \left(\frac{1+\kappa}{r} \right) G - \left(E + m + \left(\frac{3}{2}g \right)^2 \frac{\boldsymbol{\sigma} \cdot \mathbf{S}}{r} \right) F = 0, \quad (86)$$

$$\frac{dF}{dr} + \left(\frac{1-\kappa}{r} \right) F + \left(E - m + (3/2g)^2 \frac{\boldsymbol{\sigma} \cdot \mathbf{S}}{r} \right) G = 0. \quad (87)$$

Since $\boldsymbol{\sigma} \cdot \mathbf{S} = \frac{1}{2} [J(J+1) - S(S+1) - \sigma(\sigma+1)]$

$$= \frac{1}{2} (J(J+1) - 3/2), \quad (88)$$

where $\mathbf{J} = \boldsymbol{\sigma} + \mathbf{S}$; $\boldsymbol{\sigma} \cdot \mathbf{S} = 1/4$ for $J = 1$ and $\boldsymbol{\sigma} \cdot \mathbf{S} = -3/4$ for $J = 0$ it is easy to see that bound state can be formed when spins are parallel. We shall obtain solution of (71) for $J = 1$ in which case $\boldsymbol{\sigma} \cdot \mathbf{S} = 1/4$. The allowed energies are,

$$E_n = m / \left[1 + \frac{(3/2g)^2/16}{\left(\chi^2 - \frac{(3/2g)^4}{16} + n_r \right)} \right]^{1/2}, \quad (89)$$

$$= m \left[1 - \frac{(3/2g)^4}{32n^2} - \frac{(3/2g)^8}{512n^4} \left(\frac{n}{l+1/2} - \frac{3}{4} \right) \right]. \quad (90)$$

On account of the smallness of g^2 one can ignore the last term in which case the ground

state binding energy is given by

$$\text{B.E.} \cong 81mg^4/16 \times 32, \quad (91)$$

which is the Böhr formula. If we put $\alpha_g = g^2/4\pi \cong 10^{-13}$ this works out to be $\sim 10^{-21}$ eV and the Böhr radius 7×10^2 cm. For the dineutron one gets for the ground state binding energy

$$\text{B.E.} = \frac{81 Mg^4}{16 \times 64} \sim 10^{-18} \text{ eV} \quad (92)$$

and $a_0 = (8 \times 4)/(9 \times g^2 \times M) \cong 10^{-1}$ cm. These bound states cannot exist under ordinary conditions of temperature and pressure because they will be easily ionized by the thermal energy and collisions. They can exist at very low temperatures and pressures.

4.6 Aharonov-Bohm type of experiment

Aharonov and Bohm (1959) discussed the question of the effect of electromagnetic potential on the phase of the quantum mechanical wavefunction of the electron and their conjecture has been experimentally confirmed (Chambers 1960). Such effect on the quantum mechanical phase of the neutron by earth's rotation have recently been observed (Werner *et al* 1979). We shall in this section, discuss the effect of the axial photon potential on the quantum mechanical phase of the electron and propose an Aharonov-Bohm type of experiment to detect this effect. Its positive result would be definite evidence for the existence of the fifth interaction.

In the case of the electromagnetic local gauge group, the covariant derivative,

$$D_\mu \psi = (\partial_\mu + ieA_\mu) \psi \quad (93)$$

suggest that the phase of the wavefunction in the presence of the potential A_μ would be

$$\psi(x) = [\exp(ie \int_{x_0}^x dx_\mu A_\mu)] \psi_0(x), \quad (94)$$

where $\psi_0(x)$ is the wavefunction in the absence of the potential. From the covariant derivative for the fifth interaction:

$$D_\mu \psi = (\partial_\mu + 3/2(i)^2 g \gamma_5 a_\mu) \psi \quad (95)$$

it follows that,

$$\psi(x) = \left(\exp \left[+ \frac{3}{2} (i)^2 g \gamma_5 \int_{x_0}^x dx_\mu a_\mu \right] \right) \Psi_0(x). \quad (96)$$

In the Aharonov-Bohm experiment electrons from a point source are made to pass through a double slit so as to form an interference pattern on a screen. A solenoid placed between the slits and the screen which produces no magnetic field but only a vector potential outside, causes a displacement of the fringe system since the potential brings about a phase difference η given by,

$$\eta = e \int d\mathbf{S} \cdot \mathbf{B} \quad (97)$$

between the two beams. In our case we replace the solenoid by a long straight selfoc

fibre through which a circularly polarized laser beam is passing. As discussed in §3.3 this fibre would produce an axial electric field ϵ and an axial vector potential outside. It would therefore cause a fringe-shift on account of phase change

$$\eta = (3g) \int ds \cdot \epsilon. \quad (98)$$

Therefore the occurrence of a fringe-shift on passing circularly polarized light through the selfoc fibre placed between slits and the screen would be a conclusive evidence for the existence of the fifth interaction. Further, quantitative measurement of this fringe-shift would lead to a determination of the strength of this interaction.

5. Concluding remark

The introduction of the fifth interaction has been motivated by a gauge theory approach. If we believe in the philosophy that all the fundamental interactions are dictated and determined by gauge principle according to which the conserved Noether current of a global symmetry couples to the gauge field which results from the corresponding local gauge symmetry, it then follows that spin angular momentum must couple to an axial vector gauge field thereby giving rise to the fifth interaction. We have discussed two experimental evidences for the existence of such an interaction and proposed an Aharonov-Bohm type of experiment. It is found that the interaction is very weak $\alpha_g \simeq 10^{-13}$. This is why one does not easily perceive its effects in quantum electrodynamic phenomena.

The confirmation of the fifth interaction would have far reaching consequences. It promises to solve the problem of occurrence of the divergence in quantum electrodynamics. It would make the classical electron stable by providing the "Poincare stress". And last of all it provides a dynamical method of the measurement of spin in the same sense that photon provides a measurement of the charge. The superweak nature of the interaction indicates that it will play a major role in very long wavelength and low-energy domain and very long distant future of the expanding universe.

Acknowledgements

The authors would like to acknowledge fruitful discussions with the members of the theory group of the Institute of Physics, Bhubaneswar.

References

- Akhiezer A I and Berestetskii V B 1965 *Quantum electrodynamics* p. 144
 Aharonov Y and Bohm D 1959 *Phys. Rev.* **115** 485
 Bethe H A and Salpeter E E 1957 *Handbuch der Physik Atome I* (Berlin: Springer Verlag)
 Callan C G 1970 *Phys. Rev.* **D2** 154
 Carmeli Moshe 1982 *Classical fields (general relativity and gauge theory)* (New York: Wiley Interscience)
 Chambers R G 1960 *Phys. Rev. Lett.* **5** 3
 Cho Y M 1976 *Phys. Rev.* **D14** 2521
 Fritzsche H, Gel-Mann M and Leutwyler H 1973 *Phys. Lett.* **B47** 365

- Glashow S L 1980 *Rev. Mod. Phys.* **52** 539
Gell-Mann M and Low F F 1954 *Phys. Rev.* **95** 1300
Hayward J 1979 *Phys. Rev.* **D20** 3039
Kibble T W B 1961 *J. Math. Phys.* **2** 212
Naik P C 1980 *Quantum electrodynamics with axial photon*; Utkal Univ. Ph.D. thesis
Naik P C and Pradhan T 1981 *J. Phys.* **A14** 2795
Pradhan T and Lahiri A 1974 *Phys. Rev.* **D10** 1872
Salam A 1980 *Rev. Mod. Phys.* **52** 525
Sciama D W 1962 in *Recent developments in general theory of relativity* (Oxford: Pergamon Press) p. 415
Stueckelberg E C G 1938 *Hilv. Phys. Acta.* **11** 289
Symanzik K 1970 *Commun. Math. Phys.* **18** 227
Tam A C and Happer W 1977 *Phys. Rev. Lett.* **38** 278
Tam A C and Happer W 1977 in *Springer series in optical sciences, Laser spectroscopy III.* (eds) J L Hall and C L Carlsten (Berlin: Springer Verlag) p. 335
Utiyama R 1956 *Phys. Rev.* **101** 1597
Weinberg S 1980 *Rev. Mod. Phys.* **52** 510
Werner S A, Standenmann J L and Colella R 1979 *Phys. Rev. Lett.* **42** 1103