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Connectivity constant of the kagomé lattice

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Abstract. From the computation of self-avoiding walks on the kagomé lattice, its connectivity constant is found to be 2·569 ± 0·008.

Keywords. Self-avoiding walk; kagomé lattice; connectivity constant.

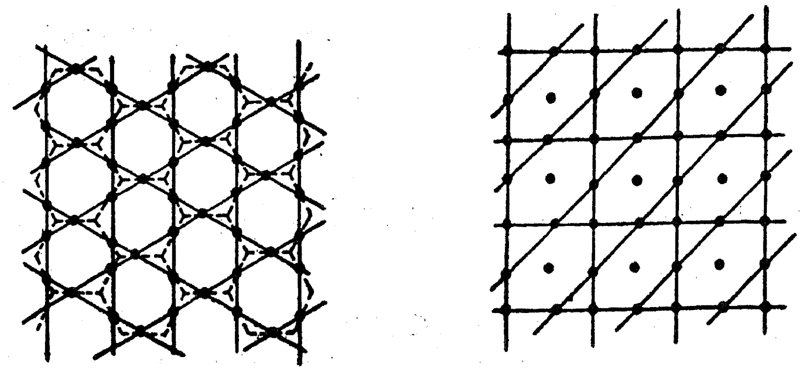
1. Introduction

The connectivity constant* (Broadbent and Hammersley 1957; Hammersley 1957)of the kagomé lattice is not reported in the literature (Shante and Kirkpatrick 1971; Watts 1975; McKenzie 1976). The coordination number of this two-dimensional lattice is four, the same as that of the simple quadratic lattice, but the overall topology is different. The results on the kagomé lattice can therefore be compared with those on the simple quadratic lattice to find the effect of the global structure of the lattice on physical quantities. For example, it is known that the transition temperature in the Ising model in these two lattices differ by 6% (Fisher 1963). Here we report the number of n-stepped selfavoiding walks (SAW) on the kagomé lattice, $n = 1$ to 16, estimate the connectivity constant of this lattice, and compare it with that of the simple quadratic lattice.

2. Calculations

The kagomé lattice is obtained by decorating the hexagonal lattice with a point at the midpoint of each side, joining these points and dissolving away the original lattice. But it is convenient for the use of a computer to distort the lattice into a square geometry (figure 1) and enumerate the neighbours. The method is explained by Dean (1963). The computer program then follows the algorithm outlined by Martin (1974). We take a 51 × 51 matrix to represent a finite part of the lattice, and count SAW of up to 16 steps. The time taken to count all the 16 step SAW is about 2hr. As a check on the computer program, we counted the SAW on the simple quadratic lattice and reproduced the first few known numbers of these walks. Table 1 lists the number of SAW counted on the kagomé lattice.

*In the literature the constant is often called the connective constant but Kesten (1980), who has done much work on self-avoiding walks, uses the term connectivity constant.



(a) (b)

Figure 1. a. The kagomé lattice structure. b. Topological structure equivalent to the kagomé lattice.

Table 1. Number of SAW on the kagomé lattice.

No. of steps n	No. of SAW c_n
1	4
2	12
3	32
4	88
5	240
6	652
7	1744
8	4616
9	12208
10	32328
11	85408
12	224640
13	589024
14	1542944
15	4039256
16	10560552

It has been conjectured that the number c_n of n-stepped SAW is given asymptotically by

$$c_n \simeq \mu^n n^g \tag{1}$$

as $n \to \infty$. Here μ is the connectivity constant and g is an index, having a value close to 1/3 for all two-dimensional lattice (Domb 1970). The numerical methods for analyzing the numbers in table 1 are well known (Domb and Sykes 1961; Martin *et al* 1967). Our best estimate for μ in the kagomé lattice is

$$\mu = 2{\cdot}569 \pm 0{\cdot}008. \tag{2}$$

Sometimes one writes $\mu = e^{\kappa}$, and the value of κ for the kagomé lattice is

$$\kappa = 0{\cdot}943 \pm 0{\cdot}003 \tag{3}$$

g turns out to be $\approx 0{\cdot}3$. The best estimate for the simple quadratic lattice is $\mu = 2{\cdot}6390 \pm 0{\cdot}0003$. Examining the detailed numbers, we find that the convergences of μ and g are somewhat poorer in the kagomé lattice than the simple quadratic lattice. Hence our estimate of the error is larger in both μ and g, as indicated above.

We conclude that the effect of the global structure *vis-a-vis* local structure on the connectivity is indeed rather small. In problems of physical interest, percolation processes or dilute magnetism, the effect of global structure may be ignored unless high accuracy is attempted in the calculation.

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