

EFFECT OF RADIATION ON THE TRANSMISSION OF TEMPERATURE DISCONTINUITY.*

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Received December 24, 1934.

THE problems of upward transmission of momentum and temperature discontinuity have been investigated by many workers. Often the two phenomena have been taken to be, and treated as, essentially similar. Momentum is transferred by convective process alone, while heat may be transferred by radiation also. The additional effect of radiation was taken into account by Brunt¹ who concluded that to a certain degree of approximation radiative heat transfer was similar to convective diffusion. The effect of radiation is felt not only by adjacent layers but also to a greater or less degree by the most distant layers, depending of course on the absorption in the intervening space. The convective process depends at least as used in theory, only on the thermal structure in the immediate neighbourhood of the point under consideration. It was shown by Roberts² that it would not be quite accurate to treat radiation as analogous to convection. It was also found by the author and Ramdas³ that, in a case when the effect of radiation could be expected to play a large part, Brunt's type of equation was inadequate. A preliminary attempt made to study the effect of radiation may be of interest.

The amount of heat that crosses a layer of unit area at height z per unit time owing to convection may be put as $-k \frac{\partial \theta}{\partial z}$ where θ is the temperature and k the eddy conductivity.

The amount of upward radiation across the same layer per unit time may be put as R . It is assumed that the levels of equal absorptive matter and levels of equal temperature are horizontal. z is the height measured vertically upwards. If ρ be the density and c_p the specific heat at constant pressure, the differential equation of thermal equilibrium would be

* Abstract was presented to the Silver Jubilee Conference of the Indian Mathematical Society at Bombay, December 1932.

¹ Brunt, *Proc. Roy. Soc. Lond.*, 1929, **124**, 201; and also 1930, **130**, 98.

² Roberts, *Proc. Roy. Soc. Edin.*, 1929-30, **50**, 225.

³ Malurkar and Ramdas, *Ind. Jour. Phys.*, 1932, **6**, 495 and also 1932, **7**, 1.

$$k \frac{\partial^2 \theta}{\partial z^2} = \frac{\partial R}{\partial z} + \rho c_p \frac{\partial \theta}{\partial t}.$$

For the purpose of this paper it may be assumed that k is sensibly constant. R depends on the distribution of temperature and the amount of absorptive matter in space. In a previous paper the form of R was found to consist of terms of type⁴

$$\int^z J(\theta_\zeta) H_2 \left(\int_z^\zeta f(\theta_\xi) d\xi \right) d\zeta.$$

Let us further assume that the maximum temperature variation in the whole space under consideration is small compared with the absolute temperature of any point. Then $\frac{\partial R}{\partial z}$ would be of form $F(\theta_z)$. Hence the above equation of heat equilibrium could be reduced to

$$k \frac{\partial^2 \theta}{\partial z^2} = F(\theta) + \rho c_p \frac{\partial \theta}{\partial t}.$$

Let us put that $\theta = \theta_0 + \phi$ and neglect the second and higher powers of ϕ , we obtain $F(\theta) = F_0 + F_1 \phi$ where F_0 and F_1 are sensibly constants in the whole layer. Then

$$k \frac{\partial^2 \phi}{\partial z^2} = \rho c_p \frac{\partial \phi}{\partial t} + F_1 \phi + F_0.$$

By the transformation $\phi = \chi e^{-F_1 t / \rho c_p} - \frac{F_0}{F_1}$, the equation reduces to

$$k \frac{\partial^2 \chi}{\partial z^2} = \rho c_p \frac{\partial \chi}{\partial t} \quad \dots \quad (A)$$

which is of the same form as the eddy equation of Taylor⁵ and Schmidt. In Taylor's equation of the above form χ is the potential temperature, and in Brunt's modified equation it is the actual temperature, and k is the modified diffusion coefficient, being the sum of convective and radiative diffusion. The solution of the equation (A) in particular cases was put by Taylor in form

$$\chi = \chi_0 \psi \left(\frac{\rho c_p z^2}{4kt} \right)$$

where ψ represented certain restricted type of functions. The boundary conditions under which the above equation was solved were either (a) χ decreases uniformly with time t , i.e., $\chi = \chi_0 - pt$, or (b) χ suddenly changes from the value 0 to χ_0 and thereafter remained constant.

In both the functions χ being a function of z^2/t only the value of χ is the same for corresponding values of z and t when z^2/t is constant, in other

⁴ See Gerland's *Beitrage z. Geophys.*, 1932, 37, 4, 411.

⁵ Taylor, *Trans. Roy. Soc. of Lond.*, 1915, 215, 1 et seq.

words, the value of χ is propagated undiminished (or without damping) to all heights under consideration.

If similar boundary conditions were adopted for the equation where the effect of radiation is considered as analogous to convective diffusion, we obtain a similar result with the modified value of k which is the sum of the two diffusivities.

For the problem in this paper the boundary conditions are taken as

$$\begin{aligned}\phi + F_0/F_1 &= 0 \text{ at } t = 0 \\ \phi + F_0/F_1 &= f(t); z = 0\end{aligned}$$

we get the solution

$$\phi + F_0/F_1 = \frac{z}{2\sqrt{k\pi}} \int_0^t f(\xi) e^{\{-z^2/4k(t-\xi) - F_1(t-\xi)\}} \cdot \frac{d\xi}{(t-\xi)^{3/2}}.$$

The integral cannot in general be evaluated. But we can study its behaviour. We assume that $f(\xi)$ has always the same sign in the ranges of values $\xi=0$ to $\xi=t$. $e^{-F_1(t-\xi)}$ is a monotonically increasing function of ξ in the same range. So we can approximate to the integral as

$$\frac{z e^{-F_1 \epsilon t}}{2\sqrt{k\pi}} \int_0^t f(\xi) e^{-z^2/4k(t-\xi)} \cdot \frac{d\xi}{(t-\xi)^{3/2}}$$

where $0 < \epsilon < 1$.

It follows that the effect of introducing the radiation term into the differential equation has been the factor $e^{-F_1 \epsilon t}$ which denotes a sort of damping or diminution function with time. This diminution is quite distinct from any effect that may be exercised by the eddy diffusion.