A SIMPLE NOMOGRAM FOR THE RATE OF ASCENT OF RUBBER BALLOONS AND IMPROVED TECHNIQUE WITH BALLOONS AND BALLOON MATERIAL*

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Nomogram

The rate of ascent of a rubber balloon filled with a lighter gas is very important for the meteorologists as they have to send up many rubber balloons daily for their work. The rubber balloons are usually filled with hydrogen and let off to determine the upper winds (pilot balloon) or to study the temperature structure of the upper air (sounding balloons—like radio sonde, etc.). By triangulating the positions of a pilot balloon, either by using two theodolites or by using the well-known tail method, the direction and speed of the upper air is determined. But the time taken for the calculation of the winds after the balloon is let off is more than what the customers of upper-air-aviators and weather forecasters—would allow. Time-saving methods are introduced and simplifications have to be made. The uniform rate of ascent method does away with the necessity of finding the height co-ordinate of the balloon and the method is used in most countries where the effect of convection is not large enough to vitiate the results. To secure uniform rate of ascent the amount of hydrogen that fills the balloon is determined. In any lot of balloons even from the same works, the weight of balloons varies so much individually that it is not possible to give the same amount of gas or give the same free-lift to the balloon. Tables have to be constructed so that the free-lift of the balloon can be determined from the rate of ascent and the dead-weight of the balloon. The formula used is

$$R = kL^{\frac{1}{2}}/(L + W)^{\frac{1}{3}}$$

where **R** is the rate of ascent of the balloon, *k* is a constant, W is the dead-weight of the balloon and **L** is the free-lift of the inflated balloon.

The Americans (U.S.A.) use practically the same formula except that instead of R, $R^{1/m}$ is used where *m* is another constant which is nearly unity.

The equation consists of three variables and the first impression is that a number of graphs are required connecting the rate of ascent, the weight

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* The work was done in 1935–36 when I was in Upper Air-Observatory, Agra.

and the free-lift of the balloons. Some have actually constructed many graphs. But even then, the disadvantage is, that if for some reason the rate of ascent or the value of the constant has to be changed, a series of new graphs will be required to be drawn. If the work can be simplified it would greatly add to the elegance and saving of avoidable work.

If in the above equation L is replaced by $(R/k)^6$ Y and W is replaced by $(R/k)^6$ X in the usual formula;

or if L and W are replaced by $(R/k)^{6/m}$ Y and $(R/k)^{6/m}$ X in the U.S.A. formula, the equations for the rate of ascent would be all reduced to a single simple formula

$$Y^3 = (Y + X)^2$$

This is an elementary equation whose graph can be constructed with the help of square and cube tables. Reading off from it could be made easy by taking an open scale. Whatever changes may be introduced in the rate of ascent, or the constants involved in the formula, the equation is unaffected, and hence the construction of a large number of graphs is rendered superfluous. The construction of the graph with suitable co-ordinates takes hardly a few minutes and the tables for any given rate of ascent take only an hour for a usual range of weight of balloons in any category. The only change that is introduced by a change in the constants or in the rate of ascent is the factor which multiplies both **Y** and **X** to give the free lift and dead weight.

Weight of Attachments to a Balloon

In a formula the dead weight of the balloon W is considered. The formula is partly empirical and partly based on general principles, of the resistance to a moving expanding sphere. At the time of discussion the question of attachments does not arise. In some countries, like England, the weight of the attachments is added to the dead-weight of the balloon before the rate of ascent is determined. In other countries notably Germany (and India before 1938) the weight of attachments is compensated by an additional free-lift equal to it. An experimental verification is hardly possible without bringing in other unknown factors. The correct procedure is probably somewhere between the two methods.

$(L + W)^{\frac{1}{3}}$ is proportional to the diameter of the inflated balloon (see below) and adding the weight of attachments to the dead-weight of the balloon means a virtual increase in the size of the sphere considered. The increase in the size of the sphere brings in more friction and surface resistance. If the weight of the attachment is compensated by an addition to the free-lift, there is no addition to the superficial resistance, contemplated. The free lift formula is really due to balancing of forces, one due to the surface resistance and the other rise due to buoyancy. If the weight of the attachments are not unduly large compared with the dead weight of the balloon, the fact that the nomogram involves a factor $(R/k)^6$ for the free-lift or the dead weight; the difference in the rates of ascent by using the alternative methods regarding the weight of the attachments is really small. In most cases it does not exceed 2·5% which is below the accuracy of our measurements.

Balloon Technique

Though a meteorologist is primarily interested in the finished rubber balloons and the upper air observations, he must be able to distinguish different consignments of balloons that he gets and utilise them most economically. The balloons made by different factories vary in size, weight, quality and consistency. Even from the same manufacturer different consignments show different results. It is not immediately necessary to go into theoretical considerations why it is so. It is sufficient if workable ideas can be put together.

Rubber balloons were obtained from various sources. Cold vulcanised rubber sheetings were also got and balloons made out of them by pasting sectors with suitable adhesives locally. A quantitative comparison of their performance and deterioration of the balloon stock would add to efficiency of work. To wait for fine weather, and let off and determine how high the balloon went was definitely unsatisfactory. For a long time there was a practice of testing a few balloons of each stock by inflating them to destruction and noting the destruction diameter. A systematisation involving simple ideas was therefore adopted.

1. In the earlier years when the type of balloon was variable, it was customary to measure the dead weight of the balloon, the diameter of the inflated balloon on the ground and also the free-lift. As the quality of the balloons had become uniform and practically spherical on inflation, the measurements of all the three quantities were unnecessary, because of the formula

$$(L + W) = \frac{13{\cdot}85 \times 1{\cdot}293 \times 273 \times \pi \times d^3}{14{\cdot}85 \times (273 + \theta) \times 6}$$

Where θ is the temperature in degrees Centigrade, d the inflated diameter measured in metres, and the free-lift and dead weight and W are given in kilograms. The volume of hydrogen gas that is required to fill the balloon at temperature θ and the atmospheric pressure is

$$0{\cdot}83\,(273 + \theta) \times (L + W)/273 \text{ in cubic metres.}$$

2. *Test of Efficiency.*—The main reason of using rubber for balloons is its power of stretching. The thinner, the rubber sheeting of the balloon can be stretched without tearing, the better is its quality. The surface area of an inflated balloon can easily be calculated and this multiplied by the thickness of the sheeting and the density gives the weight of the balloon. In the spherical balloons the measurement of the diameter is sufficient. The bursting diameter may be denoted by D. If different rubber materials had the same efficiency they would tear when the same thickness is attained after stretching. As the density of the rubber is fairly constant, the thinness is given by the weight of the balloon divided by the surface area except for the density factor. In the spherical balloons of the same efficiency, the ratio

$$D^2/W \text{ must be constant.}$$

Hence it is worthwhile noting the quantity

$$E = 6{\cdot}25\, D/W^{\frac{1}{2}} \quad \text{(D in metres)}$$

The constant 6·25, which is a submultiple of 100 was taken as the best balloons gave a value of 100 for the above factor. Others hardly touched 90, and E becomes a sort of percentage efficiency factor.

In a given sample of balloons, two or three were burst to destruction and the mean value taken as D. If the individual values differed widely, more balloons were burst to get at the mean value. The spread of the individual efficiency factors also represents the uniformity of reliability of the balloon consignment. When sheetings had to be tested, small balloons of 6·0" diameter (uninflated) were made and tested to destruction. Making small sample balloons saved a good amount of material. The variation in efficiency factor with time of balloons stored under different conditions gave an idea of the decay of the balloon material. Various material could be quantitatively compared.

3. *Height attained by the ascending balloon.**—The next important question that arises is the height upto which the balloon may be expected to rise. Given the efficiency of the particular rubber material, given by the tests of the last paragraph, and the weight of the balloon, it is possible to compute the bursting diameter of the balloon. It may be assumed that that the balloon actually bursts at this diameter. If d is the diameter on the ground of the inflated balloon and D is the bursting diameter, θ is the ground temperature, and θ_h the temperature at the bursting height, when there is no leakage of gas we have

$$d/(273 + \theta)^{\frac{1}{3}} = D/(273 + \theta_h)^{\frac{1}{3}}$$

* See also an article by L. H. G. Dines. Met. rag.—vol. 64, p. 57, 1929.

Hence to a first approximation, the balloon may be expected to burst at a height where the pressure is given by

$$1000\,(d/D)^3 \times (273 + \theta_h)/(273 + \theta) \text{ millibars}$$

The reason for saying as a first approximation is that there are many unknown factors which might change the level of bursting, the efficiency of the rubber sheet may change at the low temperatures above, the balloon may not expand till the maximum is reached and the convection currents and local heating of the balloon may carry it far higher than indicated by the formula. But as a working model it is as satisfactory as any.

The importance of the above formula is that one can determine the type of balloon that is to be used for various types of ascents. If high pilot balloon winds are required an approximate calculation would show whether a particular type of balloon is capable of attaining the height or not. Among those that can attain the height, and to safeguard accidents a couple of kilometres more, the most economical balloon can be chosen, *i.e.*, unnecessarily large balloons can be saved up. Another important use is that if a sounding balloon has to be recovered to work out the record as in Dines meteorograph in a region where the balloon would be carried off if it attained great heights, the free lift can be adjusted so that the balloon would burst at a predetermined height which ensures recovery. In N.W. India balloons were let off to burst at about 12–13 kms. for facility of recovery in 1936. It is only necessary to see that in the formula for the height attained, the pressure for about 12 kms. (average pressure estimated at that height) is taken to be that for the bursting height. It may be pointed out that fuses do not operate at such great heights.

The formula yields another result. For the same type of balloon material (with the same efficiency) D is prportional to $W^{1/2}$ and d is proportional to $(L+W)^{1/3}$. Hence $(d/D)^3$ is proportional to $(L+W)/W^{3/2}$; or if the free-lift and the dead weight of the balloon are taken to be of the same order $(d/D)^3$ is proportional to $1/W^{1/2}$ and hence to $1/d^6$. Hence it follows that it is the one-sixth power of the diameter that determines the height to which a balloon will attain. This leads to the conclusion that by a small increase in the size of a balloon, there is not much advantage gained and conversely to increase the height attained by a balloon even by a small height the increase required in the diameter of the balloon is disproportionately large. A large balloon is costly for gas also. If on the other hand the efficiency of the balloon material could be increased, there would be a good chance of attaining greater heights. As a result of this the size of the balloon can be decreased till it is able to "just not function" with its attachments like the sounding balloon equipment. In practice, a liberal allowance is always made, in the size of the balloon.

4. *Shape of the balloon.*—In an inflated rubber balloon, if r and r' are the two radii of curvature, dp is the difference of pressure inside and outside the balloon and T is the superficial strength of the rubber sheeting

$$1/r + 1/r' = dp/T$$

the usual surface tension formula.

The only surface that keeps at all its points constant total curvature and continues to keep constant (though the actual constant may vary) curvature at every stage of expansion is a sphere. If there be unequal value of the total curvature of the rubber balloon due to non-uniformity of material or due to its construction with non-spherical sectors, the thickness of the rubber material varies at various points of the surface. There would be a preferential tendency for the balloon material to give way at its thinner point. This effect will be magnified as the balloon ascends and the balloon may not attain the heights attained by spherical balloons. It follows therefore that the rubber balloons are most efficient when they are initially spherical and continue to remain spherical at all stages of expansion, *i.e.*, stretch uniformly.

5. *Cementing of Cellophane material.*—Attempts were also made to make balloons from various tissues which are non-expansible. Field* has described fully the making of gutta-percha balloons. With Cellophane sheetings (which are light and better preservable than gutta-percha) the ordinary adhesives would not give a gas-tight joint. After several tests, it was found that if a narrow strip of gutta-percha is kept between two overlapping edges of cellophane sheetings and a hot iron (just heated to about 150° C) is passed over the edge, the joint becomes gas tight and balloons can be made of the material. The joint is further moisture-proof. This cementing may have applications in other fields also.

Summary

The formula for the rate of ascent of a rubber balloon filled with a lighter gas contains two other variables, the dead weight of the balloon and the free-lift of the inflated balloon. Usually many graphs have to be drawn connecting the three variables for use. By a simple transformation it is shown that all the formulæ are reduced to $Y^3 = (Y + X)^2$ which can be

* Field. Memoirs of Ind. Met. Dept., vol. 24, p. 133.

graphed easily. The importance of this formula is explained by showing that there is little actual difference in the rate of ascent whether the weight of attachments are added to the dead-weight of the balloon or subtracted from the free-lift before applying the formula.

To utilise balloons economically and predetermine their behaviour in practice, it is necessary to make comparisons quantitatively. Simple formulæ are given.

A method of pasting two sheets of cellophane material is given which leads to a gas-tight and moisture-proof joint.