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Neutral-charged particle correlations in proton-proton collisions in the framework of a two-component model

V GUPTA* and P K MALHOTRA
Tata Institute of Fundamental Research, Bombay 400005

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Abstract. The data on $\bar{N}_0(n_-)$, the average number of neutrals as a function of n_-, the number of negatively charged particles produced, is fitted at 69, 205 and 303 GeV/c. The two-component model used for the charged multiplicity distribution P_n, is one which envisages two distinct types of collisions and is the simplest such model consistent with charge conservation. We find that the data on $\bar{N}_0(n_-)$ can be fitted reasonably well. Further, our results, based on this model for P_n, suggest that at 50 GeV/c, $\bar{N}_0(n_-)$ should increase linearly with n_- and that neutral-negative correlations should be present in the central component.

Keywords. Neutral-charge correlations; pp-collisions; two-component model of multiparticle production.

1. Introduction

Recently many attempts have been made to understand the multiplicity distribution of charged particles produced in high energy pp-collisions in terms of two-component models. The two components may correspond to two types of collisions (Nielsen and Olesen 1973, Harari and Rabinovici 1973, Van Hove 1973, Fialkowski and Miettinen 1973) or both the components may be present in varying proportions in each event (Chaudhary *et al* 1974). In this note we will study neutral and charged particle correlations in the framework of two component models in which two different types of collisions are envisaged. As is the common practice, the two types of collisions, for convenience, will be referred to as the diffractive and central (*i.e.* non-diffractive or pionization) components.

The only data on neutrals which are available at various energies, for pp-collisions, is on $\bar{N}_0(n_-)$, the average number of neutrals (actually neutral pions) for a given number n_- of negatively charged particles. The expression for $\bar{N}_0(n_-)$ depends on the neutral-negative correlation and on the charged multiplicity distribution, P_n, *i.e.* the probability for producing n charged particles. Clearly, it is necessary to have a model for P_n to study $\bar{N}_0(n_-)$. The two-component models given below were considered recently by Chaudhary and Gupta (1974).

Model I. The simplest two-component form consistent with charge conservation constraints for the probability P_n, for having n charged particles is given by

* Present Address: Theoretical Physics Division, CERN, 1211 Geneva-23, Switzerland.

$$P_n = p\,\frac{\exp(-d_{01})\,(d_{01})^{n_-}}{n_-!} + q\,\frac{\exp(-C_{01})\,(C_{01})^{n_-}}{n_-!}, \qquad p+q=1 \tag{1}$$

where $n_- = \frac{1}{2}(n - Q_{in})$ is the number of negatively charged particles produced and Q_{in} = total incoming charge. For pp-collisions $Q_{in} = 2$. Further p and q are the probabilities for the occurrence of diffractive and central type of collisions respectively. d_{01} and C_{01} are the average multiplicities of the negatively charged particles in the diffractive and central components. In this model there are no correlations present among the negatively charged particles in either of the components.

Model II. The next simplest form consistent with charge conservation gives

$$P_n = p\exp\left(-d_{01}+\frac{1}{2}d_{02}\right)\sum_r \frac{(d_{01}-d_{02})^{n_- - 2r}}{n_- - 2r\,!\,r\,!}\left(\frac{d_{02}}{2}\right)^r$$
$$+ (p \to q,\; d_{01}\to C_{01},\; d_{02}\to C_{02}) \tag{2}$$

where sum over r goes from $r = 0, 1, \ldots. n_-/2$ for even n_- and $r = 0, 1, \ldots.$ $(n_- - 1)/2$ for odd n_-. In this model d_{02} and C_{02} are the correlation integrals for two negatively charged particles in the diffractive and central components. If one puts $d_{02} = C_{02} = 0$ in eq. (2) then it reduces to eq. (1).

Model I has been considered earlier by Lach and Malamud (1973) and Rama Rao (1973) with fair success for pp-collisions from 50 to 303 GeV/c. Model II was considered by Chaudhary and Gupta (1974) and compared with model I for the energy range 50 to 405 GeV/c. It was found that model I gives an adequate description up to 303 GeV/c and that only for 405 GeV and upwards model II seems to give some improvement owing to non-zero d_{02} and C_{02}. So for the purposes of the study of neutral and charged particle correlations through $\bar{N}_0(n_-)$ we will take the charged multiplicity distribution to be given by eq. (1) for 50 GeV/c upwards.

In section 2 we introduce the generating function for the multiplicity distribution of neutrals and charged particles and derive the relevant formulae for $\bar{N}_0(n_-)$ and discuss some general consequences of various types of neutral-charge particle correlations for $\bar{N}_0(n_-)$. In section 3 we discuss the various fits to the data on $\bar{N}_0(n_-)$ for pp-collisions at 69, 205 and 303 GeV/c. We end with a summary of our results and conclusions.

2. Multiplicity distribution of neutral and charged particles

In discussing the multiplicity distribution and correlation integrals it is convenient to consider the generating function $G(h_+, h_-, h_0)$, where the variables h_+, h_- and h_0 are the generators associated with the positive, negative and neutral particles respectively. The most general form of G, incorporating the charge conservation constraints (Webber 1972) is

$$G(h_+, h_-, h_0)$$
$$= (1+h_+)^{Q_{in}}\exp\left[\sum_{b,c}\frac{f_{0bc}}{b!\,c!}(h_+ + h_- + h_+h_-)^b\,h_0^c\right] \tag{3}$$

where f_{abc} is the correlation integral for '*a*' positive, '*b*' negative and '*c*' neutral particles. Clearly, f_{ab0} are identical with the f_{ab} considered by Chaudhary and Gupta (1974) for the charged multiplicity distribution problem. Since we will be interested in the negative and neutral particles, we only need

$$G(0,\ h_-,\ h_0) \equiv G(h_-,\ h_0) = \exp\left[\sum_{b,c} \frac{f_{0bc}}{b!\,c!} h_-^b h_0^c\right] \tag{4}$$

Note that $f_{010} = \langle n_- \rangle$ and $f_{001} = \langle n_0 \rangle$ are the average number of negative and neutral particles, while f_{0bc} with $b,\ c \geq 1$ will introduce correlations among them. Further, eqs (3) and (4) also serve to define f_{0bc}. The probability of having n_- negative and n_0 neutral particles

$$P(n_-,\ n_0) = \frac{1}{n_-!}\frac{1}{n_0!}\left(\frac{\partial}{\partial h_-}\right)^{n_-}\left(\frac{\partial}{\partial h_0}\right)^{n_0} G(h_-,\ h_0)\Big|_{h_- = h_0 = -1} \tag{5}$$

Further the probability for producing n_- negative particles $P(n_-) = \sum_{n_0} P(n_-,\ n_0)$ and similarly the probability for producing n_0 neutral particles $P(n_0) = \sum_{n_-} P(n_-,\ n_0)$. Note that since $n_- = \frac{1}{2}(n - Q_{in})$, one has $P_n = P(n_-)$ for $Q_{in} \geq 0$. By definition,

$$\bar{N}_0(n_-) = \frac{\sum_{n_0} n_0 P(n_-,\ n_0)}{\sum_{n_0} P(n_-,\ n_0)},$$

or

$$P(n_-)\,\bar{N}_0(n_-) = \frac{1}{n_-!}\left(\frac{\partial}{\partial h_-}\right)^{n_-}\frac{\partial}{\partial h_0} G(h_-,\ h_0)\Big|_{h_0 = 0,\ h_- = -1} \tag{6}$$

The following general though obvious points should be noted:

(*a*) The presence of f_{00c}, $c > 1$, *i.e.* neutral-neutral correlations, do not affect $\bar{N}_0(n_-)$ but only the probability $P(n_0)$. One may keep such f_{00c} as required so that $P(n_0)$ is of the same form as $P(n_-)$.

(*b*) If only f_{0b0} and f_{00c} are non-zero, that is no neutral and negative particle correlations are present, then $P(n_-,\ n_0)$ factorizes into $P(n_-)\,P(n_0)$ giving

$$N_0(n_-) = f_{001} = \langle n_0 \rangle,$$

a constant independent of n_-.

(*c*) The presence of f_{0bc} with $b \geq 1$ but $c \geq 2$ do not affect the expression for $\bar{N}_0(n_-)$ since the expression for $\bar{N}_0(n_-)$ only involves the first derivative with respect to h_0.

(*d*) The simplest dependence on n_- is obtained if $f_{011} \neq 0$, while all other $f_{0b1} = 0,\ b \geq 2$. This gives

$$\bar{N}_0(n_-) = (f_{001} - f_{011}) + f_{011}\frac{P(n_- - 1)}{P(n_-)}, \tag{7}$$

requiring a minimum of two parameters. If $P(n_-)$ was a Poisson in n_- then this would give

$$\bar{N}_0(n_-) = A + Bn_- \tag{8}$$

where it is clear that $A = f_{001} - f_{011}$ and $B = f_{011}/\langle n_- \rangle$, in general, are functions of the energy.

Since the two-component models have had success in explaining the charge multiplicity distribution we consider the expression for $\bar{N}_0(n_-)$ for such models. For a two-component model, $G(h_-, h_0)$ will also be given by (charge being conserved separately in each component)

$$G(h_-, h_0) = p \exp\left[\sum_{b,c} \frac{d_{0bc} h_-^b h_0^c}{b!\, c!}\right] + q \exp\left[\sum_{b,c} \frac{C_{0bc}}{b!\, c!} h_-^b h_0^c\right],$$
$$p + q = 1 \qquad (9)$$

where p and q are the relative probabilities for the diffractive and central components. Further, d_{0bc} and C_{0bc} are the correlation integrals for 'b' negative and 'c' neutral particles for the diffractive and central components respectively. Clearly d_{0b0} and C_{0b0}, $b \geq 1$ are to be identified with d_{0b} and C_{0b} used in eqs (1) and (2) above.

The question arises as to which d_{0bc} and C_{0bc} to retain in eq. (9). Clearly one has to keep d_{0b0} and C_{0b0} up to $b \leq 2$ to be able to obtain $P(n_-)$ as given by eqs (1) and (2) for the models I and II for the charged multiplicity distribution. For simplicity we take only d_{011} and C_{011} as non-zero and assume that the other correlations in each component vanish. Of course the average multiplicities of neutrals d_{001} and C_{001}, in the two components, have to be non-zero. Using eqs (6) and (9) one then obtains

$$P(n_-)\bar{N}_0(n_-) = [pa_1 P_d(n_-) + qa_2 P_c(n_-)] + [pb_1 P_d(n_- - 1) + qb_2 P_c(n_- - 1)] \qquad (10)$$

where

$$a_1 \equiv d_{001} - d_{011}, \quad a_2 \equiv C_{001} - C_{011}; \quad b_1 \equiv d_{011}, \quad b_2 \equiv C_{011} \qquad (11)$$

Further, $P_d(n_-)$ and $P_c(n_-)$ denote the probabilities for producing n_- negative particles in the diffractive and central components respectively, so that

$$P(n_-) = pP_d(n_-) + qP_c(n_-) \qquad (12)$$

Since the fit to the charged multiplicity distribution determines $P_d(n_-)$, $P_c(n_-)$ and $P(n_-)$, the $\bar{N}_0(n_-)$ in eq. (10) depends on four parameters. It is clearly desirable to reduce the number of parameters before confronting eq. (10) with data. In this connection it should be noted that one has to have a minimum of two parameters since d_{001} and C_{001} are non-zero. Before we discuss the various possible reduction of parameters in eq. (10), we note that for a two-component model all the correlation integrals f_{0bc}, $b, c \geq 1$ turn out to be non-zero in general. In fact one finds from eq. (9) that

$$f_{011} = pb_1 + qb_2 + pqA_1N_1 \qquad (13\,a)$$
$$f_{021} = pq(q-p)A_1^2 N_1 + pq(A_2N_1 + 2A_1M) \qquad (13\,b)$$
$$f_{012} = pq(q-p)A_1N_1^2 + pq(A_1N_2 + 2N_1M) \qquad (13\,c)$$

etc., . . . where

$$\left.\begin{aligned} A_1 &\equiv d_{010} - C_{010},\ A_2 \equiv d_{020} - C_{020}, \\ N_1 &\equiv d_{001} - C_{001},\ N_2 \equiv d_{002} - C_{002},\ M \equiv d_{011} - C_{011} \end{aligned}\right\} \tag{14}$$

As stated in the introduction for the $P(n_-)$ we will use the fit given by the two-component model of eq. (1). In this case eq. (10) becomes

$$\bar{N}_0(n_-) = \frac{a_1 R(n_-) + a_2}{R(n_-) + 1} + \left[\frac{b_1 R(n_- - 1) + b_2}{R(n_-) + 1}\right]\frac{n_-}{C_{010}} \tag{15}$$

where

$$R(n_-) \equiv (p/q)\frac{P_d(n_-)}{P_c(n_-)} \tag{16}$$

Also note that in the notation of section 2, $d_{010} \equiv d_{01}$ and $C_{010} \equiv C_{01}$. We consider the following cases of eq. (15) with two and three parameters:

Fit A: A two parameter fit with $b_1 = b_2 = 0$. This is the simplest possibility in the sense that the neutral-negative correlations are absent in both the components. However, all the f_{0bc} are non-zero as can be seen from eqs (13) and (14). The dependence of $\bar{N}_0$ on n_- comes through $R(n_-)$ alone.

Fit B: A two parameter fit with $N_1 = N_2 = M = 0$, *i.e.* $a_1 = a_2 \neq 0$ and $b_1 = b_2 \neq 0$. This means the same multiplicity of neutrals and the same negative neutral correlations in each component. This case is interesting as eq. (15) or (10) reduces to eq. (7) with $f_{001} = d_{001} = C_{001}$ and $f_{011} = C_{011}$, all other f_{0bc}, etc. being zero as can be seen from eqs (13) and (14).

Fit C: A three parameter fit in which $b_2 = C_{011} \equiv 0$. This choice is guided by the fact that naively one expects correlations in the diffractive component while none in the central (or pionization) component in which presumably production of particles is random.

Fit D: A three parameter fit in which $b_1 = d_{011} \equiv 0$. This choice is purely phenomenological but, as we shall see, this is capable of giving a fit to $\bar{N}_0(n_-)$ which would extrapolate to lower energies where the diffractive component is absent, that is $p = 0$, and only one component, namely the central, remains.

These four cases are confronted with the data on $\bar{N}_0(n_-)$ in the next section.

3. Results and conclusions

The data on $\bar{N}_0(n_-)$ is available for various energies from 12 to 1500 GeV/c for pp-collisions. However the two component model for $P(n_-)$ given by eq. (1) fails for 12 and 19 GeV/c. Moreover, the data on $P(n_-)$ at 1500 GeV/c is not

Table 1. The values of p, d_{010} and C_{010} obtained by fitting eq. (1) to the data on $P(n_-)$ taken from Chaudhary and Gupta (1974)

P_{LAB} (Gev/c)	p	d_{010}	C_{010}
50	0	0	1·67
69	0·1	0·81	2·08
205	0·29	1·36	3·46
303	0·38	1·85	4·30

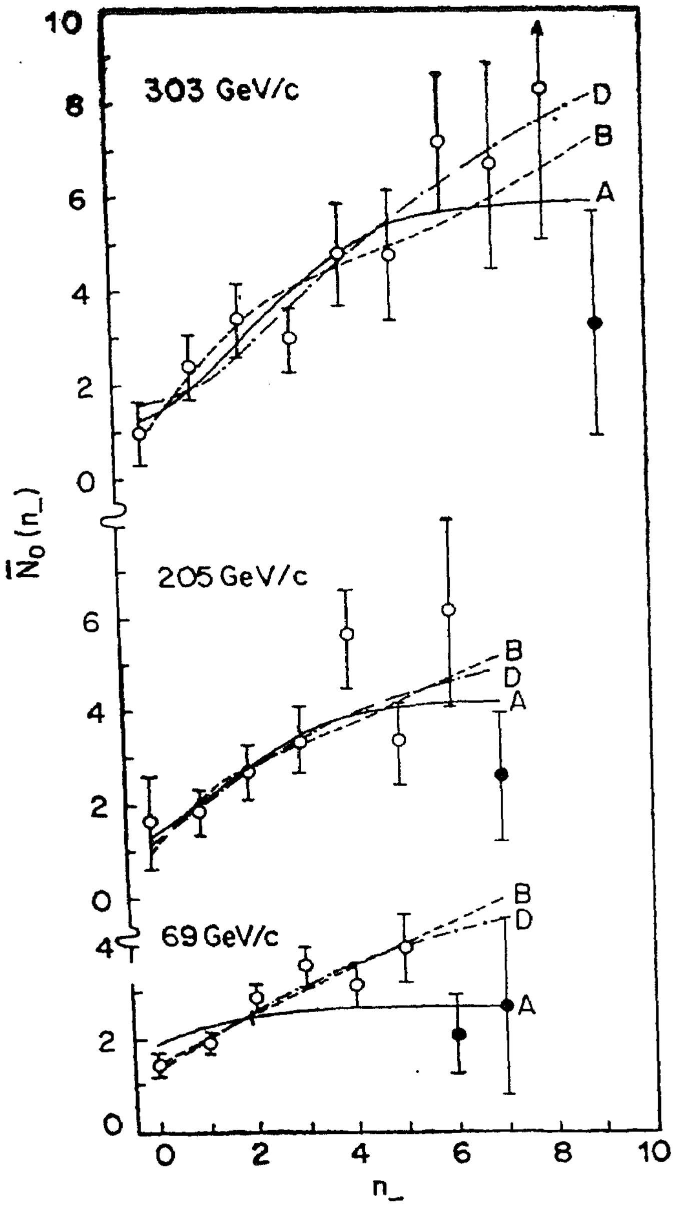

Figure 1. Average number of neutral pions as a function of the negatively charged multiplicity for the fits A, B and D discussed in the text. The experimenal data at 69, 205 and 303 GeV/c are taken from Blumenfeld *et al* (1973), Charlton *et al* (1972) and Dao *et al* (1973) respectively.

available to us. Consequently we fit the data on $\bar{N}_0 (n_-)$ at the three remaining energies, namely, 69, 205 and 303 GeV/c.

A basic feature of the $\bar{N}_0 (n_-)$ data is that the points seem to lie on a straight line and for the three energies considered here $\bar{N}_0 (n_-)$ increases with n_- and its value then drops at some large n_-, which depends on the energy. This drop in the value of $\bar{N}_0 (n_-)$ at the very largest possible n_- values (for the particular energy) is due to phase space limitations for producing such a large number of possitive, negative and neutral particles. Since this drop in the value $\bar{N}_0 (n_-)$ is purely kinematic we consequently disregard the $n_- = 6$ and 7 points at 69 GeV/c, the $n_- = 7$ point at 205 GeV/c and the $n_- = 9$ point at 303 GeV/c in fitting the data, though these points are displayed in figure 1.

Dao and Whitmore (1973) have made a purely phenomenological straight line fit given by eq. (8) with fair success. However the error on the data points is rather large at present and the n_- dependence may well be different. In fact, for the four fits we have considered the n_- dependence of $\bar{N}_0$ is different from a simple

Table 2. Values of the parameters obtained for the four fits. The last column gives the value of χ^2 over the number of degrees of freedom.

Fit	P_{Lab} (GeV/c)	Parameters				χ^2/DOF
		d_{001}	C_{001}	d_{011}	C_{011}	
A	69	$0^{+0\cdot4}_{-0}$	$2\cdot7\pm0\cdot1$	—	—	17·8/4
	205	$0\cdot27^{+0\cdot8}_{-0}$	$4\cdot3\pm0\cdot5$	—	—	4·4/5
	303	$0\cdot64\pm0\cdot6$	$5\cdot9\pm0\cdot7$	—	—	5·0/7
B	69	$2\cdot5\ \pm0\cdot13$	—	$1\cdot1\ \pm0.2$	—	4·1/4
	205	$3\cdot1\ \pm0\cdot3$	—	$2\cdot1\ \pm0.7$	—	5·1/5
	303	$3\cdot9\ \pm0\cdot4$	—	$3\cdot1\ \pm0.7$	—	4·3/7
C	69	$0^{+0\cdot4}_{-0}$	$2\cdot7\pm0\cdot1$	$1\cdot7\ \pm0.8$	—	13·7/3
	205	$0^{+1\cdot1}_{-0}$	$4\cdot4\ \pm\cdot4$	$-0\cdot7\pm1\cdot2$	—	4·1/4
	303	$0\cdot23^{+1\cdot2}_{-0}$	$6\cdot1^{+0\cdot7}_{-1\cdot0}$	$-0\cdot54^{+1\cdot4}_{-0\cdot9}$	—	4·9/6
D	69	$0^{+1\cdot7}_{-0}$	$2\cdot8\ \pm0\cdot14$	—	$0\cdot84^{+0\cdot4}_{-0\cdot2}$	3·5/3
	205	$0\cdot79^{+1\cdot4}_{-0}$	$4\cdot1\pm0\cdot6$	—	$0\cdot77\pm1\cdot6$	4·2/4
	303	$1\cdot35\pm0\cdot8$	$5\cdot6\pm0\cdot7$	—	$2\cdot4\pm2\cdot0$	3·5/6

linear dependence. The values of the parameters for the four fits at 69, 205 and 303 GeV/c which give the best χ^2 are displayed in table 2. The fits A, B and D are exhibited in figure 1. We now discuss the four fits individually.

Fit A: This two parameter (*viz.*, d_{001} and C_{001}) fit simulates a seemingly linear increase for small n_- owing to the fact that $R(n_-)$ decreases fairly rapidly with n_- and $d_{001} \ll C_{001}$. For large n_-, as $R(n_-)$ becomes negligibly small one finds that $\bar{N}_0(n_-)$ tends to C_{001}, *i.e.* a constant. Further, at 50 GeV/c the fit for $P(n_-)$ requires $p = 0$, *i.e.* only one component which implies that $\bar{N}_0(n_-) = C_{001}$ = constant for this fit for all n_- at 50 GeV/c. The over-all fit is not good, particularly at 69 GeV/c where it is poor. The calculated values are plotted in figure 1.

Fit B: In this case also one has two parameters $d_{001} = C_{001} \neq 0$ and $d_{011} = C_{011} \neq 0$, and one obtains a fairly good fit. Further, for large n_-, since $R(n_-) \to 0$, this fit requires $N_0(n_-) \to a_1 + (b_1/C_{010})\, n_-$, that is, a linear increase with n_-. For $p = 0$, *e.g.* at 50 GeV/c, this fit requires that $\bar{N}_0(n_-)$ increase linearly with n_-.

Fit C: The three parameters are d_{001}, C_{001} and d_{011}, with $C_{011} \equiv 0$. This fit gives results similar to that of fit A as can be seen from table 2. The quality of the fit is slightly better than for fit A. Further like fit A it requires $\bar{N}_0(n_-)$ constant for large n_- or $p = 0$. Thus one expects $\bar{N}_0(n_-)$ to be a constant at 50 GeV/c for this case.

Fit D: This gives a fairly good fit with the three parameters d_{001}, C_{001} and C_{011}. Further, like fit B for $p = 0$ or for large n_- it requires that $\bar{N}_0(n_-)$ becomes $a_1 + b_1 n_-/C_{010}$. Thus, for 50 GeV/c this fit like, fit B, requires a linear dependence of $\bar{N}_0(n_-)$ at 50 GeV/c.

To summarise, the fits B and D are better than the fits A and C, the latter two being particularly poor at 69 GeV/c. Since 50 GeV/c is near 69 GeV/c, we would expect that $\bar{N}_0(n_-)$ would have a linear dependence on n_- rather than be a constant. Further, the success of the fits B and D suggests that the data seem to require, neutral-negative correlations (*i.e.* $C_{011} \neq 0$) in the central component. Finally one may conclude that it is possible to understand $\bar{N}_0(n_-)$ data based on the two-component model of eq. (1) for $P(n_-)$. The n_- dependence of $\bar{N}_0(n_-)$ at 50 GeV/c depends on the model for $P(n_-)$ and it would be really interesting to have the data on $\bar{N}_0(n_-)$ at this energy.

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