FINITE QUANTUM ELECTRODYNAMICS

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Received Janaury 25, 1973

(Communicated by Prof. K. P. Sinha, F.A.SC.)

Abstract

It is shown that the ultraviolet divergences encountered in the lowest order perturbation calculations of quantum electrodynamics no longer appear if the theory is expanded so as to include the mu meson, a triplet of heavy axial vector bosons and two heavy polar vector bosons in addition to the electron and photon, and suitably chosen couplings between them are introduced.

1. Introduction

Although the divergences occurring in the perturbation theoretic calculations of physical processes in the present framework of quantum electrodynamics can be separated by the process of renormalisation and physically meaningful results can be extracted,[1] the theory is not entirely satisfactory. The relation between the bare and renormalised quantities are formal, involving divergent expressions, and can lead to various paradoxes.[2] These problems cannot be bypassed by formulating the theory in terms of renormalised field operators because then the Lagrangian contains the renormalisation constants which are, strictly speaking, meaningless in so far as they are divergent quantities. Although the method of indefinite metric[3] has circumvented these problems, it appears to have some difficulties of physical interpretation.[4]

As far as the divergence in the self-energy of the electron is concerned, it can be traced back to the classical theory where the repulsive coulomb energy diverges in the limit of a point electron. A cohesive force would keep the electron stable by compensating for the repulsive coulomb energy.

Such cohesive forces have been considered in the past[5] but they have left the theory as problematic as before.

In this paper we discuss a theory where the cohesive forces arise due to the coupling of axial vector fermionic currents with axial vector bosons which act as the cementing particles. That the axial vector currents may play a vital role in this respect was conjectured by one of us (TP). It was felt that if the Maxwell's equations were made symmetric in the electric and magnetic quantities by introducing a 'magnetic' charge-current (giving rise to no net magnetic charge but a magnetic dipole moment) then this could supply the required cohesive energy. For conservation of parity this has to be an axial vector current.[6] We know that if such a current, bilinear in the fermionic field, is to have charge conjugation -1 then it would be rather problematic. Thus, a cohesive force of the above origin in a theory with only electrons and photons is ruled out. At least one more lepton is needed to construct the magnetic current. The most natural candidate for this is the mu meson which can be used to construct a magnetic current

$$j_\lambda{}^{(1)}(x) = \frac{ig}{\sqrt{2}}\left(\bar{e}(x)\,\gamma_\lambda\gamma_5\mu(x) - \bar{\mu}(x)\,\gamma_\lambda\gamma_5 e(x)\right),$$

which has $C = -1$ and $P = +1$. This current is to be coupled to an axial vector boson (we call it $a_\lambda{}^{(1)}$) to conserve parity. However, such a coupling would have the serious consequence of violating muon number conservation. A similar trouble with strangeness conservation would arise if we had a model of $n\Lambda$ interaction mediated by the K_1 meson. This analogy suggests that $a^{(1)}$, which is analogous to K_1 here, must have its partner $a^{(2)}$, analogous to K_2. In that case we can treat the linear combinations

$$a_\lambda{}^{(\pm)} = \frac{a_\lambda{}^{(1)} \pm i a_\lambda{}^{(2)}}{\sqrt{2}},$$

rather than $a^{(1)}$ and $a^{(2)}$ themselves, as the mediating particles. The axial vector bosons $a_\lambda{}^{(\pm)}$, which are charge conjugate to each other, have muonic charge $+1$ and -1 respectively and are coupled to the currents $j_\lambda{}^{(\pm)}(x)$ given by

$$j_\lambda{}^{(-)} = ig_A\bar{e}\gamma_\lambda\gamma_5\mu,$$

$$j_\lambda{}^{(-)} = ig_A\bar{\mu}\gamma_\lambda\gamma_5 e.$$

The introduction of the interaction term $(j_\lambda{}^{(+)}a_\lambda{}^{(-)} + j_\lambda{}^{(-)}a_\lambda{}^{(+)})$ leads to finite electron and muon self-energies in the lowest order perturbation treatment if the coupling constant g_A is properly chosen, provided the electron and the muon are taken to be degenerate.

In this theory the photon self-energy gets additional contribution from the virtual muon-antimuon pairs which has a divergent part of the same sign as the contribution from the virtual electron-positron pairs. To cancel these divergences we introduce a coupling of the photon with the muonically charged bosons which contributes to its self-energy through a virtual $a^{(+)}$, $a^{(-)}$ pair. This coupling should be of the Pauli type so that the bosons do not acquire any charge. On calculation we find that the contribution of the boson pair to the photon self-energy is opposite in sign to that of the fermion-antifermion pairs and hence, the divergent parts of the two cancel for a proper choice of the coupling constants.

However, the couplings so far considered cannot remove the divergences from the self-energy of the photon and that of the muonically charged bosons simultaneously. In view of this it has been found necessary to consider, in addition to the doublet of the axial vector bosons, a doublet of polar vector bosons $A_\lambda{}^{(\pm)}$ as well. These will be taken to couple to the fermion fields in the manner

$$L_{int} = g_V\,(\bar{e}\gamma_\lambda \mu A_\lambda{}^{(-)} - \bar{\mu}\gamma_\lambda e A_\lambda{}^{(+)}).$$

The coupling of the photon with the vector and axial vector bosons will be taken to be of the Pauli type:

$$i\lambda_V A_\mu \partial_\nu\,(A_\mu{}^{(+)}\,A_\nu{}^{(-)} - A_\mu{}^{(-)}\,A_\nu{}^{(+)})$$

and

$$i\lambda_A A_\mu \partial_\nu\,(a_\mu{}^{(+)}\,a_\nu{}^{(-)} - a_\mu{}^{(-)}\,a_\nu{}^{(+)})$$

respectively. Finally, to eliminate the divergence arising in the self-energy of the polar vector bosons we introduce a muonically neutral axial vector beson a_μ with the coupling

$$L_{int} = \lambda\,(\partial_\mu a_\nu - \partial_\nu a_\mu)\,(a_\mu{}^{(+)}\,A_\nu{}^{(-)} + a_\mu{}^{(-)}\,A_\nu{}^{(+)}).$$

This coupling will give rise to a divergent self-energy of a_μ which can be cancelled by contributions arising from its coupling to the axial vector currents of the fermions. This latter coupling gives additional contributions to the self-energies of the fermions in such a manner that the earlier constraint of degeneracy of the muon and electron is no longer necessary to render their self-energies finite. Taking into account all the couplings discussed above, it is found that in addition to the fermion and boson self-energies all the vertex functions are also finite.

It must be noted that quantum electrodynamics in its present form, in spite of its unsatisfactory features, has led to excellent agreement between theory and experiment. We should therefore make sure that our modification through the introduction of the new particles does not upset this agreement. This is achieved by taking the bosons to be sufficiently heavy. It would appear that couplings of such massive vector bosons we have considered would make the theory non-renormalisable (even in a finite theory one needs renormalisation). There has been considerable discussion in the literature[7] on the ways to make the such theories renormalisable. We shall adopt here a procedure which is straightforward and has the merit of making all the vertices finite without additional constraints on the coupling constants. This method essentially consists of taking a certain limit of a local theory and arriving at an effective non-local theory where the massive bosons are coupled to non-local but conserved currents.

Our theory thus has altogether eight particles—a triplet of massive axial vector bosons, a triplet of polar vector bosons, two of which are massive—the third one being the photon—the electron and the muon. The theory can be enlarged so as to encompass leptonic weak interactions by introducing two-component neutrinos associated with the electron and the muon. The neutrino currents, when coupled to the bosons, can account for the muon decay and other leptonic weak interactions. In fact, in the limit of infinite mass of the vector bosons the leptonic weak interaction will be identical to a Fierz shuffled Fermi interaction.

2. Formulation of the Theory

The coupling scheme in our theory can be represented by the Lagrangian

$$\begin{aligned} L = L_0 &+ ie(\bar{e}\gamma_\lambda e + \bar{\mu}\gamma_\lambda \mu) A_\lambda + i(f\bar{\mu}\gamma_\lambda\gamma_5\mu + g\bar{e}\gamma_\lambda\gamma_5 e) a_\lambda \\ &- g_V(\bar{e}\gamma_\lambda \mu A_\lambda^{(-)} - \bar{\mu}\gamma_\lambda e A_\lambda^{(+)}) + ig_A(\bar{e}\gamma_\lambda\gamma_5\mu a_\lambda^{(-)} + \bar{\mu}\gamma_\lambda\gamma_5 e a_\lambda^{(+)}) \\ &+ i\lambda_V A_\mu \partial_\nu (A_\mu^{(-)} A_\nu^{(-)} - A_\mu^{(-)} A_\nu^{(+)}) + i\lambda_A A_\mu \partial_\nu (a_\mu^{(+)} a_\nu^{(-)} \\ &- a_\mu^{(-)} a_\nu^{(+)}) + \lambda(\partial_\mu a_\nu - \partial_\nu a_\mu)(a_\mu^{(+)} A_\nu^{(-)} + a_\mu^{(-)} A_\nu^{(+)}), \qquad (1) \end{aligned}$$

where e is the electric charge, f, g, g_A, g_V, λ_A, λ_V, and λ are coupling constants which are as yet arbitrary and L_0 is the kinetic energy part of the Lagrangian including the mass terms.

As pointed out earlier, because of the appearance of the massive vector and axial vector fields coupled to non-conserved currents, the theory, as defined by eqn. (1), is non-renormalisable. However, this difficulty can be overcome by introducing suitable couplings of our bosons and fermions with massless scalar bosons and adopting a limiting procedure which makes our theory renormalisable and at the same time effectively decouples these massless scalar bosons. We illustrate this procedure for the coupling of one of the vector fields, say a_λ, coupled to the axial vector current of the electrons for which the Lagrangian is

$$L = -\bar{e}(\partial + m)e - \tfrac{1}{2}\partial_\mu a_\nu \partial_\mu a_\nu - \tfrac{1}{2}M^2 a_\mu a_\mu + ig\bar{e}\gamma_\lambda\gamma_5 e a_\lambda, \qquad (2)$$

M being the mass of a_μ.

We now introduce a massless pseudoscalar boson field ϕ with the coupling

$$L_{\text{int}} = ig\epsilon\bar{e}\gamma_\lambda\gamma_5 e\partial_\lambda\phi - \epsilon^{-1}a_\lambda\partial_\lambda\phi. \qquad (3)$$

For finite ϵ this describes a local interaction. The modifications of the vertices and propagators arising out of the mixing interaction $-(1/\epsilon)\, a_\lambda\partial_\lambda\phi$ are such that in the limit $\epsilon \to 0$, ϕ is effectively decoupled leading to an effective interaction of a_λ with the conserved, but non-local, current

$$\left(\delta_{\lambda\sigma} - \frac{\partial_\lambda\partial_\sigma}{\Box}\right)\bar{e}\gamma_\sigma\gamma_5 e$$

and an effective propagator

$$-i\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\Big/\left(k^2 + M^2\right).$$

It is thus clear that the renormalisation difficulty is avoided by adopting this limiting procedure. In the following it will be understood that this procedure has been performed for all the massive vector and axial vector fields and their interactions represented in the Lagrangian given in eqn. (1). This amounts to a modification of the vertices and propagators whereby any vector or axial vector boson vertex with momentum k will acquire a factor

$$G_{\mu\nu}(k) = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$$

and the corresponding propagator will be

$$-iG_{\mu\nu}(k)\,\frac{1}{k^2 + M^2}$$

where M is the mass of the boson.

By applying these considerations we shall now see that with a proper choice of the coupling constants we can make the self-energies and vacuum polarisations finite in second order perturbation theory. It turns out that all the second order vertex functions are also finite without any further constraints on the coupling constants.

3. FERMION SELF-ENERGIES

Let us first consider the electron self-energy. The diagrams, Figs. 1 (*a*) to 1 (*d*), contribute to this self-energy. The Feynman matrix element for Fig. 1 (*a*) is (we use the Landau gauge for the photon propagator)

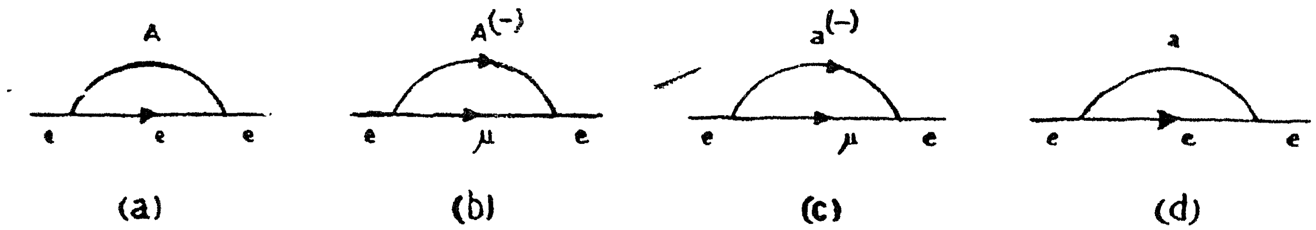

fig.1

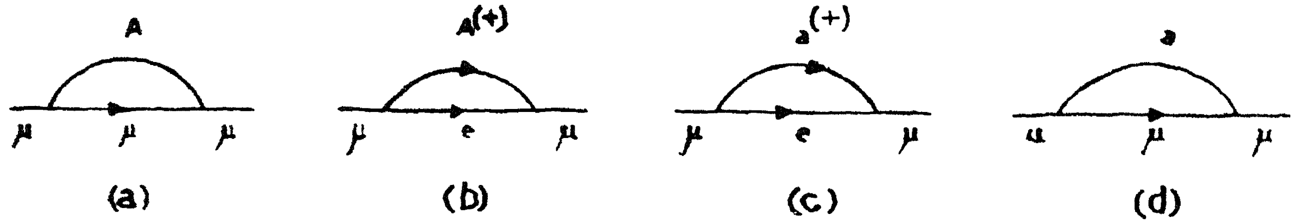

fig.2

FIGS. 1–2. Fig. 1. Feynmann diagrams for electron self-energy. Fig. 2. Muon self-energy diagrams.

$$\sum^{(a)}(p) = \frac{e^2}{(2\pi)^4} \int \frac{d^4k}{k^2} \gamma_\mu \frac{i(\hat{p}-\hat{k}) - m_e}{(p-k)^2 + m_e^2} \gamma_\nu G_{\mu\nu}(k). \qquad (4\ a)$$

By applying the rules discussed in Section 2 we get the following contributions from the other three diagrams

$$\sum^{(b)}(p) = \frac{g_V^2}{(2\pi)^4} \int \frac{d^4k}{k^2 + M_V^2} G_{\mu\nu}(k) \gamma_\lambda \frac{i(\hat{p}-\hat{k}) - m_\mu}{(p-k)^2 + m_\mu^2} \times G_{\nu\sigma}(k) \gamma_\sigma G_{\mu\nu}(k),$$

$$\sum^{(c)}(p) = \frac{g_A^2}{(2\pi)^4} \int \frac{d^4k}{k^2 + M_A^2} G_{\mu\lambda}(k) \gamma_\lambda \gamma_5 \frac{i(\hat{p}-\hat{k}) - m_\mu}{(p-k)^2 + m_\mu^2} \times G_{\nu\sigma}(\kappa) \gamma_\sigma \gamma_5 G_{\mu\nu}(k),$$

$$\sum^{(d)}(p) = \frac{g^2}{(2\pi)^4}\int \frac{d^4k}{k^2+\mathrm{M}^2}\,\mathrm{G}_{\mu\lambda}(k)\,\gamma_\lambda\gamma_5\,\frac{i(\hat{p}-\hat{k})-m_e}{(p-k)^2+m_e^2}$$
$$\times\,\mathrm{G}_{\nu\sigma}(k)\,\gamma_\sigma\gamma_5\mathrm{G}_{\mu\nu}(k),$$

$\mathrm{M_V}$, $\mathrm{M_A}$ and M being the masses of the relevant bosons and m_e, m_μ the masses of the fermions. After some simplifications, we get

$$\sum^{(b)}(p) = \frac{g_{\mathrm{V}}^2}{(2\pi)^4}\int \frac{d^4k}{k^2+\mathrm{M_V}^2}\,\mathrm{G}_{\mu\nu}(k)\,\gamma_\mu\,\frac{i(\hat{p}-\hat{k})-m_\mu}{(p-k)^2-m_\mu^2}\,\gamma_\nu, \tag{4 b}$$

$$\sum^{(c)}(p) = \frac{g_{\mathrm{A}}^2}{(2\pi)^4}\int \frac{d^4k}{k^2+\mathrm{M_A}^2}\,\mathrm{G}_{\mu\nu}(k)\,\gamma_\mu\,\frac{i(\hat{p}-\hat{k})+m_\mu}{(p-k)^2+m_\mu^2}\,\gamma_\nu, \tag{4 c}$$

$$\sum^{(d)}(p) = \frac{g^2}{(2\pi)^4}\int \frac{d^4k}{k^2+\mathrm{M}^2}\,\mathrm{G}_{\mu\nu}(k)\,\gamma_\mu\,\frac{i(\hat{p}-\hat{k})+m_e}{(p-k)^2+m_e^2}\,\gamma_\nu. \tag{4 d}$$

It is to be noted that the sign of the mass term in the neumerator of $\Sigma^{(c)}(p)$ and $\Sigma^{(d)}(p)$, which is crucial for the removal of divergence, has its origin in the occurrence of the two γ_5 matrices. By using standard techniques of Feynman parameters and symmetric integration, we see that

$$\int \frac{d^4k}{k^2+\mathrm{M}^2}\,\mathrm{G}_{\lambda\sigma}(k)\,\gamma_\lambda\,\frac{i(\hat{p}-\hat{k})}{(p-k)^2+m^2}\,\gamma_\sigma$$

is free of ultraviolet divergence while

$$\int \frac{d^4k}{(k^2+\mathrm{M}^2)\left((p-k)^2+m^2\right)}\,\mathrm{G}_{\lambda\sigma}(k)\,\gamma_\lambda\gamma_\sigma$$

is not. Thus

$$\Sigma(p) \equiv \Sigma^{(a)}(p)+\Sigma^{(b)}(p)+\Sigma^{(c)}(p)+\Sigma^{(d)}(p)$$

will be free of ultraviolet divergence if

$$m_e e^2 + m_\mu g_{\mathrm{V}}^2 = m_e g^2 + m_\mu g_{\mathrm{A}}^2. \tag{5}$$

We can treat the muon self-energy in the same manner. The diagrams, Figs. 2 (*a*) to 2 (*d*), contribute in this case and we get the condition of finiteness as

$$m_\mu e^2 + m_e g_{\mathrm{V}}^2 = m_\mu f^2 + m_e g_{\mathrm{A}}^2. \tag{6}$$

4. BOSON SELF-ENERGIES

We start with the photon vacuum polarisation diagrams. The four diagrams shown in Figs. 3 (*a*) to 3 (*d*) will contribute to this. The contribution of Fig. 3 (*a*) to the vacuum polarisation tensor is given by

$$\pi_{\alpha\beta}{}^{(a)}(k) = \frac{e^2}{(2\pi)^4} \int \frac{d^4p}{(p^2 + m_e{}^2)\left((p-k)^2 + m_e{}^2\right)} \times \mathrm{Tr}\,\gamma_\alpha \left[(i\hat{p} - m_e)\,\gamma_\beta \left(i(\hat{p} - \hat{k}) - m_e\right)\right]. \tag{7}$$

It is known that $\pi_{\alpha\beta}{}^{(a)}(k)$ can be written in the form

$$\pi_{\alpha\beta}{}^{(a)}(k) = k^2 G_{\alpha\beta}(k)\,\pi^{(a)}(k^2) + \delta_{\alpha\beta} D^{(a)} \tag{8}$$

where the constant $D^{(a)}$ containing a quadratic divergence is of no physical significance since in all observable quantities it cancels with the so-called 'sea-gull' terms. $\pi^{(a)}(k^2)$, on the other hand, contributes to physical processes, but is logarithmically divergent. By employing usual methods of calculation we get, for its divergent part $\pi_D{}^{(a)}(k^2)$,

$$\pi_D{}^{(a)}(k^2) = -\frac{4}{3}\frac{ie^2}{16\pi^2} \lim_{L\to\infty} ln \frac{L^2}{m_e{}^2}. \tag{9}$$

In a similar manner, from Fig. 3 (*b*), we get

$$\pi_D{}^{(b)}(k^2) = -\frac{4}{3}\frac{ie^2}{16\pi^2} \lim_{L\to\infty} ln \frac{L^2}{m_\mu{}^2}. \tag{10}$$

The contribution of Fig. 3 (*c*) to the vacuum polarisation tensor is given by

$$\pi_{\alpha\beta}{}^{(C)}(k) = \frac{\lambda_V{}^2}{(2\pi)^4} 2k_\nu k_\eta \int \frac{d^4q}{(q^2 + M_V{}^2)\left((k-q)^2 + M_V{}^2\right)} \times \left(G_{\alpha\beta}(q)\,G_{\eta\nu}(k-q) - G_{\alpha\eta}(q)\,G_{\beta\nu}(k-q)\right). \tag{11}$$

It can be easily seen that $\pi_{\alpha\beta}{}^{(C)}$ is of the form

$$\pi_{\alpha\beta}{}^{(C)} = G_{\alpha\beta}(k)\,[k^2\pi^{(C)}(k^2) + C], \tag{12}$$

where C is a constant which, like D in eqn. (8), does not contribute to the physical processes or to charge renormalisation and is compensated by 'sea-gull' terms and where

$$\pi_D{}^{(C)}(k^2) = \frac{i\lambda_V{}^2}{16\pi^2} \lim_{L\to\infty} ln \frac{L^2}{M_V{}^2}. \tag{13}$$

Similarly, from Fig. 3 (*d*) we get

$$\pi_D^{(d)}(k^2) = \frac{i\lambda_A^2}{16\pi^2} \lim_{L\to\infty} ln \frac{L^2}{M_A^2} \cdot \tag{14}$$

It is to be noted that the divergent parts of $\pi^{(a)}$ and $\pi^{(b)}$ are opposite in sign to those of $\pi^{(c)}$ and $\pi^{(d)}$. It is on account of this that the sum of all the contributions

$$\pi(k^2) = \sum_{i=a,b,c,d} \pi^{(i)}(k^2)$$

can be made divergence-free by suitably choosing the coupling constants. From eqns. (9) to (14) we find that $\pi(k^2)$ is free of divergence if and only if

$$\frac{8}{3}e^2 = \lambda_V^2 + \lambda_A^2. \tag{15}$$

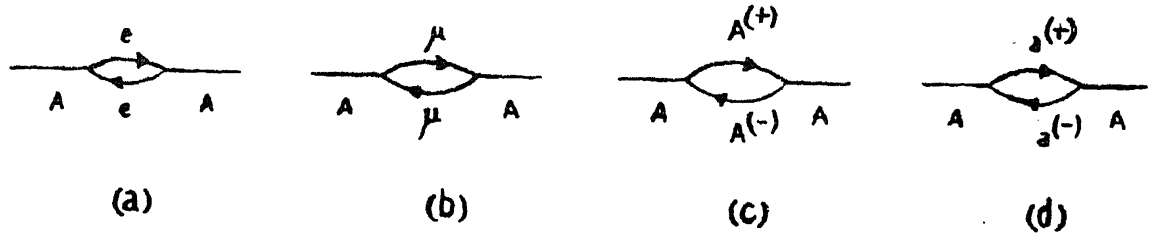

fig.3

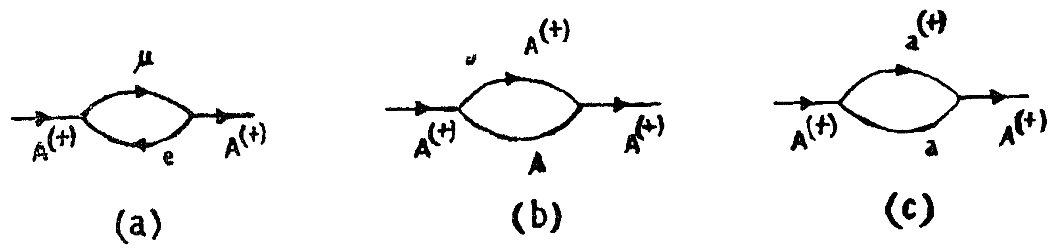

fig.4

Figs. 3–4. Fig. 3. Photon self-energy diagrams. Fig. 4. Self-energy diagrams for the massive polar vector bosons.

We now consider the self-energy of the $A^{(\pm)}$ vector bosons. Defining

$$\pi_{\alpha\beta}(k) = \frac{1}{(2\pi)^4}\int d^4x e^{ikx} \langle 0 \mid T\left(A_\alpha^{(+)}(x) A_\beta^{(-)}(0)\right) \mid 0\rangle, \tag{16}$$

we see that in the second order the three diagrams, Figs. 4 (*a*) to 4 (*c*), contribute to $\pi_{\alpha\beta}(k)$. Defining $\pi_{\alpha\beta}^{(a,b,c)}$ as the contributions of these three diagrams respectively to $\pi_{\alpha\beta}(k)$ we can easily see that they are of the form

$$\pi_{\alpha\beta}^{(a,b,c)}(k) = G_{\alpha\beta}(k)\,\pi^{(a,b,c)}(k^2), \tag{17}$$

and, dropping the quadratically divergent terms as discussed above, we get the logarithmically divergent parts as given below

$$\pi_D^{(a)}(k^2) = -\frac{ig_V^2}{16\pi^2}\frac{4}{3}\lim_{L\to\infty} ln\frac{L^2}{m^2}, \tag{18 a}$$

$$\pi_D^{(b)}(k^2) = \frac{i\lambda_V^2}{16\pi^2}\frac{1}{3}\lim_{L\to\infty} ln\frac{L^2}{M_V^2}, \tag{18 b}$$

$$\pi_D^{(c)}(k^2) = \frac{i\lambda^2}{16\pi^2}\frac{1}{3}\lim_{L\to\infty} ln\frac{L^2}{M^2}. \tag{18 c}$$

Thus, $\pi(k^2) \equiv \sum_{i=a,b,c} \pi^{(i)}(k^2)$ is free of divergence if

$$\frac{4}{3}g_V^2 = \frac{\lambda_V^2}{3} + \frac{\lambda^2}{3}. \tag{19}$$

The condition of finiteness of the self-energy of the $a_\mu^{(\pm)}$ bosons can be worked out in a similar manner and is found out to be

$$\frac{4}{3}g_A^2 = \frac{\lambda_A^2}{3} + \frac{\lambda^2}{3}. \tag{20}$$

The corresponding condition for the finiteness of the self-energy of the a_λ-field is seen to be

$$\frac{4}{3}(g^2 + f^2) = \frac{13}{6}\lambda^2. \tag{21}$$

Combining the conditions expressed by eqns. (5), (6), (15), (19), (20), (21) we see that we can choose the coupling constants in such a manner that all fermion and boson self-energies are finite. The most symmetric choice is

$$\left.\begin{aligned} g^2 &= f^2 = e^2, \\ \lambda_V^2 &= \lambda_A^2 = \frac{4}{3}e^2, \\ g_V^2 &= g_A^2 = \frac{25}{39}e^2, \\ \lambda^2 &= \frac{16}{13}e^2 \end{aligned}\right\}. \tag{22}$$

ACKNOWLEDGEMENT

The authors would like to thank Professor E. C. G. Sudarshan for a correspondence on the subject and Prof. B. Dutta–Roy for discussions. Thanks are also due to Mr. U. Basu for collaboration in the early stages of the work.

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