DIFFRACTION OF LIGHT BY SUPERPOSED ULTRASONIC WAVES

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Received September 7, 1948

(Communicated by Prof. S. Bhagavantam, F.A.SC.)

1. Introduction

The well-known phenomenon of diffraction of light by ultrasonic waves, discovered by Debye and Sears,[1] and Lucas and Biquard,[2] has been satisfactorily explained by Raman and Nath[3] for normal incidence position and at low frequencies. The most important expression giving the diffraction angle is

$$\sin \theta = \pm n\lambda/\lambda^*, \tag{1}$$

where θ is the diffraction angle, n the order of diffraction, λ the wavelength of light in vacuum and λ^* the wavelength of sound in the medium. $J_n^2(v)$ gives the intensity of the nth order spectrum, $J_n(v)$ being the Bessel function of nth order and v being a parameter involving μ, L and λ. They have not, however, studied the nature of diffraction when two or more sound waves are superposed.

The nature of diffraction when two sound waves are present simultaneously in the medium has been experimentally investigated by Bergmann.[4] He found that, when the quartz crystal is excited at two different harmonics n_1 and n_2, spectra corresponding to $n_1 \pm n_2$ were obtained besides those due to frequencies n_1 and n_2. Later Bergmann used two different crystals for generating two incommensurate frequencies of sound waves and by sending light through both the waves, summation and difference lines are obtained as in the previous case. Subsequently Bergmann and Fues[5] have again taken up the investigation and obtained a number of combination lines besides the summation and difference lines. Similar combination lines have been reported by these authors in a glass block which is excited both for the longitudinal and transverse waves. A theoretical interpretation of these observations has been given by Fues.[5] Employing the theories of Raman and Nath and Van Cittart, Fues has deduced expressions for intensities of diffraction spectra obtained when light is sent through a medium in which a number of ultrasonic waves are set up. According to Fues, the angles of diffraction orders is given by

$$\sin \theta = \frac{r\lambda}{\lambda_1^*} + \frac{s\lambda}{\lambda_2^*} + \frac{u\lambda}{\lambda_3^*} + \ldots, \tag{2}$$

where $r, s, u, \ldots$ are integers, and the intensity of the order is given by

$$\pi J_r^2(\nu_1)\, J_s^2(\nu_2)\, J_u^2(\nu_3) \tag{3}$$

It is evident from this theory that when two sound waves of frequencies n_1 and n_2 are superposed, diffraction patterns corresponding to $rn_1 \pm sn_2$ will also be obtained and that the intensity of r, sth order is given by $J_r^2(\nu_1)J_s^2(\nu_2)$. It also follows that the diffraction pattern will be perfectly symmetrical about the central image. Similar combination spectra have also been reported by Govindarao.[6] Using two different crystals excited by separate oscillators, and immersed in xylol vertically at the two opposite ends of a square glass trough, he has been able to obtain a number of combination lines everyone of which corresponded to one of the values of $rn_1 \pm sn_2$.

With a view to explain the combination lines obtained by Hiedemann and Hosch[7] in solids, Nath[8] has investigated the optical diffraction effects due to two sound waves which pass in the same direction through a glass block. To illustrate the method and justify its applicability, he has first investigated the diffraction of light by two parallel longitudinal sound waves in a liquid. There is next considered the diffraction of light in a glass block due to two parallel sound waves, one of them longitudinal and the other transverse. Results obtained showed the existence of combination lines which are in agreement with the results of Hiedemann and Hosch. Similar results have been obtained by K. Nagabhushana Rao[9] for the intensity of diffraction orders in the case of N systems of sound waves using Raman and Nath's generalised theory.

The theoretical and experimental study of the nature of diffraction has hitherto been confined to superposition of sound waves of two incommensurate frequencies or frequencies in any odd ratio. The special case of superposition of sound waves of frequencies in any even ratio and in definite phase relationship has not been so far studied theoretically or experimentally.

2. Theoretical Investigation by the Author

While studying the variation of intensity of diffraction in different spectra with a view to verify Raman and Nath's theory, the author observed a peculiar feature. There was conspicuous asymmetry in diffraction for normal incidence which was found to be due to the oscillator generating its second harmonic strongly and consequently exciting the crystal at two different frequencies one of which is an octave of the other and also in the same phase. This result led the author to investigate the phenomenon both theoretically and experimentally, Asymmetry of diffraction of light is a natural consequence of the lack of symmetry in the light wavefront emerging from the medium. By a graphical construction of the resultant wavefront obtained by superposed ultrasonic waves, it is possible to see when asymmetry will be introduced in the diffracted light. Thus, for example, when two sound waves one of which is an octave of the other are superposed in phase, the resultant wavefront will not be symmetrical which consequently leads to asymmetry in the diffraction pattern.

An analytical treatment for the calculation of intensities of diffraction spectra for some special cases of superposed waves is given here using Raman and Nath's simplified theory. The notations employed here are the same as those used in Raman and Nath's earlier publication. Considering at first two sound waves of wavelengths λ^* and $\lambda^*/2$ set up in the medium with a phase difference of Δ so as to produce maximum variations of refractive indices of μ_1 and μ_2 respectively, the refractive index $\mu(x)$ in any layer at a distance x from the origin is given by

$$\mu(x) = \mu_0 + \mu_1 \sin \frac{2\pi}{\lambda^*} x + \mu_2 \sin \left(\frac{4\pi}{\lambda^*} x + \Delta\right),$$

where μ_0 is the refractive index of the medium in the undisturbed state. The amplitude integral for the intensity at a point, whose join with the origin has its x-direction cosine as 1 is given by

$$\int_{-P/2}^{+P/2} e^{2\pi i \left\{1x - \mu_1 L \sin \frac{2\pi}{\lambda^*} x - \mu_2 L \sin \left(\frac{4\pi}{\lambda^*} x + \Delta\right)/\lambda \right\}} dx \tag{4}$$

where p is the length of the sound beam along the x-axis. Putting

$$u = 2\pi/\lambda,\ b = 2\pi/\lambda^*,\ v_1 = 2\pi\mu_1 L/\lambda \text{ and } v_2 = 2\pi\mu_2 L/\lambda \tag{5}$$

the real and imaginary parts of the integral are

$$\int_{-P/2}^{+P/2} \{\cos ulx \cos [v_1 \sin bx + v_2 \sin (2bx + \Delta)]$$
$$+ \sin ulx \sin [v_1 \sin bx + v_2 \sin (2bx + \Delta)]\}\, dx \tag{6}$$

and

$$\int_{-P/2}^{+P/2} \{\sin ulx \cos [v_1 \sin bx + v_2 \sin (2bx + \Delta)]$$
$$- \cos ulx \sin [v_1 \sin bx + v_2 \sin (2bx + \Delta)]\}\, dx \tag{7}$$

Using the well-known Bessel expansions

$$\left.\begin{aligned}
\cos(v\sin bx) &= 2\sum_{0}^{\infty}{}' J_{2r}\cos 2r\,bx\\
\sin(v\sin bx) &= 2\sum_{0}^{\infty} J_{2s+1}\sin\overline{2s+1}\,bx\\
\cos(v\cos bx) &= 2\sum_{0}^{\infty}{}'(-1)^{r} J_{2r}\cos 2r\,bx\\
\sin(v\cos bx) &= 2\sum_{0}^{\infty}(-1)^{s} J_{2s+1}\cos\overline{2s+1}\,bx
\end{aligned}\right\} \qquad (8)$$

the integral of the real part is given by

$$\int_{-P/2}^{+P/2}\Big\{\cos ulx\Big[2\sum_{0}^{\infty}{}' J_{2r}(v_1)\cos 2r\,bx \times 2\sum_{0}^{\infty}{}' J_{2s}(v_2)\cos 2s\,\overline{2bx+\Delta}$$

$$-\,2\sum_{0}^{\infty} J_{2r+1}(v_1)\sin\overline{2r+1}\,bx \times 2\sum_{0}^{\infty} J_{2s+1}(v_2)\sin\overline{2s+1}\times\overline{2bx+\Delta}\Big]$$

$$+\sin ulx\Big[2\sum_{0}^{\infty} J_{2r+1}(v_1)\sin\overline{2r+1}\,bx \times 2\sum_{0}^{\infty}{}' J_{2s}(v_2)\cos 2s\,\overline{2bx+\Delta}$$

$$+\,2\sum_{0}^{\infty}{}' J_{2r}(v_1)\cos 2r\,bx \times 2\sum_{0}^{\infty} J_{2s+1}(v_2)\sin\overline{2s+1}\times\overline{2bx+\Delta}\Big]\Big\}\,dx \qquad (9)$$

It can be easily seen that only the first term in the coefficient of cos ulx and the second term in the coefficient of sin ulx will contribute towards the intensity of even orders and the remaining terms for the intensity of odd orders. The integral for the even term of order $2m$ is given by

$r \pm m$ being even

$$\int_{-P/2}^{+P/2}\Big\{2\cos ulx\Big[\cos(2m\,bx+\Delta.\overline{r+m})\sum_{\delta}^{\infty}{}'' J_{2r}(v_1)\,J_{r+m}(v_2)$$

$$+\cos(2m\,bx-\Delta.\overline{r-m})\sum_{\delta}^{\infty}{}'' J_{2r}(v_1)\,J_{r-m}(v_2)\Big]$$

$r \pm m$ being odd

$$+\,2\sin ulx\Big[\sin(2m\,bx+\Delta.\overline{r+m})\sum_{\delta}^{\infty} J_{2r}(v_1)\,J_{r+m}(v_2)$$

$$-\sin(2m\,bx-\Delta.\overline{r-m})\sum_{\delta}^{\infty} J_{2r}(v_1)\,J_{r-m}(v_2)\Big]\Big\}\,dx, \qquad (10)$$

where δ is the least possible integer. Double dash over the summation sign indicates that the coefficient of terms $J_0(v_1)\,J_m(v_2)$ is half that of others and of $J_0(v_1)\,J_0(v_2)$ is one-fourth that of others.

Considering odd terms of $2m + 1$ order we have

$$r + m + 1 \text{ and } r - m \text{ being odd}$$

$$\int_{-P/2}^{+P/2} \{2 \cos ulx \, [\cos (\overline{2m+1}\, bx + \Delta . \overline{r+m+1}) \sum_{\delta}^{\infty} J_{2r+1}(v_1) \, J_{r+m+1}(v_2)$$

$$+ \cos (\overline{2m+1} . bx - \Delta . \overline{r-m}) \sum_{\delta}^{\infty} J_{2r+1}(v_1) \, J_{r-m}(v_2)]$$

$$r + m + 1 \text{ and } r - m \text{ being even.}$$

$$+ 2 \sin ulx \, [\sin (\overline{2m+1}\, bx + \Delta \, \overline{r+m+1}) \sum_{\delta}^{\infty} J_{2r+1}(v_1) \, J_{r+m+1}(v_2)$$

$$- \sin (\overline{2m+1}\, bx - \Delta . \overline{r-m}) \sum_{\delta}^{\infty} J_{2r+1}(v_1) \, J_{r-m}(v_2)]\} \, dx \qquad (11)$$

Special cases can be derived from these two integrals by substituting different values for the phase difference Δ.

Case I.—When $\Delta = 0$, we have, for even orders, the integral

$$r \pm m \text{ being even}$$

$$\int_{-P/2}^{+P/2} \{2 \cos ulx \cos 2m \, bx \, [\sum_{\delta}^{\infty} J_{2r}(v_1) \, J_{r+m}(v_2) + \sum_{\delta}^{\infty} J_{2r}(v_1) \, J_{r-m}(v_2)$$

$$r \pm m \text{ being odd}$$

$$+ 2 \sin ulx \sin 2m \, bx \, [\sum_{\delta}^{\infty} J_{2r}(v_1) \, J_{r+m}(v_2) - \sum_{\delta}^{\infty} J_{2r}(v_1) \, J_{r-m}(v_2)]\} \, dx \quad (12)$$

Denoting terms in square brackets by $(a + b)/2$ and $(a - b)/2$ respectively, we have

$$\int_{-P/2}^{+P/2} [a \cos (ul - 2mb) \, x + b \cos (ul + 2mb) \, x] \, dx$$

$$= P \left[a \frac{\sin (ul - 2mb) \, P/2}{(ul - 2mb) \, P/2} + b \frac{\sin (ul + 2mb) \, P/2}{(ul + 2mb) \, P/2} \right]$$

which gives the intensity of $+ 2m$ order as proportional to a^2 and of $- 2m$ order to b^2. The values of a and b given by the relations

$$\left.\begin{aligned} a &= \sum_{0}^{\infty} J_{2r}(v_1) \, J_{r+m}(v_2) + (-1)^{r-m} \sum_{0}^{\infty} J_{2r}(v_1) \, J_{r-m}(v_2) \\ b &= (-1)^{r+m} \sum_{0}^{\infty} J_{2r}(v_1) \, J_{r+m}(v_2) + \sum_{0}^{\infty} J_{2r}(v_1) \, J_{r-m}(v_2) \end{aligned}\right\} \qquad (13)$$

are different, giving different values for the intensity of $\pm 2m$ orders and thus introducing asymmetry in the pattern. Similarly for the odd orders we have expressions giving asymmetry of the pattern, the intensity of $2m+1$ order being proportional to $(a')^2$ and of $-(2m+1)$ order to $(b')^2$ where

$$a' = \sum_0^\infty J_{2r+1}(v_1)\, J_{r+m+1}(v_2) + (-1)^{r-m+1} \sum_0^\infty J_{2r+1}(v_1)\, J_{r-m}(v_2)$$

$$b' = (-1)^{r+m} \sum_0^\infty J_{2r+1}(v_1)\, J_{r+m+1}(v_2) + \sum_0^\infty J_{2r+1}(v_1)\, J_{r-m}(v_2) \qquad (14)$$

Thus for odd orders as well the pattern is asymmetric. The integral of the imaginary term for this case is zero and hence need not be taken into consideration.

Case II.—When $\Delta = \pi$, it can be easily seen by substituting this value for Δ in (10) and integrating that the integral will be same as (12) except for a sign change for the second term. This change of sign leads to the intensity of $+2m$ order being proportional to b^2 and of $-2m$ order to a^2. Similarly for the odd order spectra the intensity of $2m+1$ order comes out as proportional to $(b')^2$ and of $-(2m+1)$ order to $(a')^2$. Thus the intensities of diffraction spectra for this case are the same as those of Case I except that they are reversed, the intensities of positive orders in Case I becoming intensities of negative orders in Case II and so on. This result can be easily obtained from a geometrical construction of the resultant wavefront of the superposed waves.

Case III.—When $\Delta = \frac{\pi}{2}$, the integral of second term in (8) vanishes and hence we have the integral

$r \pm m$ being even

$$\int_{-P/2}^{+P/2} \{2 \cos ulx \cos 2m\, bx\, [(-1)^{r+m/2} \sum_\delta^\infty{}'' J_{2r}(v_1)\, J_{r+m}(v_2)$$
$$+ (-1)^{r-m/2} \sum_\delta^\infty{}'' J_{2r}(v_1)\, J_{r-m}(v_2)]\}\, dx \qquad (15)$$

which leads to the same contribution to both the $\pm 2m$ orders, thus maintaining symmetry in the pattern. But for this case the imaginary part is found to give some contribution which is found not to effect the symmetry of the pattern. Thus the case when the ratio of wavelengths is 1 : 2 is treated here for three different phase relations and found that the distribution of intensity of different spectra is different in each case.

It can be shown in a general way that similar results will follow when any even ratio of wavelengths is taken in consideration. Considering two sound waves in phase it can be easily shown that, when the ratio of wavelengths is even, the diffraction pattern will be asymmetric whereas, when they are in any odd ratio, there will be perfect symmetry in the pattern. Let the wavelengths of the sound waves, which are in phase be λ^* and λ^*/k. Following the usual method, the amplitude integral becomes

$$\int_{-P/2}^{+P/2} \{\cos ulx\, [2\sum_0^{\infty}{}' J_{2r}(v_1) \cos 2r\, bx \times 2\sum_0^{\infty}{}' J_{2s}(v_2) \cos 2s\, kbx$$

$$- 2\sum_0^{\infty} J_{2r+1}(v_1) \sin \overline{2r+1}\, bx \times 2\sum_0^{\infty} J_{2s+1}(v_2) \sin \overline{2s+1}\, kbx$$

$$+ \sin ulx\, [2\sum_0^{\infty} J_{2r+1}(v_1) \sin \overline{2r+1}\, bx \times 2\sum_0^{\infty}{}' J_{2s}(v_2) \cos 2s\, kbx$$

$$- 2\sum_0^{\infty}{}' J_{2r}(v_1) \cos 2r\, bx \times 2\sum_0^{\infty} J_{2s+1}(v_2) \sin \overline{2s+1}\, kbx]\}\, dx \qquad (16)$$

For even orders the first terms in coefficient of cos ulx and second term in coefficient of sin ulx will contribute. This leads to different expressions for the intensities of $+2m$ and $-2m$ orders. It is the remaining terms which contribute towards the intensity of odd orders and give likewise different values for the intensities of $\pm(2m+1)$ orders. Thus the pattern will on the whole be asymmetric for any even ratio of wavelengths of sound waves superposed in phase. When the value of k is odd all even terms are obtained from the coefficients of cos ulx giving same intensity for both $\pm 2m$ orders. Similarly the coefficients of sin ulx contribute towards the intensity of odd orders and give the same value for $\pm(2m+1)$ orders. Thus perfect symmetry in the pattern is maintained when the ratio of wavelengths of the sound waves is odd.

The special case when the ratio of wavelengths is 1 : 3 can be obtained be substituting $k = 3$ in (16). The treatment for this case can easily be extended for different phase relations.

3. Calculations and Discussion of the Theoretical Results

With a view to verify the theoretical conclusions, and also to show the nature and magnitude of the asymmetry introduced, intensities of diffraction spectra are calculated for Case I using relations (13) and (14). Table I gives the intensities for different values of the parameters v_1 and v_2. The relative intensities of various orders are represented in Fig. 1 to indicate the nature and extent of asymmetry introduced for different values of v_2 keeping value of v_1 constant. The test for the calculation of the intensities is to be found in the fact that the sum of the intensities of all the spectra is unity as shown in Table I, accounting for the total intensity of the incident light. Though the calculation of the intensities using (13) and (14) involve a summation of infinite series of products of Bessel functions, it is made easy due

TABLE I. $\nu_1 = 1$

I \ ν_2	0	1	2	3
I_0	0·586	0·344	0·030	0·039
I_{+1}	0·194	0·020	0·030	0·078
I_{-1}	0·194	0·299	0·119	0·001
I_{+2}	0·013	0·191	0·257	0·082
I_{-2}	0·013	0·055	0·140	0·055
I_{+3}		0·025	0·100	0·006
I_{-3}		0·053	0·161	0·131
I_{+4}		0·003	0·124	0·199
I_{-4}		0·002	0·036	0·088
I_{+5}			0·012	0·007
I_{-5}			0·040	0·117
I_{+6}			0·021	0·094
I_{-6}			0·007	0·027
I_{+7}			0·002	0·008
I_{-7}			0·004	0·034
I_{+8}			0·002	0·020
I_{-8}				0·004
ΣI	1·000	0·992	0·991	0·990

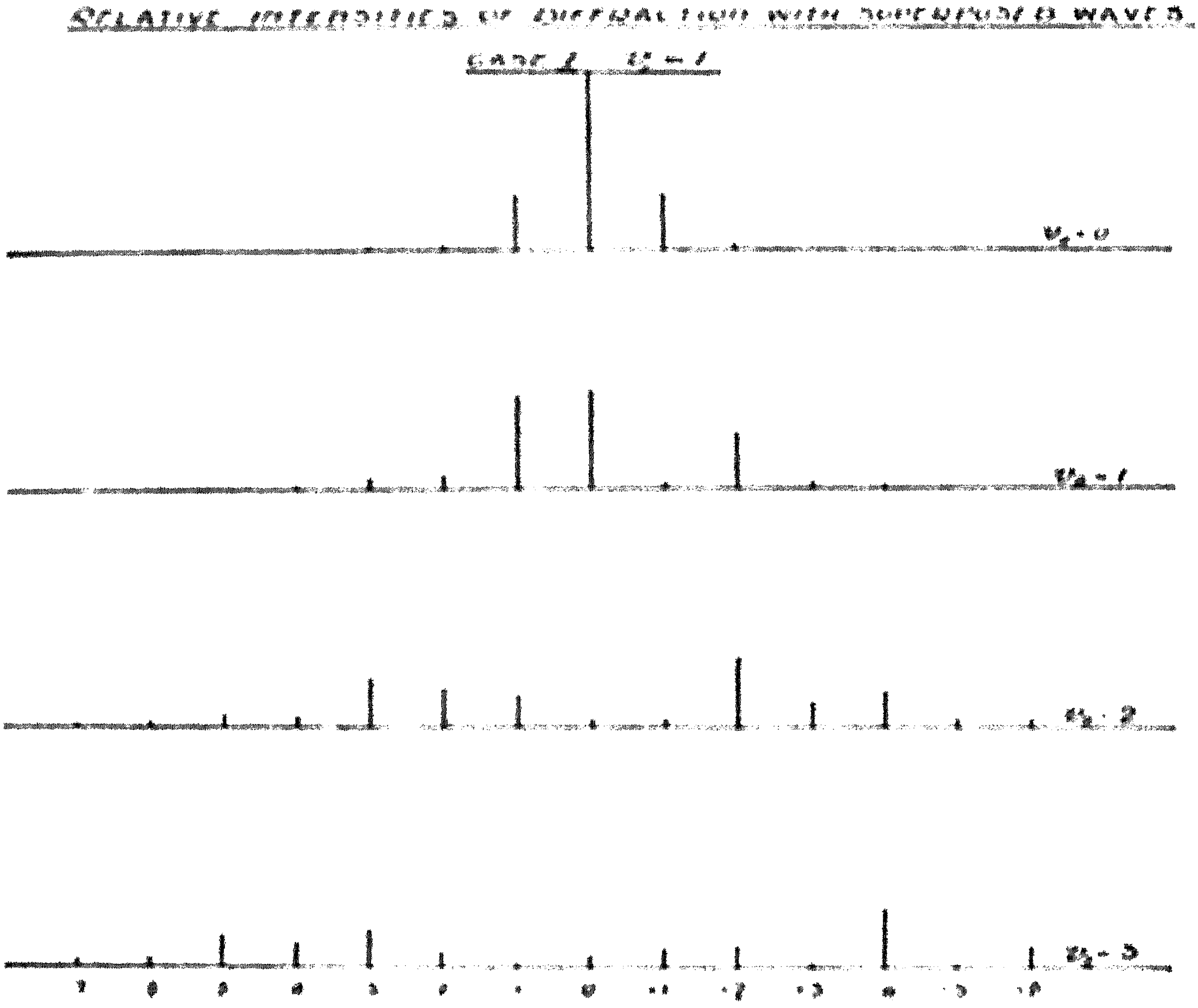


FIG. 1

to the fact that they are very rapidly convergent series so that, in most of the cases, only the first two terms of the series need be taken into account.

A remarkable feature that comes out from the calculations is that for lower values of v_1 and v_2 in general the odd order diffraction spectra are stronger on the negative side while the even orders are stronger on the positive side of the central image. Thus the -1 order is stronger than the $+1$ order, the $+2$ order is stronger than -2 order, and so on. The nature of the asymmetry introduced in this case is thus entirely different from the oblique incidence asymmetry in which most of the light will be thrown on one side of the central image making the orders on the appropriate side stronger than the corresponding orders ont the other side.

In conclusion, it may be pointed out that the same type of results can also be obtained from the rigorous theory of Raman and Nath. The theory can also be generalised to the case of diffraction of light by special types of waves like the saw-tooth wave, the diffraction effect of which can be obtained by considering it as a superposition of a number of sine waves with frequencies in integral ratios and in phase.

4. Experimental Details

With a view to verify the theoretical conclusions qualitatively, the experimental investigation is taken up by the author, the study being limited to Case I involving two sound waves with frequencies in ratio 1 : 2 and in phase. While all the previous investigators employed stationary waves it is essential for this investigation to use progressive sound waves to obtain the results predicted by the theory.

For setting up progressive sound waves in water, a long tank of $1\frac{1}{2}$ metres height is made with tin sheet and is provided with two glass windows at the top to allow the passage of light. The bottom of the tank is stuffed with felt and glass wool preventing the reflection of the sound wave and thus giving an ideal progressive sound wave. The arrangement for obtaining and recording diffraction spectra is the usual one similar to that of Debye and Sears.

Radio waves which are in phase and one of which is an octave of the other are generated by employing two different types of harmonic generator circuits. Both types of circuits are employed for obtaining the diffraction spectra but the one that is most convenient and powerful is shown in Fig. 2. The circuit employs an oscillator constructed with 813 valve whose output is fed into a regular harmonic generator circuit. The harmonic generator employs a Taylor T55 valve with a tank circuit, tuned to the second harmonic

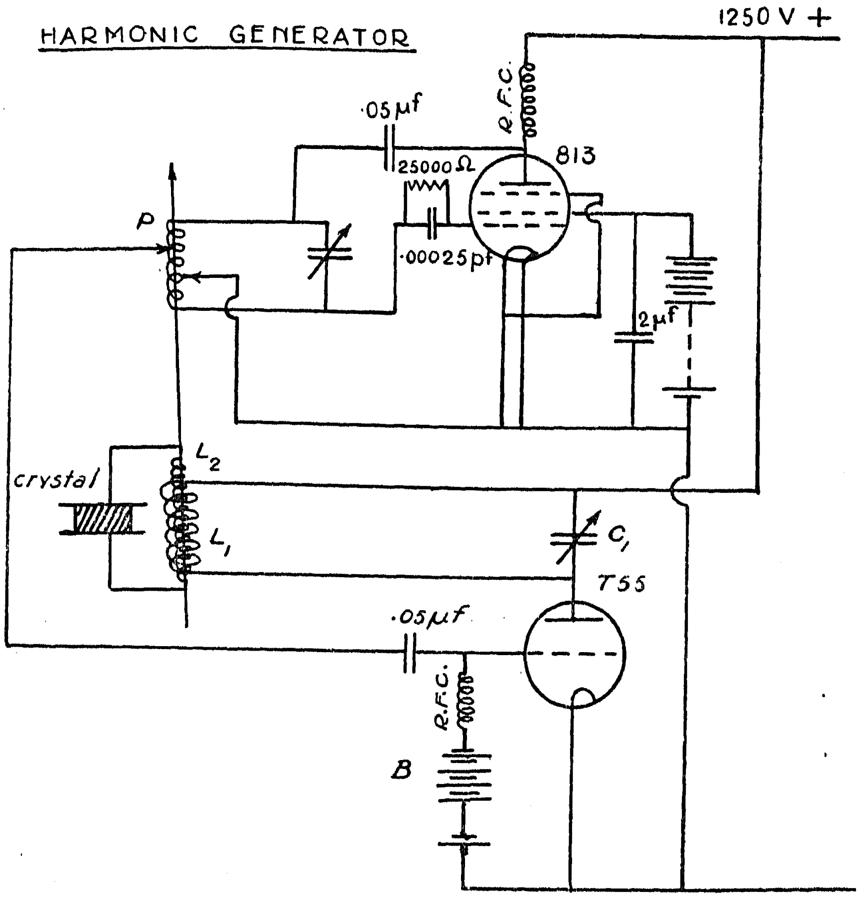


FIG. 2

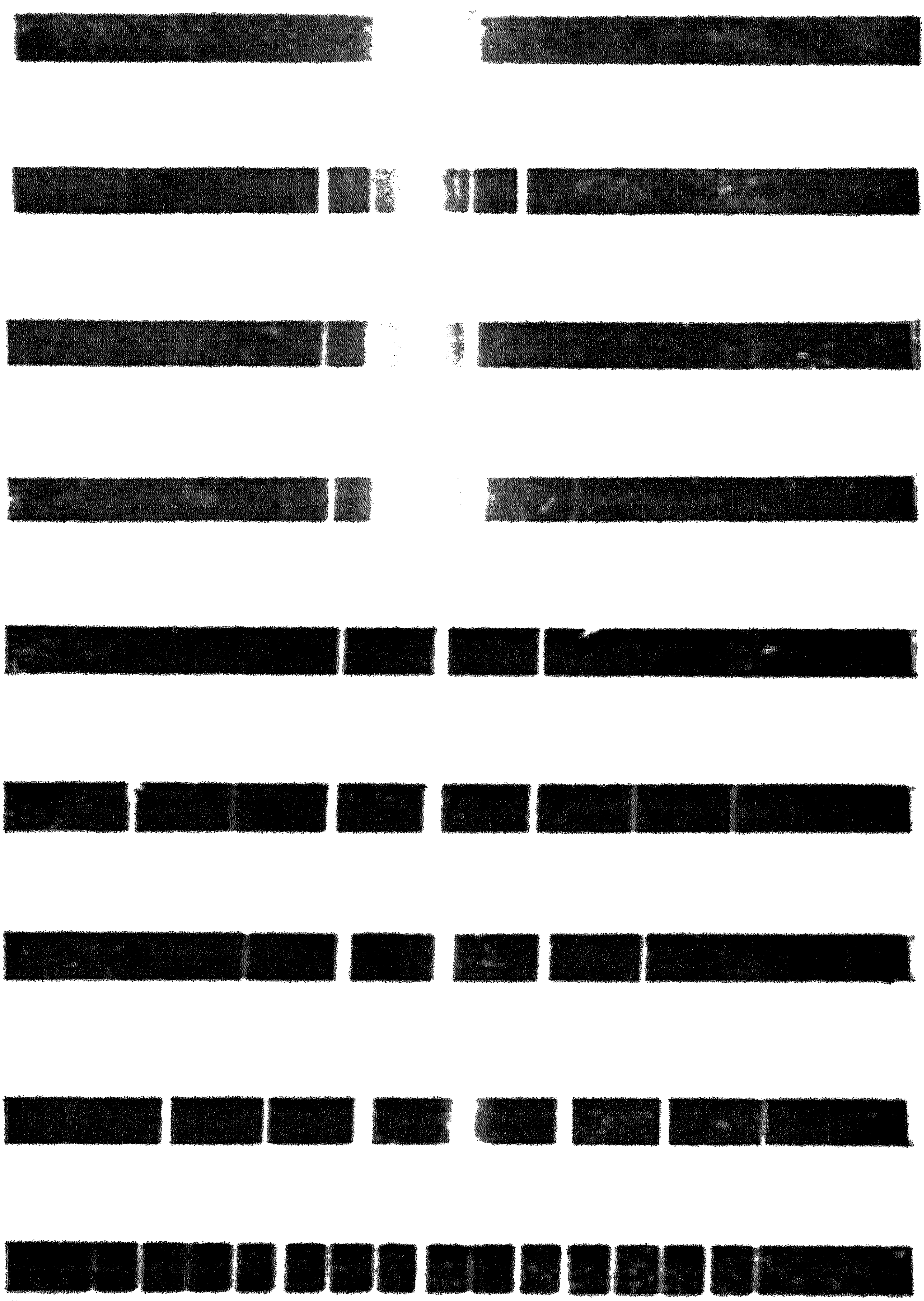
of the oscillator, as the load impedance and with a variable grid bias B obtained from a 500 volt power supply unit. The bias B and the tapping point P are adjusted to generate the second harmonic of the oscillator powerfully. Coupled to inductance coil L_1 of the harmonic generator there is another coil L_2 which is connected across the crystal. The coil L_2 being coupled to both to the oscillator and the harmonic generator draws energy from both. With this type of circuit it has been possible to obtain diffraction spectra upto a large number of orders. A quartz *x*-cut plate of thickness 2·5 mm. and a fundamental frequency of about 1·2 Mcs. is employed for the investigation. The quartz plate which is silvered uniformly and nickel plated on both its surfaces is coupled to the circuit as shown in Fig. 2.

5. Discussion of the Photographs of Diffraction Spectra

Diffraction spectra obtained for different frequencies under varying excitation conditions are shown in Plate I. Plate I (*a*) and (*b*) show patterns obtained with sound waves of frequencies 3 Mcs. and 6 Mcs. respectively. The patterns obtained by combining these two waves in phase are shown in Plate I (*c*) and (*d*). Plate I (*c*) brings out the asymmetry obtained by superposition very clearly, the 2 order being obtained very strongly compared with the -2 order and so on. Plate I (*d*) is obtained with different power of excitation of the crystal for the second harmonic and shows conspicuous asymmetry. Exciting the crystal at 6 Mcs. and its second harmonic at 12 Mcs. Plate I (*e*) to (*h*) are obtained. In all these cases it has not been possible to isolate the pattern for 12 Mcs. sound wave. In Plates I (*e*) and (*f*) obtained with loose coupling between the crystal circuit and the harmonic generator, (*e*) corresponds to 6 Mcs. sound wave and (*f*) corresponds to patterns obtained with superposed sound waves showing marked asymmetry. Patterns obtained at the same frequencies but with increased power of excitation of the crystal, are shown in Plate I (*g*) and (*h*). Though the total intensity is increased, the asymmetry is still maintained. The alternating intensities of odd and even orders, pointed out in the previous theoretical investigation, is clearly seen in both the Plate I (*f*) and (*h*). In Plate I (*h*), for example, the -1 order is stronger than $+1$ order, the $+2$ order is stronger than -2 order and so on. It must be noticed that this type of asymmetry is different from that of the asymmetry introduced by tilting where the diffracted light will be thrown on one side making each order on the appropriate side stronger than the corresponding order on the other side. With a view to show that no asymmetry will be introduced when two sound waves with frequencies in the ratio 1 : 3 are superposed, Plate I (*i*) is obtained with 6 Mcs. and 18 Mcs. sound waves. The harmonic generator is biased to give the third harmonic very strongly and the tank circuit is tuned to the third harmonic of the oscillator circuit. Pattern thus obtained showed a number of combination lines on either side of the central image without any loss in the symmetry of the pattern. Thus in a general qualitative manner the results predicted by the theoretical investigation are experimentally confirmed.

6. Summary

Employing Raman and Nath's simplified theory, the author has investigated the diffraction effects that are to be expected when the ultrasonic wave is obtained by a combination of two sound waves with frequencies in 1 : 2 ratio superposed in different phases. Intensities of individual diffraction orders have been calculated from expressions derived for the case of two sound waves of frequencies in 1 : 2 ratio superposed in phase. The type of asymmetry introduced in this case is found to be different from the oblique incidence asymmetry. The treatment is also generalised to the case of superposition of two sound waves with frequencies in any odd or even integral ratios. The theoretical results have been experimentally confirmed in a qualitative manner by employing special type of harmonic generator circuits.



In conclusion, the author wishes to express his grateful thanks to Prof. S. Bhagavantam for the keen interest he has shown during the progress of the work.

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