Cell-dynamical simulation of magnetic hysteresis in the two-dimensional Ising system

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We present results from numerical simulations using a "cell-dynamical system" to obtain solutions to the time-dependent Ginzburg-Landau equation for a scalar, two-dimensional (2D), $(\Phi^2)^2$ model in the presence of a sinusoidal external magnetic field. Our results confirm a recent scaling law proposed by Rao, Krishnamurthy, and Pandit [Phys. Rev. B **42**, 856 (1990)], and are also in excellent agreement with recent Monte Carlo simulations of hysteretic behavior of 2D Ising spins by Lo and Pelcovits [Phys. Rev. A **42**, 7471 (1990)].

I. INTRODUCTION

A proper understanding of the hysteretic response of a magnetic system to a periodic external magnetic field is of considerable technological interest since the area enclosed by the hysteresis loop is directly proportional to the energy lost in a magnetization-demagnetization cycle. Early experimental studies by Steinmetz[1] on real magnetic systems showed that the area of typical hysteresis loops A is quite accurately given by $A \approx H_0^{1.6}$ where H_0 is the amplitude of the oscillating external magnetic field, $H(t)=H_0 \sin(2\pi\omega t)$. Experimental work on the frequency (ω) dependence[2] of the area is less extensive.

Only recently have there been serious attempts to understand theoretically[3,4] the phenomenon of hysteresis in magnetic systems. The theory of Agarwal and Shenoy[3] determined a range of frequencies within which conventional hysteresis loops are obtained viz. the so-called "hysteresis window." The analysis was carried out for a single-component order parameter which exhibits Fokker-Planck dynamics within a double-well potential. Fluctuations which allow the order parameter to scale the barrier and shift from one minimum to another were provided by a temporal, Gaussian, white noise. Bounds were obtained for the derivative of the external magnetic field for which hysteresis occurs by analyzing the mean first-passage time for barrier crossing. However, in this work the role of spatial fluctuations of the order parameter was completely neglected. A systematic study of the dependence of the area of the hysteresis loops on the frequency and amplitude of the external magnetic field was also not attempted. An attempt to address both these issues was made by Rao, Krishnamurthy, and Pandit,[4] who solved the full time-dependent Ginzburg-Landau (TDGL)[5] equation for a $(\Phi^2)^2$ field theory with $O(N)$ symmetry in the $N\rightarrow\infty$ limit where the calculation essentially reduces to the solution of two coupled nonlinear integro-differential equations.[4] Quite extensive studies by these authors have brought to light the following features.

There is evidence for a "dynamical transition" in this system. At a given frequency, for low amplitudes the time averaged magnetization M_0 (averaged over an integral number of cycles of the external field) is nonzero. As the amplitude of the field is increased, M_0 goes to zero. The shapes of the hysteresis loops depend on the frequency and the amplitude of the external magnetic field. For a fixed frequency, as the amplitude is increased the loops evolve from elliptical to rectangular, and finally to typical spindle-shaped hysteresis loops seen in laboratory systems. Most important, the results suggest that the area of the hysteresis loop scales with the frequency and amplitude of the external magnetic field. The scaling law proposed by Rao, Krishnamurthy, and Pundit is $A \approx H_0^{\alpha}\omega^{\beta}$ where H_0 and ω are the amplitude and frequency of the external magnetic field, respectively; and where for the $N\rightarrow\infty$ model they obtain $\alpha \approx 0.66 \pm 0.05$ and $\beta \approx 0.33 \pm 0.03$.

For the finite values of N which are experimentally relevant, an analysis similar to that of Rao, Krishnamurthy, and Pandit is difficult because the special simplifying features of the $N\rightarrow\infty$ limit are absent. In this paper, we report results obtained by us for hysteresis in the $N=1$, Ising, universality class using a "cell-dynamical simulation" which essentially solves a space and time discretized version of the full TDGL equation. Our discretization procedure is extremely efficient, enabling us to obtain data for hysteresis over almost three decades in frequency. Using this we have confirmed the scaling law proposed by Rao, Krishnamurthy, and Pandit with the values of the exponents being now given by $\alpha \approx 0.47 \pm 0.02$ and $\beta \approx 0.40 \pm 0.01$.

After our work was completed, a paper has appeared in which Lo and Pelcovits[6] have studied hysteresis using a conventional Monte Carlo simulation of the two-dimensional (2D) Ising model. Our values for α and β are in agreement with theirs, though their data covers less than a decade in frequency compared to ours which covers three.

II. THE CELL-DYNAMICAL SYSTEM

A cell-dynamical system (CDS)[7] consists of an array of cells each of which contains a continuous dynamical variable Φ_i where i is a site index. This variable evolves according to some rule which relates Φ_i at a discretized (integer) time $t+1$ to Φ_i (local part) and to Φ_j at neighboring sites j (nonlocal part) at a time t. The cell-dynamical system is thus a continuous version of cellular automata which have been used extensively to simulate nonequilibrium phenomena in a variety of systems.[8] The CDS we use for our hysteresis study, which is a generalization of the model used by Puri and Oono[7] to study domain growth for quenches into the two-phase region in a scalar $(\Phi^2)^2$ theory in the absence of any external field, is as follows.

We have a two-dimensional square array of cells with the following update rule for the variable Φ_i,

$$\Phi_i(t+1)=\theta_i(t)+D[\hat{\mathcal{L}}\theta_i(t)] , \quad (1a)$$

and where

$$\theta_i(t)=P\tanh[\Phi_i(t)]+H(t)+Q\eta_i(t) . \quad (1b)$$

Here $\eta_i(t)$ is a Gaussian noise with zero mean and unit standard deviation. The quantities P, D, and Q are parameters specifying the thermodynamic state of the system. $H(t)$ is the external periodic magnetic field of the form $H_0\sin(2\pi t/T)$ with amplitude H_0 and integer period T. The operator $\mathcal{L}$ in Eq. (1a) is essentially the isotropized discrete Laplacian[9] with the following definition for the 2D square lattice:

$$\hat{\mathcal{L}}\theta_i=\tfrac{1}{6}(\textstyle\sum \text{ of } \theta \text{ in nearest-neighbor cells}) + \tfrac{1}{12}(\textstyle\sum \text{ of } \theta \text{ in next-nearest-neighbor cells}) -\theta_i . \quad (2)$$

For every value of the external magnetic field $H(t)$, the corresponding magnetization $M(t)$ is obtained as the average of Φ over the whole lattice. Periodic boundary conditions are used throughout the calculation. It was shown by Puri and Oono[7] that the CDS described above is equivalent to a space and time discretized version of the standard TDGL equations for the Ising universality class. The parameters P, D, and Q of the CDS are related to the parameters of the Ising TDGL equations and the spatial and temporal grid sizes. However, a knowledge of their correspondence is not essential for understanding the asymptotic behavior of the system and for calculating universal quantities. Care must be taken, however, to ensure that the CDS does not introduce artifacts of its own which are unphysical. It was shown by Puri and Oono[7] that as long as the values of P and D lie within a so-called "stability" regime, artificial effects of intabilities due to the discretization scheme do not arise. We keep the values $P=1.3$, $D=0.5$, and $Q=0.5$ throughout, which are well within this stability range.

The CDS is extremely useful because a comparatively small number of cells can be used to mimic the effect of a large number of spins since we deal with a coarse-grained field. This means that fluctuations of the order parameter are effectively averaged and accurate values for thermodynamic variables can be obtained with ease.

III. RESULTS AND CONCLUSIONS

We have done extensive simulations using the CDS described above for 30×30, 50×50, and 100×100 cells. We have found that 30×30 cells are sufficient to produce extremely accurate estimates for the areas of the hysteresis loops. Increasing the number of cells beyond 900 does not significantly improve our results (this has to be compared with Monte Carlo studies on Ising spin systems where 50×50 spins[4] did not produce data accurate enough to extract scaling forms). We start the simulation with random values for the variables Φ in each of the cells. The external magnetic field is then applied for a few cycles without accumulating any values for the magnetization, in order to get rid of transients. After this we start accumulating values for the magnetization at each step of the field cycle for many complete cycles. This enables us at the end of the simulation to calculate an averaged (over many cycles) hysteresis loop which is further smoothened using a polynomial data-smoothening routine to obtain accurate values of the area under the M-H curve. We also keep track of the errors in the magnetization which are seen to be negligible except at the points where the magnetization changes somewhat abruptly, which does not affect our estimates for the area. We obtain hysteresis loops for values of $T=50$, 100, 200, 500, 1000, 4000, and 16 000 (almost three decades in T), and for several values of H_0 for every value of T. As a result of our calculations we come to the following conclusions.

(1) For every value of T checked by us there is a range of values of H_0 for which the average magnetization within a cycle M_0 fluctuates around a nonzero average value $\overline{M}_0$, and the hysteresis loop is restricted to the upper or lower half of the M-H plane (depending on the initial condition). As H_0 increases, M_0 starts to oscillate between positive and negative values and the hysteresis loop makes significant excursions from one-half of the M-H plane to another. For larger H_0, $\overline{M}_0$ obtained by averaging M_0 over many cycles is zero. It is interesting to note that this value of H_0 where the oscillations of M_0 start is fairly independent of system size, a fact which we checked by simulating 10×10, 50×50, and 100×100 lattices. Our result for this "transition" boundary is shown in Fig. 1 and corroborates the existing evidence for a

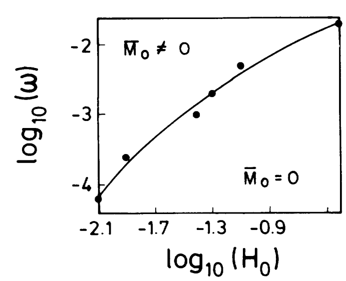

FIG. 1. Transition boundary between the dynamical states characterized by $\overline{M}_0\neq0$ and $\overline{M}_0=0$ in the H_0-ω plane. The black circles are obtained from our simulation data with the criterion $\overline{M}_0>0.001$, and the line is a guide to the eye. Note: All quantities plotted in Figs. 1–4 are dimensionless.

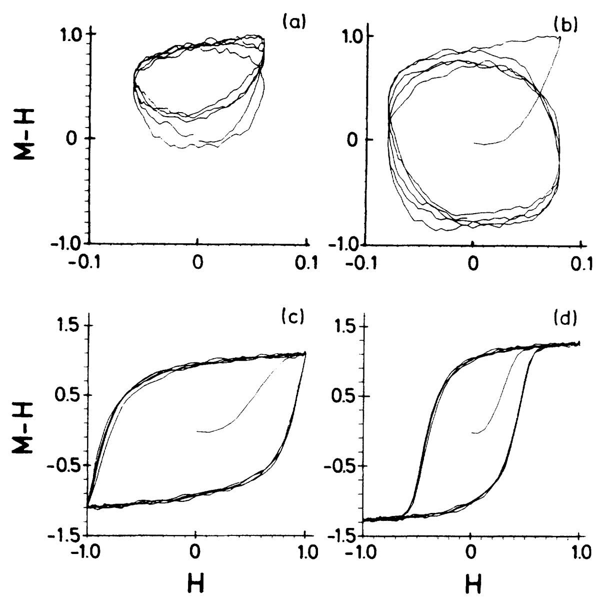

FIG. 2. (a)–(d) Typical hysteresis loops obtained for a 30×30 system with $T=100$ and $H_0=0.06$ (a), 0.08 (b), 0.20 (c), and 1.0 (d). In these figures we have plotted $M-H$ vs H since this quantity, rather than M alone, saturates for large H in our model where Φ_i is not bounded. Note that we have shown only the first few cycles for clarity. The data plotted in Figs. 4(a) and 4(b) was obtained by averaging over many such cycles; also the loops shown in (a) and (b) belong to the left of the transition boundary in Fig. 1 and were not considered in our analysis.

dynamical "transition"[4,6,10,11] in this system. The hysteresis loops change in shape as H_0 is increased for any value of T. The typical sequence of shapes we obtain in the region where $\overline{M}_0=0$ in Fig. 1 are shown in Figs. 2(a)–2(d). Elliptical hysteresis loops as obtained by Rao, Krishnamurthy, and Pandit[4] in the O(N) system with $N\rightarrow\infty$ were not seen by us in the range of T and H_0 examined.

(2) The area of the hysteresis loop A increases with increasing H_0 and T as shown in Fig. 3. A also scales with H_0 and $\omega=1/T$, in the form proposed by Rao and coworkers,[4] namely that it is a function of the single (scaling) variable $x=H_0^{\alpha}\omega^{\beta}$ with $\alpha=0.47\pm0.02$ and $\beta=0.40\pm0.01$. The scaling function can be well fitted to a quadratic form $ax+bx^2$ with the values $a=5.9\pm0.1$ and $b=30.7\pm0.9$. All error bars quoted above were calculated with 99% confidence limits. A scaling plot using the scaling function given above gives a near-complete collapse of all our data points as shown in Fig. 4(a) and 4(b). The exponents α and β are well within the error bars of the estimates for the same quantities obtained from the recent Monte Carlo calculations of Lo and Pelcovits.[6] This makes it clear that there is a dynamical universality class manifest in the magnetic hysteresis of spin systems, and that the CDS used by us belongs to the same universality class as an Ising model in two dimensions.

In summary, we have shown that our simple CDS can accurately and efficiently model the dynamics of an Ising magnet in an oscillating external magnetic field and obtain reliable results characterizing its hysteretic response. In future, we hope to extend our work to model thermal and magnetic hysteresis in ferroelectric systems[12] and hysteresis in systems with multicomponent order param-

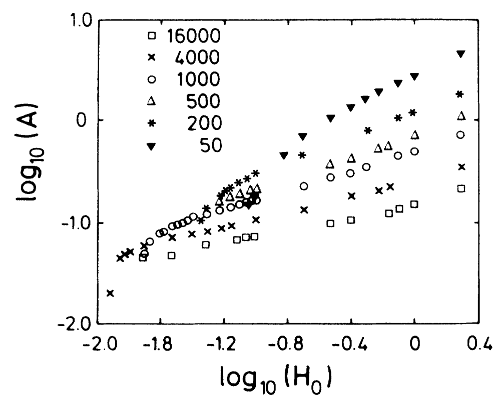

FIG. 3. Log-log plot of area of the hysteresis loops vs. H_0 for $T=50$, 200, 500, 1000, 4000, and 16,000. The linear regions of these curves are obtained for H_0 and ω well within the region characterized by $\overline{M}_0=0$ of Fig. 1.

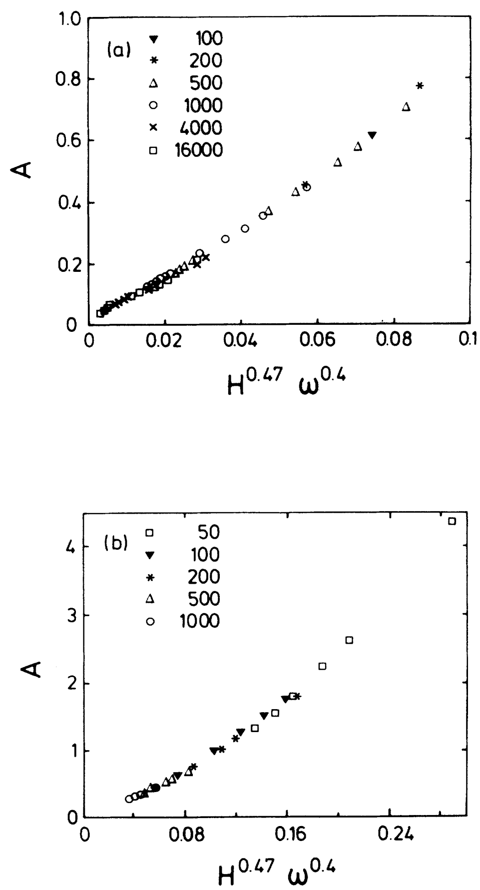

FIG. 4. (a) and (b) Area of the hysteresis loops vs $H_0^{0.47}\omega^{0.40}$. Note two different scales are used for (a) and (b) to cover data points with values of T ranging from 16000 to 50.

eters which are difficult to tackle using conventional Monte Carlo techniques.

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