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Comment on Anomalous Dispersion and Scattering Rates for Multiphonon Spontaneous Decay in He II

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We report on new measurements of the spontaneous decay threshold energy E_c *for high-frequency phonon propagation in He II at saturated vapor pressure at* $T = 0.1$ *K. Superconducting tin tunnel generators and aluminum tunnel detectors were used in this study. The measurements show that the mean free path becomes much larger than the propagation length of 1.1 mm for a value of* $E_c = 9.8 \pm 0.15$ *K. This agrees with the value originally reported* ($E_c = 9.5 \pm 0.4$ *K) by Dynes and Narayanamurti using aluminum tunnel generators, but is shown to correspond to the point where the phase velocity equals the sound velocity, when the phonons become stable, as first proposed by Pitayevski and Levinson. Evidence for* n*-phonon decay at energies lower than* E_c *is presented for* $n \gtrsim 2$ *with a short mean free path* (< 0.3 *mm) at the two-phonon decay energy. The measured values of the dispersion parameters are shown to agree closely with the spline fit to neutron data due to Donnelly, Donnelly, and Hills.*

The anomalous dispersion, i.e., the fact that the phase velocity of phonons has a maximum at finite wave vectors, is well established in He II for pressures $\lesssim 20$ bar (see Ref. 1 for a review). This characteristic of the dispersion curve allows a spontaneous decay of a phonon with $q < q_c$ into two or more phonons. Pitayevski and Levinson[2] have shown that for a dispersion relation

$$E(q) = \hbar\omega(q) = \hbar v_s q(1 + \gamma q^2 - \delta q^4 + \cdots) \tag{1}$$

where $\gamma, \delta > 0$ and v_s is the velocity of sound, the threshold wave vector against n-phonon spontaneous decay is given by

$$q_n = q_\infty(1 + 1/n^2)^{-1/2} \tag{2}$$

where $q_\infty = (\gamma/\delta)^{1/2}$ and $n \geq 2$. Originally, to explain ultrasonic attenuation

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it was believed that the decay threshold corresponded to a wave vector $q_0 = [(3/5)(\gamma/\delta)]^{1/2}$ when $v(q) < v_s$. Dynes and Narayanamurti,[3] to explain their measured spontaneous decay threshold, assumed $n = 2$ and took a value of $q_2 = [(4/5)(\gamma/\delta)]^{1/2}$. Pitayevski and Levinson, however, later showed that the phase space for scattering truly goes to zero and that the phonons become stable only at q_0, when the phase velocity becomes equal to the sound velocity. The various thresholds in the dispersion relation are shown schematically in Fig. 1. The scattering rates $\Gamma_n(q)$ for n-phonon decay are not easily calculable,[1,2,4,5] but because of the phase space argument we know that $\Gamma(q)$ vanishes as $q \to q_\infty$.

In this paper we report experimental evidence of finite scattering rates for multiphonon decay with $n > 2$. The experiments were a natural outgrowth of recent experiments on zone boundary phonon propagation in solid helium,[6] where we used tunnel junction generators of Sn, granular Al, or bulk Al covering an energy range from about 0.38 to 1.2 meV. The detector in all cases was bulk Al with a gap of 0.38 (±0.02) meV. The use of Sn junction generators allowed us to probe the region 0.38–1.2 meV through

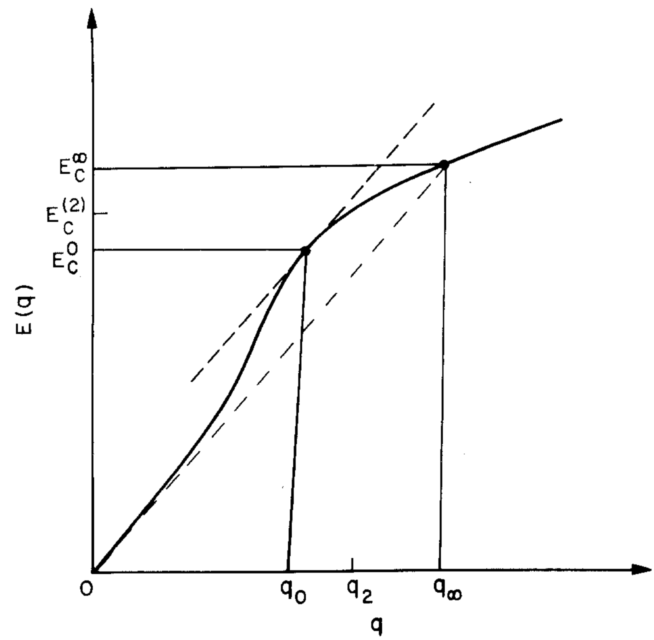

Fig. 1. Thresholds in the dispersion relation for liquid helium at SVP. At the momentum q_∞, the phonon velocity equals the phase velocity and spontaneous decay is prohibited for energies greater than E_c^∞. See text. (After Ref. 2.)

the use of the quasimonochromatic relaxation phonons (see Ref. 7 for review) produced by a modulation technique. Previous work by Dynes and Narayanamurti[3] used an Al generator where the higher frequency phonons (>0.38 meV) were generated in a small but finite amount because of incomplete reabsorption of greater than 2Δ (energy gap) phonons in the Al. The use of Sn junctions with the much larger energy gap compared to Al allowed us to obtain approximately one order of magnitude better signal-to-noise ratio in exploring the signals of interest. The phonon propagation length was 1.1 mm and the temperature T was ~ 0.1 K. The principal uncertainty in the measurement is the width of the Sn energy gap, which was about 0.1 meV in total, yielding an energy uncertainty (half-width) of ~ 0.05 meV (~ 0.6 K). The onset of the 2Δ rise of high-quality (leakage $\leqslant 1\%$) Sn junctions could, however, be measured quite precisely (~ 0.01 meV).

The data with double Al junctions were identical to those obtained earlier by Dynes and Narayanamurti[3] (hereafter referred to as DN). With Sn junction generators, however, we were able to obtain much better data at SVP, where anomalous dispersion is strong over large regions of q space well beyond the Al detector threshold. These data are shown on a highly expanded scale in Fig. 2 over the energy scale of 6–12 K. Here the abscissa is the relaxation phonon energy (in degrees kelvin) measured with respect to the *onset* of the 2Δ rise in the Sn junction generator (1.12 meV $\sim$ 13 K). The ordinate is the derivative of the detector signal with respect to the generator current (dS/dI) and is a direct measure of the strength of the relaxation phonon intensity over and above the direct recombination background.

The rapid rise in the signal level starting at $E_c = 9.8 \pm 0.15$ K is interpreted as the experimental threshold energy above which spontaneous decay becomes vanishingly small for propagation lengths ~ 1.1 mm. Also shown in Fig. 2 is the expected two-phonon decay threshold $E_c^{(2)} = E_c(q_2) = 7.89$ K according to Donnelly *et al.*,[8] who obtained their values by applying a cubic spline fit to all available neutron scattering data on the dispersion curve at SVP. The roton minimum Δ is taken as an average of available data from Greywall.[9] The experimental signal rises until a value of about 11 K due to the finite width of the Sn gap (1.2 K), flattens as expected, and then starts decaying at E_c'' due to roton scattering as discussed in a separate paper.[10] It is important to reiterate here that the sharp *onset* at E_c can be measured quite precisely and, as indicated, with an uncertainty ~ 0.15 K.

The rise of dS/dI starts definitely before our value of E_c. This slowly increasing part of the signal could originate either from propagation of rotons or from phonons having finite lifetime against n-phonon decay. However, if it were due to roton propagation, then time-of-flight measure-

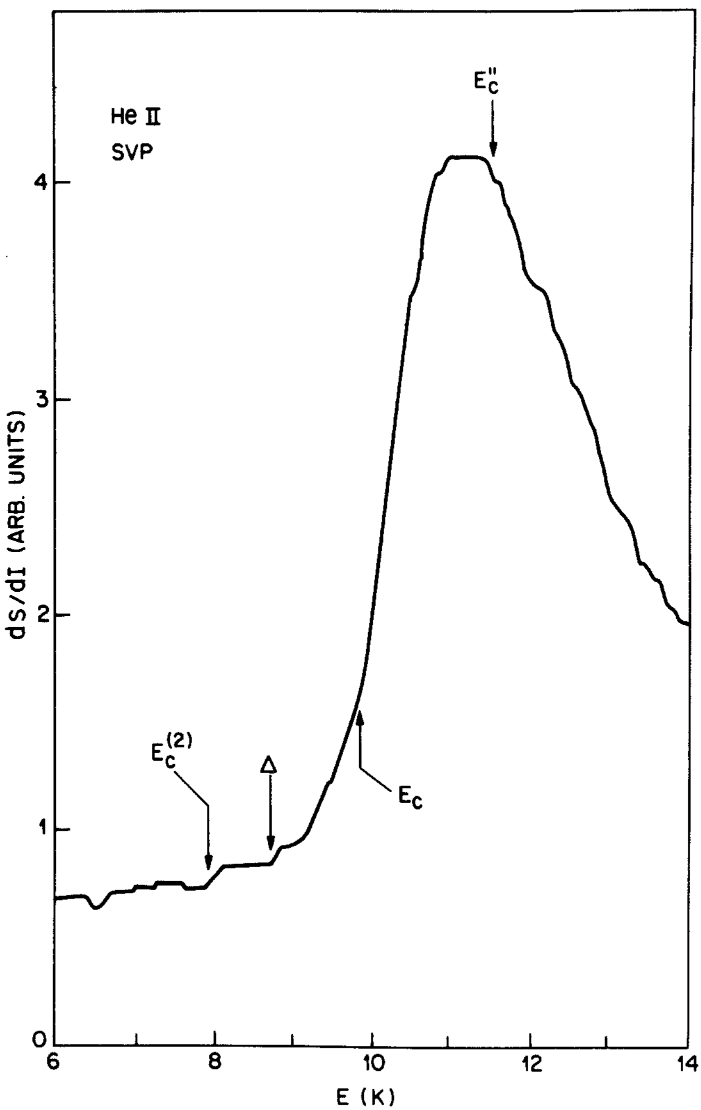

Fig. 2. Derivative of the detector signal as a function of relaxation phonon energy using a Sn tunnel junction generator and an Al tunnel detector. Propagation length 1.1 mm. Temperature $T=0.1$ K. For the definition of the various thresholds $E_c^{(2)}$, Δ, E_c, and E_c'' see text.

ments should yield a maximum group velocity[6] $v_g(q_\infty) \approx 190$ m/sec at the leading edge of the signal, since the roton group velocity does not reach this value before E_c. We have measured the leading edge velocity to be $v_g = 220 \pm 10$ m/sec, which corresponds to propagation of phonons with wave vector q_n, $n > 2$. Consequently, the observed signal at energies from Δ to E_c at SVP is more likely due to propagation of phonons than rotons. At higher pressures this kind of precursory signal was relatively much smaller.[10]

It is obvious from Fig. 2 that the difference between $E_c^{(2)}$ of Donnelly *et al.*[8] and any reasonable experimental determination of the n-phonon decay threshold E_c is much larger than the experimental uncertainty. Actually, our measurement of the onset of $E_c = 9.8 \pm 0.15$ K is close to $E_c^{(\infty)} = 9.83$ K of Donnelly *et al.*[8] and the previously reported value of 9.5 ± 0.4 K of DN. Other determinations of $E_c^{(2)}$ and $E_c^{(\infty)}$ based on analytical models due to Maris[1] and Pitayevski and Levinson[2] also suggest an almost identical difference between $E_c^{(2)}$ and $E_c^{(\infty)}$. Assuming the validity of the analytical models,[1] we see that out data suggest that the mean free path becomes truly macroscopic ($\gg 1$ mm) only for large n, with a lower bound for n being equal to 6. As suggested by Maris,[1] the precise characteristics of phonon propagation will depend on the decay rates $\Gamma^{(n)}$. Assuming an exponential damping, we find our measurements of the intensity as a function of energy suggest that $\Gamma^{2,3} \leqslant 0.3$ mm ($E_c^{(2)} = 7.9$ K, $E_c^{(3)} = 8.7$ K), $\Gamma \sim 0.4$ mm at $E_c^{(4)} \sim 9.1$ K, $\Gamma \sim 0.5$ mm at $E_c^{(5)} \sim 9.5$ K, and $\Gamma \gg 1$ mm at $n \gg 6$, i.e., close to the expected stability point when the phonon velocity equals the sound velocity.

In summary, we have studied in detail the spontaneous decay threshold in liquid He II at SVP using superconducting Sn tunnel junctions. The measurements are in excellent accord with other experimentally measured dispersion parameters provided one identifies the onset of long mean free paths to n-phonon decay with n large (≥ 6).

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