DYNAMICS OF THUNDERSTORMS*

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Thunderstorms are of such frequent occurrence and are of such great importance in meteorology, that every attempt at clarification of the processes involved would have a sufficient justification. It is but natural that the thermodynamic aspect has taken the major portion of the discussions on the subject, as energy in the form of heat plays a large role in the phenomenon. The dynamical aspect of the convective currents is also vital to the production of the thunderstorm and an enquiry into it is called for.

The preliminary background was obtained by an examination of the actual upper air soundings made at Poona. The occasions when successful soundings have been made during thunderstorms at a given place are small and the number of observations during the successive development of a thunderstorm still less. The limitation set by the number of observations at a place on days of thunderstorms can be overcome by assuming that the meteorological conditions existing at stations in the neighbourhood of places recording thunderstorms give a fairly good picture of conditions before the onset of thunderstorms. The analysis yielded the following ideas:—

(*a*) In general, the potential temperature of air increased with height, *i.e.*, the lapse rate was less than dry adiabatic. But on certain days, the rise of potential temperature with height was less marked and in some cases there was no appreciable rise for a thickness of one or two kilometres, *i.e.*, the lapse rate was or very nearly was dry adiabatic. On the days when the lapse rate was nearly dry adiabatic, there was a tendency for thunderstorms to occur in the neighbourhood of Poona.

(*b*) However, when the lapse rate was nearly adiabatic and the humidity of air small the distribution of thunderstorms was poor or absent.

(*c*) The air was not necessarily saturated in the lower layers of the atmosphere on days of thunderstorms.

The development of a thunderstorm which is essentially a convective phenomenon needs *a cause for initial convection and then a possibility of its maintenance once the convection has been started.*

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There has been considerable discussion in meteorological literature regarding the existence of potentially colder air superposed over potentially warmer air. If this superposition existed, it may be argued that the system being unstable, may initiate convection and even possibly maintain it. It is theoretically conceivable to have small isolated masses of air which may be potentially colder than the air below it. But it is not evident, whether it is ever possible to have a potentially colder stratum of air superposed over a potentially warmer one for any considerable area or over a long period. Further if such a superposition has once existed for some time, due to certain boundary conditions, it may not give rise to upward motion by itself. A slight digression on this point may not be out of place.

Rayleigh[1] showed as a result of Bernard's experiments, that in shallow layer of liquid the temperature of the lower surface could be increased above the temperature of the top surface upto a certain limit before instability set in. Brunt and Low tried to apply this result to the existence of large lapse rates near the ground on hot days. Later Jeffreys improved on Rayleigh's results and obtained similar expressions for instability, but he "could not say how far the result could apply to the atmosphere". Hales, applying the theory to the atmosphere, showed that the *form* of the result was not different from that obtained by the previous workers and deduced that only over a limited height, super-adiabatic lapse rates may occur, *i.e.*, the potential temperature cannot decrease with height for any large interval of height.

It may be argued that the top-heaviness due to the difference in the moisture content in different layers may lead to a different form of result and bring about instability. This can be disposed of without great details. The fundamental equations of instability in the thermal problem for a thin layer of air are:

$$d/dt \quad (u, v, w) - \nu\rho \; \Delta \; (u; v, w) = - (\partial/\partial x, \partial/\partial g, \partial/\partial z).p - (o, o, g\rho) \qquad (1)$$

$$d\rho/dt = \rho \; \text{div.} \; (u, v, w) \qquad (2)$$

$$d\theta/dt = k \; \Delta \; \theta \qquad (3)$$

$$\rho = \rho_0 (1 - \alpha\theta), \qquad (4)$$

[1] Rayleigh, *Phil. Mag.*, **32**, 529. *Coll. Works*, **6**, 432. Low, *Nature*, **65**, 299. Brunt, *ibid.*, 300. Jeffreys, *Phil. Mag.*, 1926, **2**, 833 and *Proc. Roy. Soc.*, 1928, **118**, 195. Low, *Proc. Roy. Soc.*, 1929, **125**, 180. Jeffreys, *Proc. Camb. Phil. Soc.*, 1931, **26**, 170. Malurkar, *Gerland's Beitr. Geophys.*, 1937, **51**, 270. Hales, *Proc. Roy. Soc.*, **151**, 624. Also Malurkar in the Abstracts Nos. 28, 29, 37 of 1937 of the Physics Section of the Indian Science Congress.

where u, v, w are components or velocity of the fluid, p is the pressure, ρ the density at any point, ν the viscosity, θ the temperature, k the thermal diffusivity, ρ_0 the intitial density and α is a constant. The z-axis is measured vertically upwards and Δ is used as usual for $\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial z^2$.

The equations (1) and (2) do not explicitly involve temperature; (1) is the general equation of motion in a fluid taking account of viscosity and (2) is the equation of continuity. But equations (3) and (4) depend on temperature. Taylor[2] showed that in eddy diffusion, the moisture content or more accurately the mixing ratio obeyed an equation similar to the equation of heat conductivity. It is also well known that the density of air and the mixing ratio have a linear relation when thin layers are considered. If a thin layer of uniform temperature is considered and r the mixing ratio is taken to decrease uniformly with height to bring about top-heaviness equations (3) and (4) are replaced by similar equations involving mixing ratio

$$dr/dt = k' \, \Delta \, r \tag{5}$$

$$\rho = \rho_0 (1 - \alpha' r). \tag{6}$$

The limits of instability can be worked out exactly as in the thermal problem except that the term mixing ratio replaces temperature at each stage of calculation.

The criterion of instability would be, in terms of density, $(\rho_1 - \rho_0)/\rho_0 < \mathrm{K}\, k\nu/gh^3$, where ρ_1 is the density of the top-layer, h the thickness of the layer and K is a constant depending on the boundary conditions. Though investigation on the analogy of Hales to the whole atmosphere, with moisture as variable, has not been carried out, the nature of the equations allow one to be confident that that the criteria of instability brought about by top-heaviness due to temperature gradient and that due to the gradient of mixing ratio are at least to a first approximation analogous.

It follows that between narrow limits, it may be possible to have top-heaviness owing to higher potential temperature at the lower levels or owing to injection of extra moisture in the lower layers. But instability rapidly sets in if the height interval is large. The top-heaviness which can exist in very shallow layers as deduced in the various papers on the basis of stability would not allow instability to set in without external stimulus; or initiation of convection without external stimulus is not likely in such systems. The cause of initial upward motion must, therefore, be sought for elsewhere

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[2] Taylor, *Phil. Trans. Roy. Soc.*, 1915, **215**, 1.

than in the general or widespread temperature and humidity distribution that already exists. The possible local causes of upward motion are:

(*a*) Strong *unequal* surface heating—If there was equal heating of the various strata in the very low layers of the atmosphere, the criteria developed previously may perhaps give rise to large lapse rates without creation of instability when thin layers are considered. But when there is unequal heating in any level such isolated stable strata or pockets of air do not form and upward motion is facilitated. In nature, the unequal heating is more likely; due to the fact that large tracts are covered by vegetation and others bare. The rivers, large water surfaces, uneven land or land not covered uniformly by vegetation or even non-uniform distribution of rocks and earth make the surfaces of equal temperature deviate largely from a horizontal position and upward motion can be initiated. The deviation of the isothermal surfaces from the horizontal would generally be great during the afternoons and early evenings. Even at other times, the non-horizontality of the isothermal surfaces brought about by differential cooling of the land and water surfaces may give rise to upward currents. When the top surface of a cloud cools by radiation and sinks, upward motion may indirectly be induced.

(*b*) If the surfaces of equal humidity are not horizontal and the gradient of humidity or mixing ratio sufficient, upward currents can be initiated. The analogy with temperature gradient can profitably be used.

(*c*) At the surface of separation of two air masses large waves may be set up and in portions of the wave, upward motion is possible.

(*d*) Gradient of wind velocity due either to (i) orography, or (ii) juxtaposition of two air masses with very different horizontal velocities. When land rises abruptly, there is an upward component of wind whenever the wind blows against the land. When two air masses meet with very different horizontal velocities the equation of continuity would indicate that large upward motion of air can take place. At a cold front or whenever there is wedge of cold air, the air adjacent to it will be lifted up.

In the large-scale operations of the atmosphere, a mass of air rising due to unequal heating may be considered as an isolated air mass obeying the well-known equation

$$\frac{d^2z}{dt^2} = g\,(\theta_r/\theta_a - 1), \qquad (1)$$

where θ_r is the temperature of the rising mass, θ_a is the temperature of the air immediately surrounding it and z is the vertical height measured upwards. The friction and the dissipative forces have been neglected. The equation cannot be solved in general. A simple case of an atmosphere with a uniform lapse rate μ can be taken. At the initial level the temperature of the free air may be θ and that of the rising air $\theta + a$. The rising air cools at the dry adiabatic rate λ. At the initial level the rising mass may be taken to have no upward extra velocity imparted to it, *i.e.*, would start from rest. Then

$$\frac{d^2z}{dt^2} = g\,(a - \lambda z + \mu z)/(\theta - \mu z) \tag{8}$$

$$\frac{1}{2}\left(\frac{dz}{dt}\right)^2 = g/\mu^2\,\{(\lambda - \mu)\,\mu z - (a\mu - \lambda\theta + \mu\theta)\log(1 - \mu z/\theta)\} \tag{9}$$

dz/dt would be zero at $z = 0$ and again at the point given by

$$\mu\,(\lambda - \mu)\,z = (a\mu - \lambda\theta + \theta\mu)\log(1 - \mu z/\theta). \tag{10}$$

As θ is large compared with λ, μ or z the equation can be approximately solved and dz/dt is equal to zero at

$$2a\Big/\left(\lambda - \mu - \frac{a\mu}{\theta}\right) \tag{11}$$

while $d^2z/dt^2 = 0$ at $a/(\lambda - \mu)$.

The level to which a mass of air rises due to its being lighter than its surroundings at any stage is not limited by the fact that at the terminal position the density must be equal to that of the surrounding air. The acceleration alone is reduced to nothing at the point of hydrostatic equilibrium. The momentum developed by the rising current would be able to carry the mass to a level considerably above the level of hydrostatic equilibrium. When this rising mass, after having overshot the hydrostatic equilibrium level, comes to rest at the hydrodynamic equilibrium level, it would be colder or denser than its immediate surroundings and would descend. Similarly during descent, the mass would go quite well below the level of hydrostatic equilibrium and find itself warmer or lighter than its surroundings. It will rise again and would execute oscillations. The initial amplitude would be small if the rise of potential temperature with height was large as the rising mass has nearly a constant potential temperature. If the increase in potential temperature with height is relatively smaller the amplitude of the oscillations would correspondingly be greater.

In any particular problem, however, the successive amplitudes diminish, perhaps rapidly, due to the dissipative forces like resistance and mixing. For the problem on hand, it is not necessary that oscillations *should take place* but that the rising mass must be able to go above the level of static equilibrium.

In the thermodynamic treatment, the actual mode of rise of air is not very material. And often, the treatment is more or less static. The importance of the extra ascent between the levels of static and dynamic equilibrium is that though a mass of unsaturated air may not reach the stage of condensation before the static level, yet it may attain saturation before the dynamic level of equilibrium is reached. In the above equations it is easily seen that the extra height between the levels of static and dynamic levels is nearly of the same order as that between the initial level and the level of static equilibrium. In other words a mass of air which would have cooled by 10° C. by rising one kilometre to reach its static level of equilibrium may cool nearly 15 to 20° C. before the dynamic level of equilibrium is reached. The difference involved is obvious. The dynamical treatment allows the condensation point to be attained for a rising mass which is much drier than the semi-static treatment. If the condensation is reached before the level of static equilibrium is reached, no special complications arise.

When the rising mass attains its saturation point and condenses, latent heat, which is comparatively large for water vapour, is liberated and it heats the air immediately surrounding the rising mass. This heated air has the impetus to rise due to this extra energy and may in turn initiate further upward motion. The system becomes self-regenerating and, if conditions are favourable, may grow at a continuously accelerated pace to result in a thunderstorm.

If the initial upward movement is due to unequal distribution of humidity and there is sufficient gradient of humidity upwards, arguments similar to the ones above can be used by replacing lighter for warmer and denser for colder. Once the condensation point is reached and the latent heat warmed the surrounding air, the further development would be similar to the upward motion due to extra heat imparted to the rising mass.

When the upward movement is due to horizontal gradient of wind, either due to orography or frontal action, the density of air at the initial place of vertical motion is not lighter than the surrounding air. Hence the upward motion of air due to gradient of wind may be considered as analoguous to the stage described in the previous paragraphs when the lighter air had passed the level of static equillibrium but not yet come to rest, *i.e.*, still possessed upward momentum. If this air reached its condensation point before the level of momentary rest is reached, the liberated latent heat would give the surrounding air impetus for further vertical motion after which the previous arguments may be applied.

In nature the conditions described as of distinct types do not occur individually. More than one condition is operative. The orography may be helped by extra surface heating and bringing in of moisture in the lower levels or relative drying up of higher levels.

Incidentally, it may be mentioned that during the process of condensation and vertical movement, strong horizontal winds would drift away the products of condensation and may inhibit the development of a thunderstorm. Weaker winds at higher levels would help the growth of a thunderstorm. The inversions may also help to conserve or localise the accumulation of energy. Every small addition of heat or moisture in the lower layers can be dissipated away by convection currents. The inversion by preventing these small currents would allow sufficient energy to be accumulated in the lower layers and when the convection is brought into play to counteract the effect of inversion, it may do so with explosive violence.

When a uniform plate is heated from below the convection currents that are set up form isolated upward streams. There is no general upward transference of air over the whole plate. The actual places where these small rising currents are set up probably depend on irregularities on the surface. Even otherwise the points of ascent would perhaps be determined by the physical characteristics of the plate, *e.g.*, the points of ascent would somewhat be similar to the nodes of the Chladni's figures on the plate. The problem in nature, the places of occurrence of heat thunderstorms, may be compared to the patterns of nodes and anti-nodes in a loaded plate with suitable boundary conditions corresponding to the rivers, hills and other items of orography. This analogy was thought out during a conversation with L. P. Cohen and may profitably be followed up.

From a practical point of view, the above discussion leads one to the useful conclusion in the case of heat thunderstorms: if at higher levels fresh air which is relatively cooler or potentially cooler than the one existed before or if at lower layers relatively warmer or potentially warmer and more moist air than that which existed earlier is brought about, the conditions become favourable for thunderstorms. If the convective currents are started these conditions would help to maintain convection and may give rise to thunderstorms.

Summary

The development of a thunderstorm which is essentially a convective phenomenon needs a cause for initial convection and then conditions for its maintenance once started. The possibility of the inherent instability due to potentially colder air superposed on potentially warmer air or the

analogous case of extra injection of moisture in the lower layers is shown to be not applicable in nature and that causes for initial convection must be found elsewhere than in the potential density distribution. Some of the causes are non-horizontality of surfaces of equal temperature and equal humidity; and gradient of wind velocity. It is shown that upward rise of air produced by unequal heating of the ground, does not stop where the rising mass of air attains a density equal to that of the environment (hydrostatic equilibrium), but continues to rise higher till the momentum developed is reduced to zero. The extent of over-shooting is nearly of the same order as the height between the initial level and the level of hydrostatic equilibrium. Once the condensation level is reached, it is well known that the convection will become regenerative due to the evolution of the latent heat of condensation. In the usual treatment of the problem, the ascending parcel of air is expected to condense at or before it reaches the level of hydrostatic equilibrium if it is to develop into a thunderstorm. The dynamical treatment outlined in this paper takes into consideration the overshooting of the parcel of air and thereby allows much drier air to reach condensation and thereafter maintain convection: and can thus account for a larger number of thunderstorms.