

Auction Based Mechanisms for Electronic Procurement

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Abstract—Auction based mechanisms are extremely relevant in modern day electronic procurement systems since they enable a promising way of automating negotiations with suppliers and achieving the ideal goals of procurement efficiency and cost minimization. This article surveys recent research and current art in the area of auction based mechanisms for e-procurement. The survey delineates different representative scenarios in e-procurement where auctions can be deployed and describes the conceptual and mathematical aspects of different categories of procurement auctions. We discuss three broad categories: (1) *single item auctions*: auctions for procuring a single unit or multiple units of a single homogeneous type of item; (2) *multi-item auctions*: auctions for procuring a single unit or multiple units of multiple items; and (3) *multi-attribute auctions* where the procurement decisions are based not only on costs but also on attributes such as lead times, maintenance contracts, quality, etc. In our review, we present the mathematical formulations under each of the above categories, bring out the game theoretic and computational issues involved in solving the problems, and summarize the current art. We also present a significant case study of auction based e-procurement at General Motors.

A Note to Practitioners— Since the burst of the dot com bubble, many procurement professionals and purchasing managers have begun to question the ability of the Internet to redefine procurement processes within their firms. In this article we set out to show that this would be a misplaced sense of *deja vu* because the Internet along with a milieu of decision technologies based on Game Theory and Optimization is proving to be a significant tool in the hands of procurement professionals. Sans all the hype, the dot com phenomena has left behind useful ideas including that of e-platforms for on-line auctions. Building upon this core conceptual construct familiar to most procurement professionals, we set out to survey the exciting field of research this has opened up with a vast potential for immediate and gainful applications. We review the existing state-of-the-art in this field, track its recent developments and classify the models available for different procurement scenarios. We also provide pointers to areas that require further fundamental as well as applied research which calls for the attention of not just academic researchers but also practicing professionals.

Index Terms—E-Procurement, negotiations, auctions, single item auctions, volume discount auctions, multi-item auctions, combinatorial auctions, multi-attribute auctions, game theory, mechanism design, NP-hard problems.

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ACRONYMS

PR	Purchase Request
ICT	Information and Communication Technologies
RFQ	Request for Quote
LP	Linear Program
IP	Integer Program
MIP	Mixed Integer Program
MILP	Mixed Integer Linear Program
GP	Goal Programming
CA	Combinatorial Auction
GVA	Generalized Vickrey Auction
VCG	Vickrey-Clarke-Groves (mechanism)
NP	Non-Deterministic Polynomial Time
FPTAS	Fully Polynomial Time Approximation Scheme
MAUT	Multi-Attribute Utility Theory
MCDA	Multi-Criteria Decision Analysis
IRDA	Iterative Reverse Dutch Auction

I. INTRODUCTION

THE purchasing function and the associated procurement processes in organizations, large and small, have traditionally been an important area of operations affecting business performance. In many organizations, procurement costs constitute a major part of the total costs. For example, about 60 percent of the cost of a car is attributed to procurement costs. Recent trends in the business environment suggest that the importance of procurement is being both reinforced through the emergence of global supply chains and amplified by the growing incidence of outsourcing in many industrial sectors. Simultaneously, the procurement function itself has undergone much transformation. In one large global firm that the authors have worked with, decentralized, factored purchasing processes have given way to uniform, centralized purchasing practices, with worldwide purchasing decisions being coordinated by a single centralized organization. These changes can, in part, be attributed to the influence that information and communication technologies (ICTs) have had in reshaping procurement processes both within and between organizations.

The procurement process itself may be hierarchically decomposed and a first level decomposition yields four distinct stages: *supplier search and analysis*, *supplier selection stage*, *automated transactional stage*, and *supply chain planning and control*. Many aspects of procurement, in each of the four stages, have benefited from the application of ICTs and decision technologies. However, *negotiations*, which form a crucial part of the supplier selection stage have so far relied on human based processes with little technological support. Also, *Negotiation Theory*, a framework for reaching decisions through consensus whenever a person, organization, or any other entity

cannot achieve its goals unilaterally [1], [2], has traditionally been a topic of research within the economic and social science community. However, with recent advances in ICTs, including the rapid growth of the Internet, an opportunity exists to automate negotiations that happen at the supplier selection stage. The design of such automated negotiations requires an operations research (OR) and computer science/information systems perspective to feed back into the negotiation models and procedures devised by the economists and social scientists.

This interdisciplinary research has contributed to three closely related approaches to automated negotiation systems: (1) development of negotiation support systems; (2) software agents for negotiation; and (3) auction mechanism design and on-line auction platforms. Each of these approaches addresses the requirements of a wide variety of negotiation scenarios which is captured in [3] as integrative and distributive negotiations. For descriptions of the first two approaches, we refer the reader to [4], [5], [6]. Here our focus is on the use of automated auctions for electronic procurement.

A. Representative Procurement Scenarios

We first describe a few practical procurement scenarios, based on our experience in working with several major procurement organizations. The scenarios progressively illustrate the range of complexity in procurement situations: from buying the proverbial *pin (very simple) to a plane (very complex)*.

1) *Scenario A: Single Item Procurement:* A Purchase Request (PR) to buy a specific cutting tool is generated by a user department, such as Production. This request is communicated to a central buying department called Purchasing. The buyer responsible for this category of items acknowledges the request. If the item is in one of the standard price lists of vendors already supplying to the organization and a blanket order already exists then a spot buy order is sent to the supplier. Otherwise a request for quote (RFQ) is prepared by the buyer and sent to a selection of suppliers who respond with quotes. These quotes are analyzed and a sourcing recommendation is made based on the prices that are quoted. This is a very simple buying situation where the decision to buy a single item is made on the basis of a single attribute - price per unit.

Now, consider the same situation as above except that multiple units of the item are to be bought. The suppliers are now able to exploit economies of scale and hence provide bids with volume discounts thereby introducing one level of complexity in the buyer's supplier selection or bid selection problem also sometimes called the winner determination problem.

In the scenario above, very often it is observed that the same tool is required by many plants across a geographical region or across all plants worldwide. In such cases the buyer creates a blanket order which can be used by all the plants. The blanket order specifies a single price and a delivery point which is the nearest pickup location operated by a third party logistics provider on behalf of the manufacturer. Clearly in this case the sourcing decision involves a greater degree of complexity since the total cost of procurement - the cost of the item(s) plus the shipment cost - is to be optimized by the buyer.

The auction mechanisms motivated by this scenario are discussed in Section III of this paper.

2) *Scenario B: Multi Item Procurement:* Here is an extension to scenario A. Purchase requests may be raised by user departments across the organization for many types of cutting tools. These requests are processed by a single buyer responsible for the procurement of cutting tools. The requests are aggregated and in some cases bundled and an RFQ is generated. The suppliers respond with bids for bundles which could indicate volume discounts as well as point prices. The buyer has to make a decision of allocating the orders so as to minimize his total cost of procurement as well as restrict the number of suppliers who receive the orders so as to reduce management overhead. The decision problem in this case can be challenging, and in some cases if the number of suppliers is large then the computations involved can overwhelm the buyer.

The auction mechanisms relevant for this scenario are discussed in Section IV.

3) *Scenario C: Multi-Attribute Procurement:* While it may be possible to buy cutting tools off-the-shelf by negotiating along only one dimension - price, buying a machine tool for a specific machining requirement depends on many attributes. These may include at least the following: quality of machine tool, lead time to manufacture and deliver, availability of spares, maintenance spares to be held in inventory, etc. It may be possible to attach a monetary value to some or all of the attributes that influence the purchase decision. In some cases, additional features and options could be purchased as add-on components to the basic machine tool from some or all of the suppliers. Grappling with such multi-attribute procurement decisions is the *raison d'être* of purchasing departments.

The auction mechanisms motivated by the above scenario are discussed in Section V.

B. Auctions and Electronic Procurement

The field of auctions, as a sub-field of mechanism design, is concerned with the design of the *rules of interaction*, using the tools of game theory and mechanism design [7], [8], for economic transactions that will yield some desired outcome. In the context of negotiations for procurement we require rules governing: (1) bidding for contracts, (2) the issues and attributes that will be considered to determine winner(s) of the contract, (3) determination of winning suppliers, and (4) the payments that will be made. English auctions, Dutch auctions, and sealed bid contracts are well understood, widely used economic mechanisms in the procurement context. Since the *rules of interaction* in these auctions are well laid out, they have been a natural target for automation, and as a result have formed the core conceptual constructs on which on-line auctions, like those seen in www.ebay.com, www.onsale.com, www.freemarkets.com, etc., have been based. In order to address the software requirements of such on-line auctions, firms such as Ariba, Emptoris, Frictionless Commerce, etc. have appeared. These firms have now expanded their software product portfolio to include more general e-procurement tools. See <http://www.research.ibm.com/absolute/> for more details on these software vendors.

Many organizations have started using automated auction technology for procurement. Examples include: Compaq Com-

puter Corp, General Dynamics, Dutch Railway, General Electric, Sears Logistics, and Staples Incorporated [9], [10], [11]. For instance, General Electric has adopted on-line auctions for many of its procurement operations, procuring more than \$6 billion worth of goods and services in on-line auctions in 2000 [10], which led to the *Internet Week* magazine awarding it the title "E-Business of the Year 2000". Many more firms, for example, GlaxoSmithKline [12], Metro [13], Volkswagen [14], and Bechtel [15] have already started using auctions for a significant share of their procurement operations (up to 30 percent in many of them). A study conducted by the Center for Advanced Purchasing Studies [16] shows that more than a third of the firms interviewed by them are procuring goods in excess of \$ 100 million through electronic procurement auctions. However, in many cases, the only information that can be gleaned about these implementations is from articles appearing in the popular press. Detailed case studies, apart from a few exceptions [17], [18], [19], [11], [20], [21], [22], [23] have been hard to come by possibly because procurement is considered to be a key source of strategic advantage and hence organizations have been unwilling to put the details of implementation into the public domain.

C. Contributions and Outline of the Paper

The primary motivation for this paper comes from the key role that auctions have come to play in electronic procurement where they promise a faster convergence to high quality procurement contracts, even in complex B2B industrial procurement settings. The secondary motivation arises from the fact that there are many key results in this area but these results are spread across a wide body of literature. Motivated by the above two observations, our goal in this paper is to provide a comprehensive review of the issues, mathematical formulations, and the current art in this area, in a self-sufficient way.

This paper differs from other survey articles that have appeared on related topics in the following ways.

- This paper is not a survey on auctions in general. There are widely popular books (for example, by Milgrom [24] and Krishna [25]) and surveys on auctions (for example, [26], [27], [28], [29], [30]) which deal with auctions in a comprehensive way.
- This paper is not a survey on combinatorial auctions (currently an active area of research). Exclusive surveys on combinatorial auctions include the articles by de Vries and Vohra [31], [32], Pekec and Rothkopf [33], and Narahari and Pankaj Dayama [34]. Cramton, Ausubel, and Steinberg [35] have recently brought out a comprehensive edited volume containing expository and survey articles on varied aspects of combinatorial auctions.
- This paper is not a survey on dynamic pricing. There are excellent surveys on dynamic pricing [36], [37]. As is well known, auctions represent a particular mechanism for dynamic pricing.

The focus of our paper is on auction based mechanisms which can be used as part of an e-procurement process to improve the efficiency of the process. It is therefore closer in its

content to the papers by Elmaghraby and Keskinocak [18], Elmaghraby [38], Bichler, Davenport, Hohner, and Kalagnanam [23], and Bichler *et al* [37]. While the papers [18], [38] deal with combinatorial auctions for procurement together with a description of a case study, the papers [23], [37] deal with volume discount auctions and multi-attribute auctions. These papers deal with certain specific categories of procurement auctions. In contrast to these papers, our paper provides an umbrella view of the entire area in a self-sufficient way.

The rest of the paper is structured as follows.

Section II: Issues in Electronic Auctions: This section is intended to present the basic intuition behind auctions and game theoretic modeling. This section is structured as follows: [a] Types of Auctions, [b] Design of Auctions [c] Properties Desired from an Auction [d] Design Space for Auction Mechanisms [e] Computational Complexity Issues [f] Relevance of Game Theoretic and Incentive Issues in E-Procurement.

Sections III, IV, and V: In these three sections, we discuss three major, broad categories of procurement auctions:

- *Single Item Procurement Auctions:* Procurement of single unit or multiple units of a single item based on a single attribute (Section III)
- *Multi-Item Procurement Auctions:* Procurement of single unit or multiple units of multiple items based on a single attribute (Section IV)
- *Multi-Attribute Procurement Auctions:* Procurement of multiple units of a single item based on multiple attributes (Section V)

In each subsection, we begin by discussing the main issues involved followed by a detailed discussion of a few best practice mechanisms from the literature. We conclude each section with a summarizing discussion on the state-of-the-art and provide a tabular listing of important current papers relevant to the topic.

Section VI: A Case Study from General Motors: In this section, we discuss a real world case study of electronic procurement at General Motors. The last four authors of this paper were part of a team that developed an auction/optimization model for this procurement scenario. This case study provides a solid, recent application of procurement auctions and optimization.

Section VII: Summary and Future Work: We summarize the contents of this paper and we conclude with a discussion of some issues that need to be addressed with the current genre of models and directions for future work.

II. ISSUES IN ELECTRONIC AUCTIONS

In this section, we first briefly discuss different types of auctions. Next we provide an overview of a game theoretic and mechanism design setting for design of auctions and present certain key properties that are required from an auction. We then present certain important impossibility and possibility results from mechanism design theory which define the design space for auction mechanisms. After this, we look at some computational issues in implementing auctions. Finally, we bring out the need for and importance of game theoretic modeling in design of procurement mechanisms. This section

builds the necessary background for our discussion of procurement auction mechanisms in Sections III, IV, and V. For a more comprehensive and detailed treatment of auction theory in general, reader is referred to [26], [27], [29], [39], [24], and [25].

A. Types of Auctions

An Auction is a mechanism to allocate a set of goods to a set of bidders on the basis of bids and asks. In a classical auction, there is an auctioneer who wishes to allocate a single indivisible item to a buyer from among a group of bidders. There are four basic types of auctions described in the literature [26], [27], [29]: Open cry or English auction, Dutch auction, first price sealed bid auction, and second price sealed bid auction (also called the Vickrey auction). An English auction is an iterative auction where the bidders submit monotonically increasing bids. This iterative process continues until a price is reached where there is just a single bidder who is willing to buy. The item is awarded to this buyer at the final bid price. The Dutch auction is the reverse of the English auction where the price is monotonically decreased by the auctioneer until there is a buyer who is willing to buy at the currently announced price. In both these auctions one must note that they are iterative in nature and price signals are continuously being fed back to the bidders. The first and second price sealed bid auctions however are single round auctions where bidders submit sealed bids. The winner of the contract is the bidder who submits the highest bid. The payment that the winner makes in the First Price sealed bid auction is the bid price itself. In the second price sealed bid auction also called the *Vickrey Auction* [40], the payment that the winner makes is the second highest bid. This specific payment rule makes truthful bidding a best response for the bidders [40]. There is an important result called the *revenue equivalence theorem* [26], [27] which states that although these auction mechanisms are vastly different from each other, they yield the same expected revenue for the auctioneer when one item is being sold, under the following set of assumptions:

- 1) The bidders are *risk neutral*, in the sense of having linear utility functions.
- 2) The valuations of bidders follow the *independent private values model* (this means that the players draw their valuations for the item from mutually independent probability distributions)
- 3) The bidders are *symmetric* (that is, the valuation distributions are identical)
- 4) Payments depend only on bids.

Building on these basic types, auctions have evolved rapidly to include multiple resources, business level constraints, and more complex market structures. Kalagnanam and Parkes [30] have suggested a framework for classifying auctions based on six major factors as outlined below:

(1) *Resources*: Resources are the entities over which the negotiation in an auction is conducted. The resources could be a single item or multiple items, with a single or multiple units of each item.

(2) *Market Structure*: There are three types of market structures in auctions. In *forward auctions*, a single seller sells resources to multiple buyers. In *reverse auctions*, a single buyer attempts to source resources from multiple suppliers, as is common in procurement. Auctions with multiple buyers and sellers are called *double auctions* or *exchanges*.

(3) *Preference Structure*: The preferences define an agent's utility for different outcomes in the auction. For example, when negotiating over multiple units agents might indicate a decreasing marginal utility for additional units. An agent's preference structure is important when negotiation happens over attributes for an item, for designing scoring rules used to signal information, etc.

(4) *Bid Structure*: The structure of the bids within the auction defines the flexibility with which agents can express their resource requirements. For a simple single unit, single item commodity, the bids required are simple statements of willingness to pay/accept. However, for a multi-unit identical items setting bids need to specify price and quantity. This introduces the possibility for allowing volume discounts. With multiple items, bids may specify all-or-nothing, with a price on a bundle of items.

(5) *Winner Determination*: Other phrases which are used synonymously with *winner determination* are *market clearing*, *bid evaluation*, and *bid allocation*. In the case of forward auctions, winner determination refers to choosing an optimal mix of buyers who would be awarded the items. In the case of reverse auctions, winner determination refers to choosing an optimal mix of sellers who would be awarded the contracts for supplying the required items. In the case of an exchange, winner determination refers to determining an optimal match between buyers and sellers. The computational complexity of the winner determination problem is an important issue to be considered in designing auctions.

(6) *Information Feedback*: An auction protocol may be a *direct* mechanism or an *indirect* one. In a direct mechanism such as a sealed bid auction, agents submit bids without receiving feedback, such as price signals, from the auction. In an indirect mechanism, such as an ascending-price auction, agents can adjust bids in response to information feedback from the auction. Feedback about the state of the auction is usually characterized by a *price signal* and a *provisional allocation*, and provides sufficient information about the bids of winning agents to enable an agent to redefine its bids.

In this paper, our interest is in *procurement auctions* with a specific attention on three broad types: [a] Single item procurement auctions (single unit or multi-unit), [b] Multi-item procurement auctions (single unit or multi-unit), and [c] Multi-attribute procurement auctions.

B. Design of Auctions

The design of auctions can be viewed as a problem of designing a *mechanism* that implements a *social choice function*. Designing a mechanism, in turn, can be viewed as a problem of designing a *game with incomplete information* having an equilibrium in which the required social choice function is implemented.

Consider a set of agents or players $N = \{1, 2, \dots, n\}$ with agent i having a type set Θ_i ($i = 1, 2, \dots, n$). The type set of an agent represents the set of perceived values of an agent (also called private values). For example, in an auction, the type of an agent may refer to the valuation that the agent has for different items up for auction. The two most common models of valuation used in the context of auction design are: *independent-private-values model* and *common-value model* [26]. In the *independent-private-values-model* model, each bidder knows precisely how much she values the item. She does not know the valuations of other bidders for this item but perceives any other bidder's valuation to be a draw from a probability distribution. Also, she knows that the other bidders regard her own valuation as being drawn from a probability distribution. In the *common-value model*, the item being auctioned has a single objective value but nobody knows its true value. The bidders, having access to different information sources, have different estimates of the item's valuation.

Let Θ be the Cartesian product of all the type sets of all the agents (that is Θ is the set of all type profiles of the agents). Let X be a set of outcomes. An outcome, in the context of auctions, corresponds to an assignment of auction items to bidders and the payments to be made to or received by the bidders. A social choice function is a mapping from Θ to X and associates an outcome with every type profile. A social choice function in an auction corresponds to a desirable way of producing outcomes from given type profiles. Let S_i denote the action set of agent i , that is S_i is the set of all actions that are available to an agent in a given situation. A strategy s_i of an agent i is a mapping from Θ_i to S_i . That is, a strategy maps each type of an agent to a specific action the agent will choose if it has that type. In an auction, a strategy corresponds to the bid the agent will place based on its observed type. Let S be the Cartesian product of all the strategy sets. A mechanism μ is basically a tuple $(S_1, S_2, \dots, S_n, g(\cdot))$, where g is a mapping from S to X . That is, $g(\cdot)$ maps each strategy profile into an outcome. A given mechanism can always be associated with a game with incomplete information, which is called the game induced by the mechanism. For details, refer to [7], [41].

We say that a mechanism $\mu = (S_1, S_2, \dots, S_n, g(\cdot))$ implements a social choice function f if there is an equilibrium strategy profile $(s_1^*(\cdot), s_2^*(\cdot), \dots, s_n^*(\cdot))$ of the game induced by μ such that $g(s_1^*(\theta_1), s_2^*(\theta_2), \dots, s_n^*(\theta_n)) = f(\theta_1, \theta_2, \dots, \theta_n)$ for all possible type profiles $(\theta_1, \theta_2, \dots, \theta_n)$. That is, a mechanism implements a social choice function $f(\cdot)$ if there is an equilibrium of the game induced by the mechanism that yields the same outcomes as $f(\cdot)$ for each possible profile of types. Depending on the type of equilibrium we qualify the implementation. Two common types of implementations are *dominant strategy implementation* and *Bayesian Nash implementation*, corresponding respectively to *weakly dominant strategy equilibrium* and *Bayesian Nash equilibrium*. The weakly dominant strategy equilibrium is an extremely robust solution concept that ensures that the equilibrium strategy of each agent is an optimal strategy regardless of the strategies of the rest of the agents. The Bayesian Nash equilibrium is a weaker solution concept that only guarantees that the

equilibrium strategy of each agent is optimal with respect to the equilibrium strategies of the other agents. For a detailed discussion of these, of these, the reader is referred to [7].

A *direct revelation mechanism* corresponding to a social choice function $f(\cdot)$ is a mechanism of the form $\mu = (\Theta_1, \Theta_2, \dots, \Theta_n, f(\cdot))$. That is, the strategy sets are the type sets itself and the outcome rule $g(\cdot)$ is the social choice function itself. A social choice function is said to be incentive compatible in dominant strategies (or strategy proof or truthfully implementable in dominant strategies) if the direct revelation mechanism μ implements $f(\cdot)$ in a weakly dominant strategy equilibrium where the equilibrium strategy of each agent is to report its true type. Similarly a social choice function is said to be Bayesian Nash incentive compatible if the direct revelation mechanism μ implements $f(\cdot)$ in a Bayesian Nash equilibrium where the equilibrium strategy of each agent is to report its true type. The *revelation principle* [7] states that if a social choice function can be implemented in dominant strategies (or in Bayesian Nash equilibrium), it can also be truthfully implemented in dominant strategies (or in Bayesian Nash equilibrium). The revelation principle enables one to focus attention only on incentive compatible mechanisms.

The mechanism design problem is to come up with a mechanism that implements a *desirable* social choice function. Some desirable properties which are sought from a social choice function and hence from the implementing mechanism (and in the present case, from procurement auctions) are described next [7], [41], [33].

C. Properties Desired from an Auction

We now present an intuitive discussion of properties that an auction designer looks for while designing an auction mechanism. For a detailed treatment of these, the reader is referred to [7], [8].

Solution Equilibrium: The solution of a mechanism is in equilibrium, if no agent wishes to change its bid, given the information it has about other agents. Many types of equilibria can be computed given the assumptions about the preferences of agents (buyers and sellers), rationality, and information availability. They include: *Nash equilibrium*, *Bayesian Nash Equilibrium*, *weakly dominant strategy equilibrium*.

Efficiency: A general criterion for evaluating a mechanism is *Pareto efficiency*, meaning that no agent could improve its allocation without making at least one other agent worse off. Another metric of efficiency is *allocative efficiency* which is achieved when the total utility of all the winners is maximized. When allocative efficiency is achieved, the resources or items are allocated to the agents who value them most. These two notions are closely related to each other; in fact, when the utility functions take a special form (such as quasi-linear form [7]), Pareto efficiency implies allocative efficiency.

Incentive Compatibility: A mechanism is said to be incentive compatible if the agents optimize their expected utilities by bidding their true valuations of the goods. Depending on the equilibrium achieved by truthful bidding, an incentive compatible mechanism is qualified as Bayesian Nash incentive

compatible or dominant strategy incentive compatible. In the latter case, the mechanism is said to be *strategy proof*. If a mechanism is strategy proof, each agent's decision depends only on its local information and there is no need whatsoever for the agent to model or compute the strategies of the other agents.

Individual Rationality: A mechanism is said to be individually rational (or is said to have voluntary participation property) if its allocations do not make any agent worse off than if the agent had not participated in the mechanism. That is, every agent gains a non-negative utility by participating in the mechanism.

Budget Balance: A mechanism is said to be *weakly* budget balanced if the sum of monetary transfers between the buyer and the sellers is non-negative while it is said to be *strongly* budget balanced if this sum is zero. In other words, budget balance ensures that the mechanism or the auctioneer does not make losses.

Revenue Maximization or Cost Minimization: In an auction where a seller is auctioning a set of items, the seller would like to maximize total revenue earned. On the other hand, in a procurement auction, the buyer would like to procure at minimum cost. Given the difficulty of finding equilibrium strategies, designing cost minimizing or revenue maximizing auctions is not easy.

Solution Stability: The solution of a mechanism is stable, if there is no subset of agents that could have done better, coming to an agreement outside the mechanism.

Low Transaction Costs: The buyer and sellers would like to minimize the costs of participating in auctions. Delay in concluding the auction is also a transaction cost.

Fairness: Winner determination algorithms, especially those based on heuristics, could lead to different sets of winners at different times. Since there could be multiple optimal solutions, different sets of winners could be produced by different algorithms. This creates a perception of unfairness and can influence bidders' willingness to participate in auctions. Bidders who lose even though they could have won with a different algorithm could end up feeling unfairly treated.

D. Design Space for Auction Mechanisms

Ideally, one would like to put in place an auction mechanism that satisfies all the properties described above, while simultaneously achieving computational tractability. Unfortunately this is not always possible, as shown by several landmark results in mechanism design theory which rule out the possibility of mechanisms simultaneously satisfying certain combinations of properties. Fortunately, there do exist certain other combinations of properties which can be simultaneously satisfied. We indicate below some of the important results germane to the discussion on procurement mechanisms.

- Arrow [7] first pointed out the impossibility of implementing a Pareto efficient social choice function unless it is *dictatorial* (a dictatorial social choice function is a very special type of social choice function that requires the presence of a distinguished agent such that the function always chooses a top ranked outcome of this agent). The

Gibbard-Satterthwaite theorem [7] is also similar in spirit to Arrow's theorem. These impossibility results are in the context of settings where the agents simply have ordinal preferences over outcomes.

- However, when we are in quasilinear settings (where the preferences may be captured by utility functions and furthermore utilities are comparable across agents), it was shown by Groves [42] and Clarke [43] that allocatively efficient and strategy proof mechanisms are possible. These mechanisms are known by the name *VCG mechanisms*. Clarke's mechanisms are in fact a special class of Groves mechanisms. The GVA (Generalized Vickrey Auction) mechanism [44] is basically the Clarke mechanism applied to the case of combinatorial auctions. The famous Vickrey auction is a special case of the Clarke mechanism applied to the case of auction of a single indivisible item.
- The Groves mechanisms are in general not budget balanced even in quasilinear settings. There are two useful settings where they do satisfy budget balance.
 - The first setting arises when the type information of one of the agents is completely known or when the agent does not have any preferences over the allocations. In this setting, by defining monetary transfers appropriately, we can achieve the strategy proof property, allocative efficiency, and weak budget balance. The GVA mechanism is an example of one such situation. In fact, the GVA mechanism satisfies four properties simultaneously: allocative efficiency, individual rationality, weak budget balance, and strategy proofness.
 - The second setting arises when we settle for the weaker notion of Bayesian Nash equilibrium, as opposed to the much stronger dominant strategy equilibrium. In this setting, it was shown by Arrow [45] and d'Aspremont and Gerard-Verat [46] that, under quasi-linear preferences, it is possible to have a mechanism (which is called the dAGVA mechanism) that is allocatively efficient, Bayesian Nash incentive compatible, and budget balanced.
- The positive result indicated in the second setting above may not however be sustained when we insist on individual rationality. The Myerson-Satterthwaite [47] theorem articulates this negative result: Even in the simplest of trading situations such as in bilateral trade (a simple exchange setting), it is impossible to design a social choice function or a mechanism that is incentive compatible, allocatively efficient, budget balanced, and individually rational.
- Myerson [48] showed that revenue maximization, individual rationality, and Bayesian incentive compatibility can be achieved simultaneously if we sacrifice allocative efficiency. However, this result is true only while auctioning a single indivisible item.

For more details on these results, we refer the reader to [41], [7], [49], [25], [30]. The above discussion gives an idea of how the design space for auctions is constrained.

Another key factor that constrains the design space further is the computational complexity at the agent level and at the mechanism level. The central problem of a procurement auction designer is to devise the best possible mechanism that is contained in this constrained design space and that is computationally tractable.

E. Computational Complexity Issues in Auction Design

In an economic mechanism where resource allocation is based on decentralized information, computations are involved at two levels: first, at the agent level and secondly at the mechanism level. The complexity questions involved are briefly indicated below. For a more detailed discussion refer to [41], [30].

Complexity at the Agent Level:

- *Strategic Complexity*: Should agents model other agents and solve game theoretic problems to compute an optimal strategy? For instance, in a sealed bid procurement contract scenario, sellers will need to not only take their valuation of the contracts into consideration but also the bidding behavior of their competitors. This requires sophisticated bidding capability.
- *Valuation Complexity*: How much computation is required to provide preference information within a mechanism? For instance, in a multi-item procurement scenario where the items exhibit cost complementarities, estimating a bid for every possible permutation of the bundle of items is costly.

Complexity at the Mechanism Level:

- *Winner Determination Complexity*: How much computation is expected of the mechanism infrastructure to compute the winning agents given the bid information of the agents. It turns out in many common auction problems that the winner determination problem is NP-hard [41], [30], [31].
- *Payment Determination Complexity*: How much payment do the winners of an auction make or receive? In many situations, determining the price or payment also turns out to be an NP-hard problem [41], [30].
- *Communication Complexity*: How much communication is required between agents and the mechanism to compute an outcome. For instance, in an English auction, where individual valuations are revealed progressively in an iterative manner, the communication costs could be high if the auction were conducted in a distributed manner over space and/or time.

The point to be noted is that even if we are able to design a mechanism that satisfies many desired properties, the computational complexity at the mechanism level and at the agent level may prohibit the use of this mechanism. An example here is that of GVA applied to multi-item procurement auctions. GVA satisfies allocative efficiency, weak budget balance, strategy proofness, and individual rationality. However, the winner determination problem turns out to be a set covering problem which is NP-hard. The payment determination problem is also a set covering problem. In fact, the payment determination problem is to be solved for each winner separately!

F. Relevance of Game Theory and Incentive Issues in E-Procurement

It may appear, at first glance, that there is a disconnect between game theory and mechanism design on the one hand and actual, real-world procurement negotiations on the other. In our opinion, such an interpretation would be specious and misleading for the following reasons.

- Firstly, procurement professionals are acutely aware of the information asymmetry that exists in negotiations and as a result the need to bargain hard. The analysis carried out by procurement professionals before such bargaining often implicitly involves game theoretic reasoning. Witness is provided by the plethora of auction formats that procurement professionals employ in practice. A formal game theoretic analysis only helps to place such reasoning on a firm basis. In the absence of sound game theoretic analysis, naive auction strategies may actually result in disastrous consequences. For evidence of this we point the reader to a discussion of spectrum auctions [28].
- Secondly, often it is argued that human agents fail to measure up to the strict requirements of the fundamental assumptions of rationality and intelligence that game theory makes. Hence the inapplicability of game theoretic analysis. However if we extrapolate the vector of development in this area of application and assume that at some point in the not too distant future, automated negotiations carried out by software agents will become a reality then a game theoretic analysis would be eminently applicable before such systems are designed and built [44].

In summary, game theoretic considerations are already used implicitly in procurement situations. Providing appropriate incentives to the suppliers is an immediate example of application of game theoretic analysis. Therefore, sound use of game theoretic principles explicitly in designing automated procurement mechanisms will enhance the efficacy of e-procurement. This provides a compelling motivation for using auction based mechanisms for e-procurement.

III. SINGLE ITEM PROCUREMENT AUCTIONS

We discuss here models related to the procurement of a single item with price as the only consideration for decision making under a variety of different business contexts. Specifically, we review: (a) auction mechanisms for a single unit of an item, (b) auctions for multiple units of a single item with volume discount bids, and (c) auction mechanisms for procuring multiple units of a single item for multiple manufacturing points taking logistics costs into account.

A. Procurement Auctions for Single Unit of an Item

In many procurement scenarios for a single unit of an item, the English auction or a *sealed bid tender* process is generally used to decide the *winner* of the contract. In the later case the *winning* rule is generally a first price rule. The second price auction is rarely used because bidders feel that the information may be used to the unfair advantage of the buyer. However,

in Internet based buying, where anonymity of buyers and sellers and non disclosure of prices may be assured, the second price auction may yet be gainfully employed. While, we have seen from the revenue equivalence theorem, that the expected revenues from all the basic auctions may be the same, there are significantly different implications when the auctions are to be automated. We summarize the differences below.

1) *Computational Implications of the four Basic Auction Mechanisms*: As indicated in the previous section, complexity can be analyzed at the level of the agent - strategic and valuation complexities, and the mechanism - winner determination and communication complexities.

Strategic Complexity: Clearly for an agent participating in a Vickrey auction, which is incentive compatible, the strategic complexity is reduced to just bidding one's true valuation without any consideration for the strategies that would be followed by other agents. However, agents in the Dutch and first price sealed bid auctions require to base their bids not only on their private valuations but also condition them on the valuations of their competitors. This requires them to process additional information - the number of competitors, the probability distributions of their valuations, etc. which increases their computational overhead. The English auction has certain benefits relative to the Dutch and First Price Sealed auctions. In an English auction, if all bidders bid up to their true valuations, no single bidder can gain by unilaterally deviating from this *truthful* strategy.

Communication Complexity: The communication overhead, and hence the processing overhead, is clearly much higher in multi round mechanisms like the English and Dutch auctions as compared to single shot mechanisms such as the Vickrey and first price sealed bid mechanisms.

B. Volume Discount Auctions

In a procurement context when a single buyer and multiple sellers who wish to exploit scale economies are present, a *volume discount auction* is appropriate. Here suppliers provide bids as a function of the quantity that is being purchased [19], [50]. The winner determination problem for this type of auction mechanism is to select a set of winning bids, where for each bid we select a price and quantity so that the total demand of the buyer is satisfied at minimum cost.

The winner determination here is fairly straightforward if there are no business constraints such as maximum/minimum number of units that are to be purchased from a supplier, minimum/maximum number of winning suppliers, etc. The computational problem is to simply find the supplier who offers the *best price* and buy the entire quantity from this seller. However, procurement problems in practice rarely are without business constraints. See [51] for a comprehensive tabulation of business constraints that are observed in practice. With inclusion of business constraints, the winner determination problem becomes a mixed integer programming problem (MIP) [19], [52]. In the following subsection, we first present the mathematical formulation of a volume discount auction problem adapted from [50], [52] and then discuss the computational issues that arise in determining the winning bids.

TABLE I
NOTATION FOR VOLUME DISCOUNT AUCTIONS

Q	quantity of item
K	number of suppliers
k	index for the suppliers ($k = 1, \dots, K$)
B_k	supply curve (bid) from supplier k
M_k	number of price-quantity pairs in bid B_k
j	index for price-quantity pairs, $j = 1, \dots, M_k$
P_{kj}	unit price the supplier k charges if the number of units bought from this supplier is within the j^{th} interval $[Q_{kj,low}, Q_{kj,high}]$
x_{kj}	decision variable that takes value 1 if the buyer buys a quantity in the range $[Q_{kj,low}, Q_{kj,high}]$
z_{kj}	a continuous variable that specifies the exact number of units bought

1) *Mathematical formulation of volume discount auctions*: Table I provides the notation for the volume discount procurement auction model presented below. [50], [52].

- The buyer needs to procure a quantity Q of an item.
- The buyer identifies a list of potential suppliers $k = 1, \dots, K$ who can bid in the auction.
- Each supplier responds with a bid composed of a supply curve. A supply curve from supplier k given by a bid B_k consists of a list of M_k price quantity pairs, $\{(P_{k1}, [Q_{k1,low}, Q_{k1,high}]), \dots, (P_{kM_k}, [Q_{kM_k,low}, Q_{kM_k,high}])\}$. Each price quantity pair $(P_{kj}, [Q_{kj,low}, Q_{kj,high}])$ specifies the price P_{kj} that the supplier charges per unit of the item if the number of units bought from this supplier is within the interval $[Q_{kj,low}, Q_{kj,high}]$. It is assumed that the quantity intervals for the supply curve are all pairwise disjoint. Also, note that different unit prices are used for different ranges within the overall quantity Q_{kj} (that is, if a quantity spans multiple intervals, the unit price for different spanned intervals will be taken as the designated unit prices in the intervals).

The MIP formulation is as follows:

- A decision variable x_{kj} is associated with each price-quantity pair $(P_{kj}, [Q_{kj,low}, Q_{kj,high}])$ for each bid B_k . This is a 0 – 1 variable taking on the value 1 if we buy some number of units within the price range and 0 otherwise.
- A continuous variable z_{kj} is associated with each price-quantity pair, which specifies the exact number of units of the item that are purchased from the bid b_k within this price-quantity pair.

The formulation is (see Table I):

$$\min \sum_{k=1}^K \sum_{j=1}^{M_k} z_{kj} P_{kj} + \sum_{k=1}^K \sum_{j=1}^{M_k} x_{kj} C_{kj} \quad (1)$$

subject to:

$$z_{kj} - (Q_{kj,high} - Q_{kj,low}) x_{kj} \leq 0 \quad \forall k = 1, \dots, K; j = 1, \dots, M_k. \quad (2)$$

$$\sum_{j=1}^{M_k} x_{kj} \leq 1 \quad \forall k = 1, \dots, K. \quad (3)$$

$$\sum_{k=1}^K \sum_{j=1}^{M_k} (z_{kj} + x_{kj} Q_{kj,low}) \geq Q \quad (4)$$

$$x_{kj} \in \{0, 1\} \quad \forall k = 1, \dots, K; \forall j = 1, \dots, M_k.$$

$$z_{kj} \geq 0 \quad \forall k = 1, \dots, K; \forall j = 1, \dots, M_k.$$

The coefficient C_{kj} is a constant and computed a priori as:

$$C_{kj} = \sum_{l=1}^{j-1} P_{kl}(Q_{kl,high} - Q_{kl,low}) \quad (5)$$

In the formulation provided by [50], it is assumed that N types of items are being bought, which is a more generalized problem. However even with a single item, the MIP formulation is *NP-hard*. Additional side constraints such as a limit on the number of winning suppliers and quantity constraints at the level of the supplier increase the complexity of the decision problem.

2) *Computational Issues*: The model is developed to address the winner determination problem with the implicit assumption that agents participating in the auction have a bidding strategy in place, perhaps with the use of game theoretic analysis. This being the context, we restrict our attention to computational issues that arise at the mechanism level and neglect analysis of any computational issues at the agent (bidder) level. The MIP formulation in the previous section is a variation of the multiple choice knapsack problem which is *NP-Hard*. Although Knapsack Problems, from a theoretical point of view, are almost intractable as they belong to the family of *NP-hard* problems, several of the problems may be solved to optimality in fractions of a second [53] and can be considered as the simplest among *NP-Hard* problems in combinatorial optimization. This is because the knapsack problem exhibits special structural properties which makes it easy to solve. Dynamic programming techniques, branch-and-bound algorithms and polynomial time approximation schemes have been proposed to solve knapsack problems. For a detailed exposition of several of these methods refer to [53]. Kameshwaran [54] shows that multi-unit procurement with volume discount bids leads to piecewise linear knapsack problems and proposes a variety of exact and heuristic techniques to solve this class of problems.

C. Single Item, Multi-Unit Procurement to Minimize Total Cost of Procurement (Supply Chain Auction)

In the previous subsections, we examined auction mechanisms in automating supplier selection situations which coincide with a fairly operational view of procurement. When the supplier selection process within a supply chain setting is viewed, at a higher level of granularity, through a strategic lens, a procurement manager is concerned with a larger set of issues: What is the impact of a sourcing decision on the total cost of procurement? How does one structure the sourcing pool

so that the total cost of procurement is minimized? A partial list of important cost components that contribute to the total cost of procurement could include: price per unit, logistics cost, inventory holding costs, and lead time costs. Classical auction literature, however, has focused on price and ignored the impact of other costs on the sourcing decision.

A first step in incorporating these other cost components while making a strategic sourcing decision is taken by Chen, Janakiraman, Roundy, and Zhang [55]. They provide the design of single item, multi-unit auction mechanisms that achieve overall supply chain efficiency while taking into account production and transportation costs. The concern is to bring together the business requirements within a global supply chain, economic/game theoretic desiderata, and computational issues in building the decision model. We call this the *supply chain auction*.

The procurement problem addressed is as follows: A buyer has requirements, called consumption quantities, for a certain component at a set of geographically diverse locations. It is assumed that the buyer has private valuations of consumption quantities at the demand locations, which forms the consumption vector and that she will act strategically to maximize her utility. Multiple suppliers, each owning a set of production facilities, are available to satisfy this demand. Every supplier has a production cost, which can be described by a convex function of the quantities that he produces at his production locations. It is also assumed that each supplier is a rational, self interested player who is trying to maximize his payoff (payment received minus the production cost). In addition the buyer knows the per unit transportation cost along each link in the supply network and she pays for all shipments along the links.

A key requirement in such resource allocation problems is to ensure that the allocation is efficient, in the sense that contracts are allocated to suppliers who provide the best value. To do this the buyer needs to know the true production costs of the suppliers. So, efficient allocation and incentive compatibility are two of the key desiderata. There are three key mechanisms for multi unit auctions: Pay-as-you-bid, uniform-price, and VCG auctions. From auction literature, it is known that VCG based mechanisms are both incentive compatible and efficient. The auction models presented below for supply chain procurement essentially invoke the VCG structure.

1) *Model Formulation*: Chen, Janakiraman, Roundy, and Zhang [55] present three separate auction mechanisms - Auction T, Auction R, and Auction S. The first two mechanisms incorporate transportation costs explicitly whereas in the third model, only production costs are considered for the allocation decision and transportation decisions are made subsequently. This provides a means to compare the two approaches and the implications for total cost of procurement strategies. Table II provides the notation for the models introduced below.

2) *Auction T*: In this auction mechanism, the buyer submits a fixed consumption vector \mathbf{q} to the auctioneer. Supplier k submits to the auctioneer a bid function $\mathcal{F}_k(\mathbf{x}_k)$ for supplying \mathbf{x}_k units, for which he incurs a production cost $C_k(\mathbf{x}_k)$, $\mathbf{x}_k \in \mathcal{R}^{|N^k|}$. The supplier may or may not see the consumption vector.

TABLE II
NOTATION FOR AUCTION T, AUCTION R AND AUCTION S

K	number of suppliers
k	index for suppliers, $k = 1, \dots, K$
N	total number of supplier production facilities
n	index for production facilities, $n = 1, \dots, N$
M	total number of buyer locations
m	index for buyer locations, $m = 1, \dots, M$
N^k	set of production facilities owned by supplier k
k^n	index for supplier that owns production facility n
q_m	consumption at demand center m , $q_m > 0$
x_n	production quantity at production facility n
\mathbf{x}_k	vector of size $ N^k $
y_{nm}	quantity shipped from the production facility n to demand center m
τ_{nm}	cost of transporting unit quantity from production facility n to demand center m
z_{km}	$\sum_{n \in N^k} y_{nm}$, total quantity shipped to demand center m by supplier k
$C_k(\mathbf{x}_k)$	production cost function for supplier k ($\mathcal{R}^{ N^k } \rightarrow \mathcal{R}$)
$F_k(\mathbf{x}_k)$	bidding function from supplier k ($\mathcal{R}^{ N^k } \rightarrow \mathcal{R}$)

The auctioneer decides the quantities awarded to each supplier, and the amounts transported from suppliers' production centers to buyer's demand locations by solving the following winner determination problem.

$$\min \sum_{k=1}^K \mathcal{F}_k(\mathbf{x}_k) + \sum_{n=1}^N \sum_{m=1}^M \tau_{nm} y_{nm} \quad (6)$$

subject to:

$$\sum_{n=1}^N y_{nm} = q_m, m = 1, \dots, M. \quad (7)$$

$$\sum_{m=1}^M y_{nm} = x_n, n = 1, \dots, N. \quad (8)$$

$$y_{nm} \geq 0, m = 1, \dots, M; n = 1, \dots, N. \quad (9)$$

A Vickrey based payment rule, belonging to the more general truth-inducing VCG family described in [56], is used. The rule, in words, essentially is:

$$\begin{aligned} \text{Payment to} &= \text{Bonus payment on} &+ & \text{Bid of} \\ \text{vendor } k & \text{account of the value} & & \text{vendor } k \\ & \text{that the vendor } k \text{ adds} & & \\ & \text{to the system by} & & \\ & \text{participating in the} & & \\ & \text{auction.} & & \end{aligned}$$

That is, if $\pi(\mathbf{q})$ is the optimal value of the objective function for a given \mathbf{q} ; $\mathcal{Q} = \{\mathbf{q} : \mathbf{q} > \mathbf{0}, \pi(\mathbf{q}) < \infty\}$ and we restrict $\mathbf{q} \in \mathcal{Q}$ to ensure sufficient supply capacity. If $(\mathbf{x}^T, \mathbf{y}^T)$ is an optimal solution, and $\pi^{-k}(\mathbf{q})$ is the optimal value of the objective function with the additional constraint that the supplier k does not participate in the auction ($\mathbf{x}_k = \mathbf{0}$), the buyer will pay supplier k :

$$\psi_k^T(\mathbf{q}) = \pi^{-k}(\mathbf{q}) - \pi(\mathbf{q}) + \mathcal{F}_k(\mathbf{x}_k^T) \quad (10)$$

While it is true that the payment rule induces rational suppliers to bid their true costs, $\mathcal{F}_k(\mathbf{x}_k) = C_k(\mathbf{x}_k)$, irrespective of other suppliers' bids, it also results in adverse effects for the buyer. In situations where the production capacity is tight, and the production costs are sharply convex, the contribution that a supplier makes to the system can be significantly high resulting in disproportionately high payments even when production is at a low cost. Alternately, as pointed out by the authors, when suppliers' capacities are asymmetric, removing a supplier to solve the optimization problem may result in computing an infinite value for the contribution that is made, which is an undesirable consequence. To rectify these problems, the authors propose *Auction R*.

3) *Auction R*: In this auction mechanism, the winner determination (optimization) problem solved by the auctioneer retains the essential structure of *Auction T* except that the payment rule is modified. This modification results in much lower payments, compared to *Auction T*, by the buyer while simultaneously ensuring that the sellers bid their true costs. In essence, incentive compatibility is retained.

The key idea introduced to achieve this result is as follows: Here, the buyer submits a utility function $W(\mathbf{q})$, which is a proxy for the true consumption utility $U(\mathbf{q})$, in place of a consumption vector as in *Auction T*. The auctioneer solves a modified version of the winner determination (optimization) problem using a new payment rule. This rule follows the VCG payment structure but also incorporates the utility function $W(\mathbf{q})$ in computing the payments. The utility function essentially serves the role of a reservation price function for each unit of the item that the buyer might acquire. While in most auctions the payment made to a supplier is based on *bids from all suppliers*, in *Auction R*, part of the payment is determined by the proxy utility function acting as a reservation price function. This prevents unusually large payments.

There is however a *price to pay* for this improvement. In order to submit an optimal utility function $W^*(\mathbf{q})$, the buyer needs to know the suppliers' production cost functions. However this may not hold in reality. So the buyer will have to expend effort to at least get a probabilistic belief of the cost functions. This causes uncertainty in both payments and consumption quantities. Numerical examples to illustrate this effect are provided in [55].

4) *Auction S*: In order to compare the cost savings, if any, that could be achieved by taking an integrated view of sourcing decisions, the authors formulate *Auction S*. Here the buyer's sourcing decision is based only upon the bids submitted by the sellers, and the transportation costs are determined subsequently.

The buyer submits a consumption vector \mathbf{q} as in *Auction T*. The auctioneer solves two optimization problems, one to determine allocation and payments and the other to optimize transportation costs, separately.

Winner Determination Problem:

$$\min \sum_{k=1}^K \mathcal{F}_k(\mathbf{x}_k) \quad (11)$$

subject to:

$$\sum_{n=1}^N y_{nm} = q_m, m = 1, \dots, M. \quad (12)$$

$$\sum_{m=1}^M y_{nm} = x_n, n = 1, \dots, N. \quad (13)$$

$$y_{nm} \geq 0, m = 1, \dots, M; n = 1, \dots, N. \quad (14)$$

If $\pi_S(\mathbf{q})$ is the optimal objective value, \mathbf{x}^S the production vector, and $\pi_S^{-k}(\mathbf{q})$ the optimal objective value without supplier k , the buyer's payment to supplier k is given by the rule:

$$\psi_k^S(\mathbf{q}) = \pi_S^{-k}(\mathbf{q}) - \pi_S(\mathbf{q}) + \mathcal{F}_k(\mathbf{x}_k^S) \quad (15)$$

We can observe that this payment rule still retains the VCG structure, and hence the suppliers will submit their true cost functions. Subsequently the transportation costs are determined by solving the following optimization problem:

Transportation Problem:

$$\min \sum_{n=1}^N \sum_{m=1}^M \tau_{nm} * y_{nm} \quad (16)$$

subject to:

$$\sum_{n=1}^N y_{nm} = q_m, m = 1, \dots, M. \quad (17)$$

$$\sum_{m=1}^M y_{nm} = x_n^S, n = 1, \dots, N. \quad (18)$$

The buyers total outflow, $\kappa_S(\mathbf{q})$, is given by $\kappa_S(\mathbf{q}) = \sum_{k=1}^K \psi_k^S(\mathbf{q}) + \sum_{n=1}^N \sum_{m=1}^M \tau_{nm} * y_{nm}$. The numerical experiments clearly show that *Auction S* minimizes total production costs but leads to higher supply chain costs than *Auctions T* and *R*.

5) *Computational issues:* The crucial contribution of this model has been to take an integrated view of the sourcing problem by combining the pricing and the transportation decisions. The mechanism incorporates game theoretic considerations in supply chain formations when products are sourced globally for demand points that are distributed geographically. Since the mechanism is incentive compatible for the suppliers, the complexity of evolving an optimal bid strategy is simply reduced to reporting the actual production costs, thereby eliminating the need for complex modeling of competitors' behavior. At the mechanism level however, $(K+1)$ optimization problems need to be solved to decide the allocations and payments. Further, since the mechanism is not incentive compatible for the buyer, the buying agent needs to solve an optimization problem to decide an optimal bidding strategy. Such an optimization problem is known to be challenging and hence a numerical approach may be warranted.

Also, from the analysis presented in this paper, it is clear that in designing auction mechanisms for complex supply chains, focus on truth revealing auction mechanisms is to be augmented by pragmatic considerations of limiting the total payout of the buyers. This can be achieved by using the idea of a reservation price suitably expressed as a utility function.

D. Summary and Current Art

In this section, we reviewed models proposed to handle the procurement of both single unit and multiple units of a single item, with a single attribute - price as the criterion for decision making. In the first model (III.A), we illustrated the game theoretic considerations that influence the bidding behavior of agents. In the second model (III.B), in the absence of game theoretic requirements, we pointed out the computational complexity of the winner determination problem in a more complex sourcing situation. The third model (III.C) brought together the game theoretic as well as computational issues that complicate the sourcing problem. This was done to illustrate the fact that in the absence of proper mechanism design, the buyer could end up leaving money on the table.

We now briefly describe some recent work in this area. A successful case study of multi-unit procurement with volume discount bids is reported in [19]. The computational challenges involved in the winner determination problem there are described in [52], [50]. Kameshwaran [54], in his recent work, has shown that single item, single attribute, multi-unit procurement with volume discount bids leads to piecewise linear knapsack problems to be solved. In his work, Kameshwaran has developed several algorithms (exact, heuristic-based, and fully polynomial time approximation schemes) for solving such knapsack problems.

Kothari, Parkes, and Suri [65] consider single-item, single attribute, multi-unit procurement auctions where the bidders use marginal-decreasing, piecewise constant functions to bid for goods. The objective is to minimize cost for the buyer. It is shown that the winner determination problem is a generalization of the classical 0/1 knapsack problem, and hence NP-hard. Computing VCG payments is also addressed. The authors provide a fully polynomial time approximation scheme (FPTAS) for the generalized knapsack problem. This leads to an FPTAS algorithm for allocation in the auction which is approximately strategy proof and approximately efficient [65]. It is also shown that VCG payments for the auctions can be computed in worst-case $O(T \log n)$ time, where T is the running time to compute a solution to the allocation problem.

Dang and Jennings [64] consider multi-unit auctions where the bids are piece-wise linear curves. Algorithms are provided for solving the winner determination problem. In the case of multi-unit, single-item auctions, the complexity of the clearing algorithm is $O(n(S+1)^n)$ where n is the number of bidders and S is an upper bound on the number of segments of the piecewise linear pricing functions. The clearing algorithm therefore has exponential complexity in the number of bids.

To summarize, the two main issues in single item, multi-unit, single attribute auctions are (1) to reduce the complexity of the winner determination problem and (2) to make the mechanism strategy proof. Table III provides an appropriate taxonomy for this type of auctions and gives a listing of relevant papers under the various categories.

IV. MULTI-ITEM PROCUREMENT AUCTIONS

In the preceding section, we reviewed mechanisms that support single item procurement. In one of the mechanisms,

Scenarios in Single Item, Single Attribute Procurement	Math. Programming Models	Game Theoretic Analysis	Case Studies and Surveys	Other Relevant References
Basic Auction Types		[5], [24], [48], [57] [60], [61]	[58], [16], [12], [13] [14], [15], [3], [4] [24], [9], [39]	[26], [27], [28], [59] [30], [29], [25]
Volume Discount Auction	[54], [62], [62]	[63], [19], [64], [65]	[19]	
Supply Chain Auction		[55], [66]		

TABLE III
CLASSIFICATION OF SINGLE ITEM, SINGLE ATTRIBUTE PROCUREMENT AUCTIONS

we discussed the inclusion of logistics costs as an additional element influencing the procurement decision. Clearly such supply chain criteria are of crucial importance to procurement managers responsible for large global supply chains. Typically purchase requests within such large purchasing organizations provide opportunities to exploit complementarities in logistics costs and often in production costs too. For instance, in one of the negotiation scenarios depicted in Section II.A, a family of cutting tools may be procured by a series of sequential reverse auctions, one for each of the items. This has at least two consequences: Firstly, the price that a supplier may be willing to offer may depend in complicated ways on what other items he wins, and since there is uncertainty associated with this, the suppliers have no incentive to bid aggressively. Secondly, the suppliers do not have an opportunity to express their unique complementarities in production costs and hence the buyer cannot exploit the economies of scope associated with bundling the demands. In such cases it may be beneficial to allow the suppliers to bid on combinations of items rather than on single items. Such auctions are called *combinatorial auctions*.

Simply defined, a combinatorial auction (CA) is a mechanism where bidders can submit bids on combinations of items. The winner determination problem is to select a winning set of bids such that each item to be bought is included in at least one of the selected bids, and the total cost of procurement is minimized. In this section we first present the crucial issues that arise when CAs are applied to procurement settings. We then review approaches to modeling multi-item procurement in supply chain settings which take business and capacity level constraints into consideration.

A. Issues in Multi-Item Procurement

Important issues in combinatorial auctions are well surveyed in [31], [33], [18], [38], [34]. In the multi-item procurement case, when suppliers are allowed to respond with combinatorial bids, the winner determination problem becomes a weighted set covering problem, which is known to be NP-hard. In addition, since the bidders can submit bids on combinations of items, the representation of the bid is also a crucial implementation issue. Bidding language issues are discussed in [67], [68].

As opposed to single item procurement scenarios, in multi-item procurement scenarios, the range of issues to be considered is vastly expanded because practical business level issues and constraints need to be included in the decision model.

TABLE IV
NOTATION FOR COMBINATORIAL PROCUREMENT

N	total number of items to be procured
i	index for items, $i = 1, \dots, N$
d^i	number of units of item i demanded
K	number of suppliers
k	index for suppliers, $k = 1, \dots, K$
M	maximum number of bids allowed for any supplier
j	index for a bid, $j = 1, \dots, M$
B_{kj}	bid j of supplier k
a_{kj}^i	0-1 variable which takes value 1 iff B_{kj} will supply the entire lot corresponding to item i
p_{kj}	price associated with bid B_{kj}
$W_{k,min}$	minimum quantity that should be allocated to supplier k
$W_{k,max}$	maximum quantity that can be allocated to supplier k
x_{kj}	decision variable that takes value 1 iff B_{kj} is allocated
y_k	indicator variable that takes value 1 iff supplier k is allocated any lot
S_{min}	minimum number of winners required
S_{max}	maximum number of winners allowed

These could include exclusion constraints (for example, item A cannot be procured from supplier X), aggregation constraints (for example, at least two and at most five suppliers need to be selected for goods of category B), and exposure constraints (for example, not more than 25% of the procurement value should be assigned to any one supplier) [69], [19]. By including these constraints in the decision model, a sensitivity analysis of side constraints may also prove to be a useful tactical input to procurement planners.

B. A Single Round Combinatorial Procurement Mechanism

As indicated above, the reverse combinatorial auction problem is a set covering problem, which is NP-hard. Additional side constraints make a fundamental impact on the problem. Even finding a feasible solution when exposure constraints are in place is NP-hard. The basic formulation of the problem and the additional side constraints are presented below and solution techniques are discussed thereafter. The following formulation is from [50], [19]. Table IV provides the notation.

1) *Mathematical Formulation*: For each item type $i = 1, \dots, N$ there is a demand d^i . Each supplier $k \in K$ is allowed up to M bids indexed by j . Associated with each bid B_{kj} is a zero-one vector a_{kj}^i , $i = 1, \dots, N$ where $a_{kj}^i = 1$ if B_{kj} will supply the entire lot corresponding to item i and zero otherwise. Associated with each bid B_{kj} is price p_{kj} at which the bidder is willing to supply the combination of items in the

bid. A mixed integer programming (MIP) formulation can be written as follows:

$$\min \sum_{k=1}^K \sum_{j=1}^M p_{kj} x_{kj} \quad (19)$$

$$s.t. \quad \sum_{k=1}^K \sum_{j=1}^M a_{kj}^i x_{kj} \geq 1 \quad \forall i = 1, \dots, N. \quad (20)$$

$$x_{kj} \in \{0, 1\} \quad \forall k = 1, \dots, K. \quad \forall j = 1, \dots, M.$$

$$W_{k,min} y_k \leq \sum_{i=1}^N \sum_{j=1}^M a_{kj}^i d^i x_{kj} \quad \forall k = 1, \dots, K. \quad (21)$$

$$\sum_{i=1}^N \sum_{j=1}^M a_{kj}^i d^i x_{kj} \leq W_{k,max} y_k \quad \forall k = 1, \dots, K. \quad (22)$$

$$\sum_{j=1}^M x_{kj} \geq y_k \quad \forall k = 1, \dots, K. \quad (23)$$

$$S_{min} \leq \sum_{k=1}^K y_k \leq S_{max} \quad (24)$$

$$y_k \in \{0, 1\} \quad \forall k = 1, \dots, K.$$

$W_{k,min}$ and $W_{k,max}$ are the minimum and maximum quantities that can be allocated to any supplier k . Constraints (21) and (22) restrict the total allocation to any supplier to lie within $(W_{k,min}, W_{k,max})$. y_k is an indicator variable that takes the value 1 if supplier k is allocated any lot. S_{min} and S_{max} are respectively the minimum and maximum number of winners required for the allocation and constraint (24) restricts the winners to be within that range.

2) *Solution Approach and Discussion:* Although the decision model is NP-hard, integer programming techniques are known to be effective in solving problems with 500 items and up to 5000 bids [19]. For the purpose of this paper, the authors use IBM's OSL (Optimization Solutions and Library) to solve the IP formulation. From the experiments that were conducted, the following conclusions could be derived:

- Firstly, varying the aggregation constraint seems to have a large impact on the computation time. This is because, as we commented earlier, the problem of finding a feasible solution itself is NP-complete.
- Secondly, the min-max exposure constraints also have a significant effect on the computational time. In some sense, the exposure constraint could itself be a proxy for the aggregation constraint and hence the behavior appears similar.

C. An Iterative Reverse Dutch Auction for Combinatorial Procurement

In the previous section, we discussed a purely computational view of the multi-item procurement problem without looking at economic desiderata. It is however important for a variety of reasons to lend credence to economic issues in the modeling and analysis of multi-item procurement. Firstly, we would like to ensure that procurement contracts are allocated to those that *value them most*. In the absence of economic analysis, the game is open for suppliers to indulge in strategic bidding behavior in an effort to extract the maximum possible surplus utility from the transaction. This is especially true when single shot mechanisms are used. Secondly, even if multi-round combinatorial auctions are used where allocation and pricing information is disclosed at the end of each round, it is not clear to suppliers as to how they need to reformulate their bids. The two issues together may prompt wasteful usage of resources by each supplier in trying to *outsmart* the buyer and other suppliers. One way to amend the situation is to design an incentive compatible mechanism.

Generalized Vickrey Auctions (GVA) generalize the single item second price auction (Vickrey auction) proposed in [40]. This is an incentive compatible mechanism for combinatorial auctions, which can be applied to the procurement context [70]. However, one issue that needs to be considered is that the GVA requires an optimal solution to the allocation problem which is NP-hard. In the absence of an optimal solution to the allocation problem, the resulting mechanism, whose allocation is obtained through some approximation scheme, is no longer incentive compatible [56]. However, iterative algorithms for combinatorial auction problems have been proposed by [71] and [72] which try to resolve the tension between economic and computational efficiencies. In a similar vein, Biswas and Narahari [70] have proposed an iterative reverse Dutch auction scheme for combinatorial procurement auctions which we discuss next.

1) *Integer Programming Formulation for the Reverse Dutch Auction:* The basic idea behind this approach is to formulate the procurement problem as a weighted set covering problem and use the Dutch auction format to develop an iterative solution scheme. This approach reduces the computational complexity by breaking up one large GVA into several smaller GVAs. The iterative algorithm itself is motivated by [73], and involves a two step process: (1) progressively increasing the *average reserve price* of the items in each round, and (2) allocating items for which bids satisfy the reserve price. Classical (forward) Dutch auctions are decreasing auctions which have been typically used for multi-unit homogeneous items. In the single unit Dutch auction the auctioneer begins at a high price and incrementally lowers it until some bidder signals acceptance. Similarly in the multi-unit case the price is incrementally reduced till all the items are sold or the seller's reserve price is reached.

In the reverse auction for procurement the buyer has a procurement budget and tries to procure a bundle of items at minimal cost not exceeding the procurement budget. She starts with a low initial willingness to pay (say equal to zero)

and keeps on increasing the willingness to pay until the total bundle is procured or the budget limit is reached. This total budget cannot be divided linearly into budget for each item because of the complementarities involved.

The iterative mechanism proposed in [70], [74] consists of multiple bidding rounds denoted by $t = 0, 1, \dots$, ($t = 0$ is the initial round). The buyer sets $W(B_t)$, maximum willingness to pay for the remaining bundle B_t to be procured in round t . The pricing of items is not linear, therefore the cost of the allocated bundles cannot be divided into price of individual items. Therefore we compute p_t , the average willingness of the buyer to pay for each item in round t .

$$p_t = \frac{W(B_t)}{|B_t|}, \text{ where } B_t \neq \phi \quad (25)$$

The payment made by the buyer for the subset S_t in iteration t is $V^*(S_t)$. The average price paid by the buyer for each item procured is $v_t = \frac{V^*(S_t)}{|S_t|}$. Iterations in which no items are procured are ignored.

The reserve price of the seller for any bundle S in iteration t is $|S|v_{t-1}$. The bundle procured in round t is denoted by S_t . Therefore the integer programming formulation of the GVA problem with reserve prices in iteration t becomes

$$\begin{aligned} V^*(S_t) &= \min \sum_{k=1}^K \sum_{S \subseteq G} v_k(S)y(S, k) \\ \text{s.t.} \quad &\sum_{S \ni i} \sum_{k=1}^K y(S, k) \geq 1 \quad \forall i \in G \\ &\sum_{S \subseteq G} y(S, k) \leq 1 \quad \forall k = 1, \dots, K. \\ &v_k(S)y(S, k) \geq |S|v_{t-1} \quad \forall S \subseteq G, \forall k = 1, \dots, K. \\ &\sum_{k=1}^K \sum_{S \subseteq G} v_k(S)y(S, k) \leq W(B_t) \quad \forall S \subseteq G \\ &y(S, k) = 0, 1 \quad \forall S \subseteq G, \forall k = 1, \dots, K. \end{aligned} \quad (26)$$

The notation for the Iterative Reverse Dutch Auction (IRDA) algorithm is provided in Table V.

The IRDA Algorithm: As opposed to a naive approach, the IRDA algorithm relies on choosing an increment in each round such that it is based on the size of the bundle to be procured.

- 1) Suppose the buyer's initial willingness to pay for the entire bundle B_0 is zero i.e. $W(B_0) = 0$. Therefore the willingness to pay for each item is also zero i.e. $p_0 = 0$. Since no seller is likely to be interested in bidding at this price we have

$$V^*(S_0) = v_0 = 0$$

- 2) Increment the average willingness to pay for each item by ϵ to $p_1 = v_0 + \epsilon$. This actually means that the buyer's willingness to pay for the bundle B_1 is changed to $W(B_1) = |B_1| \times p_1$. We assume that the increment ϵ in every iteration is constant. The reserve price of any bundle S for the sellers becomes $|S|v_0$.

TABLE V
NOTATION FOR ITERATIVE REVERSE DUTCH AUCTION ALGORITHM

G	set of items to be procured
i	index for an item to be procured, $i \in G$
S	a subset of items, $S \subseteq G$
K	total number of suppliers
k	index for suppliers, $k = 1, \dots, K$
t	iteration number
B_t	bundle remaining to be procured in iteration t
(S_t)	set procured in iteration t
$V^*(S_t)$	buying price of the set S_t in iteration t
v_t	average buying price of each item in iteration t
p_t	average price set by the auctioneer in iteration t
$W(B_t)$	maximum price set by the auctioneer for the bundle B_t in iteration t
$v_k(S)$	valuation of set S to seller k
$y(S, k)$	indicator variable that takes value 1 iff bundle $S \in G$ is allocated to agent k
ϵ	increment in buyer's willingness to pay

- 3) Solve the allocation problem if there are any bids i.e., for iteration $t = 1$ solve Eq. 26 and calculate v_1 . This is again a combinatorial optimization problem. But this is much smaller than the complete problem.
- 4) Allocate the subsets to the winners. Remove the allocated items from the set to be procured and increment the average willingness to pay for each item to $p_2 = v_1 + \epsilon$, i.e., the maximum willingness of the buyer to pay for the remaining bundle B_2 is $W(B_2) = |B_2| \times p_2$. The new reserve price of any bundle S of items for the sellers is $|S|v_1$.
- 5) Go to step 3 and repeat until the buyer can procure the entire bundle or the upper limit i.e., the total procurement budget is reached. In any iteration t the following condition should be satisfied:

$$\text{total procurement budget} \geq W(B_t) + \sum_{i=0}^{t-1} V^*(S_i)$$

This approach to the combinatorial procurement problem, solves the problem of capturing cost complementarities. However, in typical strategic procurement settings capacity planning at suppliers is a crucial task meriting detailed attention if supply lines are not to be disrupted. This planning of capacity and allocation of contracts can be significantly difficult when multiple contracts are to be established simultaneously. We discuss this issue in the next subsection.

D. An Iterative Procurement Mechanism for Capacity Constrained Environments

In some strategic sourcing settings, where suppliers are looked upon as extensions of the enterprise, it is imperative to engage in prudent capacity management. Failure to do so could result in production line disruptions and high overtime costs. So when making multi-item procurement decisions, even when the product attributes do not show complementarities in costs, but share resources it is necessary to factor in capacity planning at least at the aggregate levels. One recent effort to incorporate this aspect into auction based procurement mechanisms has been due to Gallien and Wein[66].

TABLE VI

NOTATION FOR PROCUREMENT UNDER CAPACITY CONSTRAINTS

N	number of components (items) to be procured
i	index for components, $i = 1, \dots, N$
q_i	number of units of component i to be procured
K	number of suppliers
k	index for resources (suppliers), $k = 1, \dots, K$
c_k	capacity of supplier k
a_{ki}	amount of resource k required for component i
t	index that represents auction rounds
$b_{ki}(t)$	bid submitted in round t by supplier k for any quantity between 0 and q_i
$x_{ki}(t)$	decision variable for bid $b_{ki}(t)$

The procurement scenario that they analyze is as follows (see Table VI for notation): The buyer wants to procure q_i units of component $i \in \{1, \dots, N\}$. There are K suppliers willing to supply some or all of these quantities subject to overall capacity constraints. Overall capacity constraints of resource (supplier) k is c_k and the usage can be described by a linear model where a_{ki} is the amount of resource k required for component i . An auctioneer acts as an intermediary between the buyer and the seller thereby decoupling the information between the two, much like what happens on <http://www.freemarkets.com/>.

1) *The Auction Mechanism:* The mechanism is designed as a multi round auction where in each round t the supplier k submits a bid $b_{ki}(t)$ to sell any quantity between 0 and q_i of component i . Further non-renegeing and minimum bid decrement rules are in place. In each round of the auction a potential allocation $\mathbf{x}(t) = (x_{ki}(t))_{k \in \{1, \dots, K\}, i \in \{1, \dots, N\}}$ is obtained by solving the linear program given below.

$$\min \sum_{k=1}^K \sum_{i=1}^N b_{ki}(t) x_{ki}(t) \quad (27)$$

$$s.t \quad \sum_{i=1}^N a_{ki} x_{ki}(t) \leq c_k \quad \forall k = 1, \dots, K. \quad (28)$$

$$\sum_{k=1}^K x_{ki} = q_i \quad \forall i = 1, \dots, N. \quad (29)$$

$$x_{ki}(t) \geq 0 \quad \forall k = 1, \dots, K; \quad \forall i = 1, \dots, N.$$

At the end of each round, the potential allocation $\mathbf{x}_k(t)$ is displayed to the supplier k which can be used to better the bid in the following round. The supplier can choose to follow a *myopic best response (MBR)* strategy embedded within a software based bid suggestion device. To use this feature, the supplier k is required to submit his actual production costs v_{ki} for component i to the auctioneer. The next bid $\mathbf{b}_k^*(t+1)$ to be submitted by supplier k is such that he maximizes his potential payoff $\Pi_k(\mathbf{b}(t)) \equiv \sum_{i=1}^N (b_{ki}(t) - v_{ki}) x_{ki}(t)$ in round t by carrying out a suitable computation [66].

E. Summary and Current Art

In this section we reviewed mechanisms designed to support multi-item procurement scenarios. In the first part (V.B), we

discussed a purely computational approach to the procurement decision problem to illustrate the computational problems that arise. In the next section we introduced economic concerns and discussed the limitations to achieving both computation and economic efficiencies. Finally we discussed the implications of capacity constraints at suppliers on decisions made through combinatorial auction models. Also, we deliberated on one approach to providing the suppliers with a bid suggestion device to help in reformulating bids in a complex combinatorial environment.

Currently, combinatorial auctions constitute a very active research area. There are several surveys that have appeared on this topic, for example, see [31], [32], [33], [18], [34], [41]. There is also a recent comprehensive book by Cramton, Shoham, and Steinberg [35]. There have been numerous efforts in solving the winner determination problems arising in complex combinatorial auctions. The reader is referred to the papers [79], [86], [87], [77], [75], [76], [78], [91], [64]. Another topic of active research is design of truthful combinatorial auctions. Please refer to [81], [85], [56], [32] for more details. Design of iterative combinatorial auctions [82], [83], [41], [84], [90], [74], [88], [89] is one more direction in which a fair amount of research activity is in progress. Multi-attribute combinatorial auctions, however, have received little attention due to the intrinsic difficulties involved.

Table VII provides an appropriate taxonomy of combinatorial procurement auctions and gives a listing of relevant papers under each category.

V. MULTI-ATTRIBUTE PROCUREMENT AUCTIONS

The procurement mechanisms of the two previous sections involve a single attribute based on price. In practice, these mechanisms address only a limited band of the negotiation spectrum, whereas sourcing decisions involve multiple criteria - both quantitative and qualitative [92]. Benyoucef, Ding, and Xie [51] present an exhaustive, hierarchical list of criteria that are used in evaluating and selecting suppliers. Incorporating these criteria into an automated negotiation tool to support sourcing decisions has been the holy grail of purchasing professionals, industrial engineers, and computer scientists. Many software solution vendors now support multi attribute reverse auctions in their e-sourcing solutions. Weighted, multi-parameter, multi-line item request-for-quotes (RFQs) and reverse auction capabilities are provided by Ariba, freemarkets, and Procuri; i2 Technologies goes one step further and claims to support all these within an optimization module that allows business level constraints to be incorporated [9]. Since these are commercial products, it is not possible to independently verify the mechanics of the automated sourcing solutions. However, we conjecture that they use one or more of the known approaches to multiple criteria decision analysis (MCDA) such as additive value models, analytic hierarchy process (AHP), lexicographic ordering, multi-attribute utility theory (MAUT), simple multi-attribute rating technique (SMART), and traditional weight assessment. For a detailed treatment of these approaches, we refer the reader to [93].

In this section, our discussion focuses on single item, multi-attribute procurement problems to investigate (1) the

Scenarios in Multi-Item Procurement	Math.Programming Models	Game Theoretic Analysis	Case Studies, Surveys, References
Combinatorial Auction	[75], [76], [77], [78], [81], [82], [83], [41], [84]	[79], [23], [18], [65], [80] [70], [74], [85], [86], [87], [88], [89], [64] [90], [74], [88], [89]	[19], [17], [38], [11], [20] [21], [22], [31], [32] [33], [34], [35], [23]
Volume Discount Auction	[63], [62], [54]	[63], [62]	[19]

TABLE VII
CLASSIFICATION OF MULTI-ITEM PROCUREMENT AUCTIONS

problem scenarios addressed, (2) solution approaches, and (3) computational and game theoretic issues.

A. Issues in Multi-Attribute Procurement

Simply defined, multi-attribute procurement refers to the decision process related to the determination of a contract by considering a variety of attributes involving not just price but also aspects such as quality, delivery time, contract terms, warranties, after sales service, etc. In practice, multi-attribute procurement scenarios come in various hues and shades. To aid in analysis, we group these problems into three distinct categories which we believe adequately reflect the issues to be considered from a computational / automation point of view. The groups and their characteristics are:

- Multiple attributes are known a priori and are uncorrelated; individual attributes have point values. The suppliers provide *point bids* where each bid has a single price and each attribute has a single value, either provided by the supplier or computed by the buyer.
- Multiple attributes are known a priori and are uncorrelated; each attribute can take an individual value from a domain of possible values for the attribute. The suppliers provide *configurable bids* which specify multiple values and price markups for each attribute.
- The multiple attributes are not known a priori and they are uncorrelated. The suppliers may provide *point bids* or *configurable bids*. This is not unlike many negotiation situations where the buyer has only a rough idea of her requirements but relies on suppliers to educate him about attributes relevant to the procurement decision.

A further level of complexity is involved when the attributes are correlated in each of the above cases. Also, each of the above scenarios can occur in combinations.

In the first scenario above, if we assume that there exists some decision rule which, as a function of all the multiple attributes, ranks the bids, then the winner determination problem is relatively straightforward. In the latter two cases, however, the options are combinatorial in nature and hence the winner determination problem will have exponential complexity. This raises the issue of compactness of information or bid representation. We now focus on methods to *develop* the decision rule for ranking of bids.

Traditional approaches to developing the decision rule have relied on either a hierarchical elimination process or on weighted average techniques. Since determining the hierarchy or the choice of weights is not always straightforward,

especially in the face of a large number of attributes, many sophisticated approaches, including the use of optimization tools, have been proposed which we review next.

1) *Elimination Methods*: In this method, at each level, we eliminate from the bid list, bids that do not satisfy the selection rule. The selection rule may be a *conjunctive rule* or a *lexicographic rule*. In either case, a hierarchy of the attributes needs to be established in the order of their importance, which is fuzzy for most decision makers. To overcome this handicap, optimization based approaches have been devised. We briefly describe these techniques below:

Lexicographic Ordering: Here, on the first level, we select the most significant criterion and we compare bids based on this. If a bid satisfies this criterion much better than the other suppliers, then it is chosen, otherwise the bids are compared with respect to the second criterion, and so on.

Satisficing: In this technique we set minimum levels for every attribute except one, which is the target attribute, such as price. We select bids which satisfy the minimum levels and choose the one with the optimal value of the target. It is also possible to iterate through the minimum levels.

Analytic Hierarchy Process (AHP): This is an analytical tool, supported by simple techniques, that enables people to explicitly rank tangible and intangible factors against one another for the purpose of resolving conflict or for setting priorities. The process involves structuring a problem from a primary objective (e.g. selecting the best bid) to secondary levels of objectives (e.g. performance objectives, quality needs, etc.). Once these hierarchies have been established, a pairwise comparison matrix of each element within each level is constructed.

Goal Programming: This is a way to handle multiple objectives in what would otherwise be a LP. The basic concept is to set "aspiration levels" (targets) for each objective and prioritize them. One would then optimize the highest priority objective with respect to the original ("hard") constraints. Next, a constraint is added saying that the first objective function's value must be at least as good as what was achieved or the aspiration level, whichever is worse. Now the second objective is optimized, turned into a constraint, etc.

2) *Weighted Average Methods*: The weighted average methods essentially rely on the introduction of a *virtual currency* which expresses the overall utility of a bid to the buyer. The computation of the virtual currency is based upon the utility function specified by the buyer and bids that achieve the best overall utility are declared winners. It is obviously of interest to the buyer to specify the *best possible* utility

TABLE VIII
NOTATION FOR ORDINAL RANKING OF ALTERNATIVES

B	Set of Bids, $B = \{B_1, \dots, B_K\}$
k	index for supplier bids ($k = 1, \dots, K$)
n	number of attributes
i	index for attributes, $i = 1, \dots, n$
Q	vector of attributes
x_k^i	level of attribute i in bid k
\mathbf{x}_k	vector of attributes of bid B_k
$U_i(\mathbf{x}_k^i)$	utility function for attribute i in bid B_k
S_k	score of Bid B_k
w_i	weight for attribute i

function. We briefly describe some of these techniques below:

Traditional weighted average technique: Purchasing professionals have traditionally relied upon assigning *weights* to individual attributes and using a simple additive rule to derive the virtual currency. This is not dissimilar to approaches in micro-economic theory for developing utility functions. This has been further refined by the swing weighting approach described in [94]. The application of this approach can be severely restricting when the number of attributes to be considered is large.

Weight Determination Based on Ordinal Ranking of Alternatives (WORA): This approach recognizes the pitfalls in using the weighted average technique and hence relies on linear programming techniques to compute the optimal weights for each of the attributes. We detail this approach in the next section.

Inverse Optimization Methods: This technique is an improvement over WORA, in the sense that it does not rely on a single central agency (the buyer) to provide inputs to compute the optimal decision rule. Rather it uses a novel and intelligent technique based on inverse optimization to gather information distributed among various agents (sellers) in the marketplace to develop the optimal scoring (decision) rule.

In the following sections, we emphasize two different approaches that have been proposed to develop multi attribute procurement mechanisms.

B. Additive Value Model Based on Ordinal Ranking of Alternatives

A straightforward approach to multi-attribute procurement is to assume that the utilities of each of the attributes are additive in nature and hence a virtual currency, like the stock market index can be developed. Formally, we have a vector Q of relevant attributes of a bid. We index the attributes by i and the set of bids $B = \{B_1, \dots, B_K\}$ by k . See Table VIII for notation. A vector $\mathbf{x}_k = (x_k^1, \dots, x_k^n)$ is specified, where x_k^i is the level of the attribute i in bid k . In the simple case of an additive utility function $U(\mathbf{x}_k)$, each attribute is evaluated through a utility function $U_i(\mathbf{x}_k^i)$ and the overall utility is the sum of all weighted utilities. This produces a virtual currency to be used in mechanisms of the type proposed for single attributes. The crucial issue however is the following: *How are the weights or the utility function decided?* Naive approaches include the elicitation of buyers preferences through the design

of smart web forms [1]. More recently techniques based on decision analysis have been proposed by [94] and [95] which we describe next.

The WORA technique has been developed as part of an application framework to provide buyer side decision support for e-sourcing [94]. It is based on the realization that the buyer can make deductions about the superiority of bids through simple pairwise comparisons. By making many such comparisons, not entirely exhaustive, it is possible to build a larger information set to elicit *intelligently* the scoring function. This is done in a two step process.

In the first step, sample ordinal rankings of bids are provided by the buyer. They are of the form $B_1 \succ B_2 \succ B_3$. These rankings are checked for *intransitive preferences* and *dominance violations*. These sample rankings are then transformed into constraints to a linear program, which then generates estimates for the decision maker's weights. The LP is formulated as follows:

Let $B = \{B_1, B_2, \dots, B_t\}$ be a subset of bids that are ranked in the order $B_1 \succ B_2 \succ B_3 \succ \dots \succ B_t$. The score S_k of each bid B_k is computed as:

$$S_k = \sum_{i=1}^n w_i U_i(x_k^i) \quad \forall k \quad (30)$$

where the weights w_i are unknown and i is the number of the attribute, and these satisfy the bid rankings. The LP is:

Maximize 0

subject to:

$$S_1 \geq S_2$$

$$S_2 \geq S_3$$

⋮

$$S_{t-1} \geq S_t$$

$$\sum_{i=1}^n w_i = 1, \quad w_i \geq 0 \text{ for each } i$$

A feasible solution to this LP can be found and results in a set of weights to be used in the scoring rule. The authors also provide some numerical experiments to show the efficacy of the technique. These weights are then used within a standard single attribute like procurement mechanism with the *virtual currency*, obtained by combining price and all other attributes, substituting for *price*.

While this technique is an improvement over ad-hoc weight assignment models, it still relies on a single buying agent to indicate an ordinal ranking in order to come up with an optimal scoring function. This approach does nothing to exploit the cost complementarities that production systems of the suppliers may exhibit in providing certain attribute levels. In such cases it may be beneficial for the buyer to understand the nature of the suppliers' cost functions in terms of the non-price attributes. Understandably, it is not likely to be in the interest of suppliers to part with this information. In the next section we discuss one approach which tries to overcome this problem.

TABLE IX
NOTATION FOR INVERSE OPTIMIZATION TECHNIQUES

K	number of suppliers
k	index for suppliers ($k = 1, \dots, K$)
n	number of attributes
i	index for attributes, $i = 1, \dots, n$
P	number of cost parameters
p	price specified in a bid
r	index for rounds of auction ($r = 1, \dots, P + 1$)
x_i	magnitude of non-price attribute i
θ_{ikp}	magnitude of parameter p for attribute i of supplier k
ψ_{ip}	magnitude of parameter p affecting attribute i of buyer
ϕ_{ipr}	magnitude of parameter p affecting attribute i in round r of the auction
$f_i(x_i)$	unparameterized scoring rule in round $P + 1$

C. Additive Value Model with Inverse Optimization Techniques

In some procurement scenarios where the establishment of a contract depends upon multiple attributes (price, quality, delivery time, features and options, etc), a Request-for-Quote (RFQ) process is preferred. In this process, the buyer announces a scoring rule in terms of the bid price and the various attributes to be considered. It is not uncommon for the buyer to change the scoring rule to reflect any new information that has been gleaned during the RFQ/negotiation process. This idea is formalized in the eRFQ mechanism in [95] which is designed to address a procurement scenario with the following characteristics and assumptions (see Table IX for notation):

- A single item with multiple attributes is to be procured. The attributes are indexed by $i = 1, \dots, n$; k indexes the K suppliers; and $r = 1, \dots, P + 1$ indexes the rounds of the auction, where P is the number of cost (and utility) parameters.
- Each bid submitted by a supplier is of the form (p, x_1, \dots, x_n) where p denotes the price and x_i is the magnitude of non-price attributes which are continuous, nonnegative variables.
- Supplier k 's cost function $\sum_{i=1}^n c_{ik}(x_i, \theta_{ik1}, \dots, \theta_{ikP})$, is additive across attributes and c_{ik} is increasing, convex and twice continuously differentiable in x_i .
- The auctioneer is assumed to know the form of the suppliers' cost function but not the actual parameter values $(\theta_{ik1}, \dots, \theta_{ikP})$
- $\sum_{i=1}^n v_i(x_i, \psi_{i1}, \dots, \psi_{iP}) - p$, is the true utility function of the buyer and the scoring rule is $\sum_{i=1}^n v_i(x_i, \phi_{i1r}, \dots, \phi_{iPr}) - p$, in round $r = 1, \dots, P$, which is increasing and concave in x_i . Further, to guarantee existence of solutions to the optimization problem, described later, a set of technical requirements are imposed upon the cost, scoring and valuation functions.

The eRFQ mechanism is based upon an open ascending auction format consisting of $P + 1$ rounds. The first P rounds enable learning of the P parameters, through *Inverse Optimization*, and the last round is with an optimized scoring rule. Ahuja and Orlin [96] describe Inverse Optimization in the following manner: "A typical optimization problem is a forward problem because it identifies the values of observable

parameters (optimal decision variables), given the values of the model parameters (cost coefficients, right-hand side vector, and the constraint matrix). An inverse optimization problem consists of inferring the values of the model parameters (cost coefficients, right-hand side vector, and the constraint matrix), given the values of observable parameters (optimal decision variables)".

In each round of the auction, the buyer announces a scoring rule in response to which suppliers submit bids. Activity rules and transition rules are imposed to move from one round to the other. In each round the buyer ranks the bids according to the latest scoring rule and announces the rankings without revealing the bidders or the actual bids. The winner determination, at the end of $P + 1$ rounds, is based simply upon an English auction like rule, with the bidder providing the highest utility at the end of $P + 1$ rounds being offered the contract at his bid price.

The key questions that arise are: (1) what is the likely bidding behavior of the suppliers; (2) how does the buyer estimate the suppliers' cost functions; and (3) how is the optimal scoring rule determined after *learning* the cost functions.

Bidding Behavior of Suppliers: The supplier is assumed to follow a *myopic best response* bidding behavior which is in line with the approaches used in [97] and [66]. Here, the supplier chooses his bid such that he maximizes his current profit with the assumption that other suppliers do not change their bids. The bid is chosen by solving the following Non-Linear Optimization problem:

$$\max_{p, x_1, \dots, x_n} p - \sum_{i=1}^n c_{ik}(x_i, \theta_{ik1}, \dots, \theta_{ikP}) \quad (31)$$

subject to

$$\sum_{i=1}^n v_i(x_i, \phi_{i1r}, \dots, \phi_{iPr}) - p = K + \epsilon \quad (32)$$

In the final round however, the constraint would be:

$$\sum_{i=1}^n f_i(x_i) - p = K + \epsilon \quad (33)$$

While the bidding behavior does not include a game theoretic analysis of competing suppliers, it still requires bidders to be sophisticated enough to solve optimization problems. This is striking a middle ground with respect to the bidders' rationality.

Estimating cost functions of the suppliers: By virtue of the auction design, the auctioneer at the end of P rounds has for each attribute i and each supplier k , P equations. These P equations obtained by solving the first order conditions for $i = 1, \dots, n$,

$$\frac{\delta v_i(x_i, \phi_{i1r}, \dots, \phi_{iPr})}{\delta x_i} = \frac{\delta c_{ik}(x_i, \theta_{ik1}, \dots, \theta_{ikP})}{\delta x_i} \quad (34)$$

form a set of simultaneous linear equations. By solving this set of equations for each supplier k the true cost parameters for each attribute i can be obtained.

Computing the optimal scoring rule: After learning the suppliers' cost functions, the optimal scoring rule for round

$P + 1$ is computed by solving the following optimization problem:

$$\max_{f_i} \left\{ \sum_{i=1}^n v_i(x_{i1}^*, \psi_{i1}, \dots, \psi_{iP}) - \sum_{i=1}^n f_i(x_{i1}^*) \right\} + \left\{ \sum_{i=1}^n f_i(x_{i2}^*) - \sum_{i=1}^n c_{i2}(x_{i2}^*, \theta_{i21}, \dots, \theta_{i2P}) \right\} \quad (35)$$

subject to:

$$S_k = \sum_{i=1}^n f_i(x_i^*) - \sum_{i=1}^n c_{ik}(x_i^*, \theta_{ik1}, \dots, \theta_{ikP}) \quad (36)$$

$$S_2 > \epsilon \quad (37)$$

$$S_1 > S_2 + \epsilon \quad (38)$$

The objective function maximizes the buyer's utility with the last three terms of (24) making up the winning suppliers price; (36) is supplier s 's maximum drop-out score; (37) and (38) ensure that the top two suppliers participate in round $P + 1$.

In the model described above, assumptions about (1) undistorted bidding behavior by suppliers and (2) knowledge of the form of the suppliers' cost functions may be untenable in practice. The authors propose several extensions to the basic model in order to make it more robust. First, in developing the scoring rule for the last round, the authors envisage three problems: (a) a difficult mathematical programming problem needs to be solved, (b) the scoring rule may turn out to be too complex, and (c) the scoring rule may force the losing supplier to submit bids with negligible values of non-price attributes. To overcome these problems the authors propose (1) changing the method of finding the best competitor, (2) providing the scoring rules in graphical form, and (3) introducing lower bound constraints on the attribute levels in the final round. Secondly, the bidding behavior of suppliers may not follow the *undistortedness* assumption either because of strategic intentions of the supplier or the lack of sophistication on their part. This would result in an inconsistent set of simultaneous linear equations. The authors suggest using a weighted average least squares procedure to reduce the effects of bid distortion. We however would like to emphasize that this approach does not absolve buyer of need to compute true utility.

D. Summary and Current Art

In this section we reviewed mechanisms designed for the procurement of items with multiple attributes. We briefly reviewed the approaches to multi-criteria decision making and identified two broad categories of techniques - elimination methods and weighted average methods. The crucial issue in multi criteria procurement is the assignment of weights to each of the attributes to facilitate the development of a scoring function which captures the buyers' utility. Two *intelligent* approaches - the first relying on a central agency to indicate

a pairwise preference among a sample of received bids and the second based upon estimating the suppliers cost functions, were detailed.

Currently, developing efficient approaches for multi-attribute procurement is an active research area. The reader is referred to [101], [98], [99], [69], [54] for some recent work. The papers [101], [99], [98] build upon the approaches described above. Kameshwaran and Narahari [69] have proposed an approach based on goal programming. Goal programming (GP) is one of the tools of choice for multi criteria decision analysis [102]. In [69], the authors show that GP can be used to model procurement scenarios where suppliers provide bids with *configurable offers*. Here the bids are assumed to be piece-wise linear and the buyer has a hierarchy of goals or aspiration levels which are to be satisfied. The authors propose the use of Weighted GP, Lexicographic GP, and Interactive Sequential GP techniques to solve the multi-attribute procurement problem.

There are many issues that remain unresolved in multi-attribute procurement. No single approach seems to work uniformly well and it is intrinsically a challenging problem. Much work remains to be done in all the areas: winner determination algorithms, payment rules, and achieving truth revelation.

We summarize the discussion on multi-attribute procurement auctions by presenting Table X which gives snapshot of the possible scenarios and a listing of relevant references.

VI. A CASE STUDY FROM GENERAL MOTORS

Procurement business professionals need to collect large numbers of bids from prospective suppliers. They then determine an allocation of awards to bids that minimizes the procurement cost subject to a variety of constraints such as vendor award count on commodities or subsets of commodities, logistics costs and supplier delivery risk. This need is present during the initial stage of awarding business to suppliers on new products, and is also present when primary suppliers are unable to deliver supplies (e.g., in the case of a strike, natural disaster, financial default, or other event that causes a work stoppage) to existing products.

A team of researchers from General Motors Research (which included the last four authors of this paper) recently used an auction-optimization approach similar to those described earlier in this paper to solve this problem. This approach, soon to be deployed as a web application within General Motors Corporation (GM Corp), allows business users to determine an optimal allocation of awards to bids using the application over the company's intranet. Because the optimization approach is math driven, automatic, and can be rolled out over the businesses intranet, business users can now focus on the business constraints that drive the award process. Business users can dynamically adjust the business constraints to understand the impact of the stated constraints on the award allocation. That is, they can understand the sensitivity of the awards to the constraint settings. This process can happen in real time, thus making the procurement business professionals' task much faster, thereby allowing them to focus on developing

Scenarios in Multi-attribute Procurement	Math.Programming Models	Game Theoretic Analysis	Case Studies, Surveys, References
Additive Value Models	[98], [69], [99]	[1], [100], [94], [23], [101],	[51], [92], [2]
Inverse Optimization Models		[95]	[96]

TABLE X
CLASSIFICATION OF MULTI-ATTRIBUTE PROCUREMENT AUCTIONS

the relationship with suppliers rather than spending their time crunching numbers in spreadsheets in an effort to discover the optimal award allocation using more primitive manual approaches.

The sourcing corresponds to that of an important raw material for automotive manufacturing at GM. The overall commodity sourcing process is shown in Figure 1. Within GM, a huge amount of this commodity is sourced every year. To gain maximum cost savings (at a sufficiently high level of desired quality), GM uses a centralized demand aggregation and reselling application for the whole supply chain. This application attempts to combine the individual commodity requirements of its processors with GM's direct commodity requirements to create large orders. These larger orders often qualify for significant volume discounts with the commodity suppliers. GM then resells a portion of the purchased commodity to its processors to cover their material needs. Some of the processing is performed by external processors, and the rest is done by GM's internal processing group. So, the overall process is very complex and manual approaches for determining an allocation of awards to the suppliers require enormous effort. Moreover, the solution may not be optimal. By using an optimization model similar to those described earlier in this paper, superior solutions can be obtained with less effort.

A. System Overview

A general commodity sourcing application in the automotive industry will look like the one shown in Figure 2. Here, the overall process is the same as shown in Figure 1. The requirements for the commodities are aggregated within a centralized system (sourcing tool) and an effective bundling of the commodity is found. The tool looks at the catalogs of the approved suppliers and sends RFQs to each of the suppliers for those bundles that they are capable of supplying. Each of these suppliers submits a list of configurable bids to the tool in response to the RFQ. The tool evaluates the bids and feeds them into the optimization model. The optimization model is also provided with plant specific constraints and other business constraints. The tool outputs a cost effective allocation to the suppliers.

The main elements of a configurable bid submitted by a supplier are shown in the tree-like structure shown in Figure 3. A configurable bid gives either a base price for a bundle and quantity, or a volume discount price, which is a function of quantity. The bid consists of various attributes, which can be fixed attributes, range attributes, or optional attributes. Fixed attributes are simple attributes with one corresponding value.

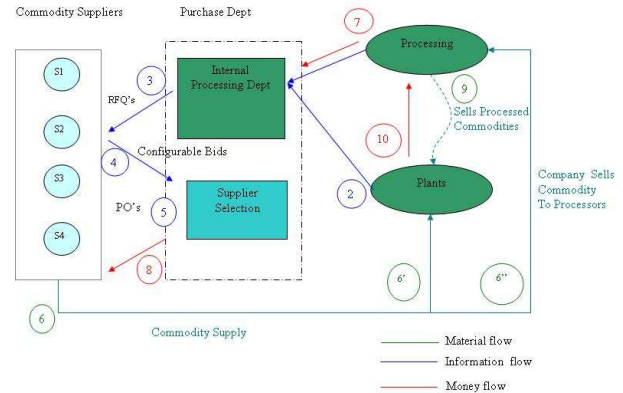


Figure 1: Overall Business Process

Fig. 1. Overall Business Process

Range attributes allow suppliers to specify an attribute as a step function, where each step has a fixed impact on price. Optional attributes can either be included or not. A supplier can also specify some logical rules for assigning some discounts to specific combination of selected attributes.

B. A Model for Minimizing Procurement Spend

The commodity requirements of all the plants are aggregated and bids are invited from the suppliers. Each plant has a set of preferred suppliers for procuring the commodities. Also, suppliers are capable of supplying certain commodities to one or more particular plants. Each bid submitted has variable and fixed cost components. They are submitted for multiple units of multiple heterogeneous commodities destined for multiple plants. There are some bounds on the number of units of a particular commodity that a supplier can supply to a plant. Also, there are some restrictions on the number of suppliers that can be selected for a plant and also on the number of suppliers that can be awarded business for a commodity. All of these constraints are determined by taking into consideration the suppliers' capacities, their disruption risk, their overhead costs, etc. The model that the team created to solve the problem determines an efficient allocation of awards to bids so that overall procurement cost is minimized, and the various constraints on the number of suppliers for a plant/commodity, bounds on the award to each supplier, plant preferred suppliers, and other constraints are satisfied.

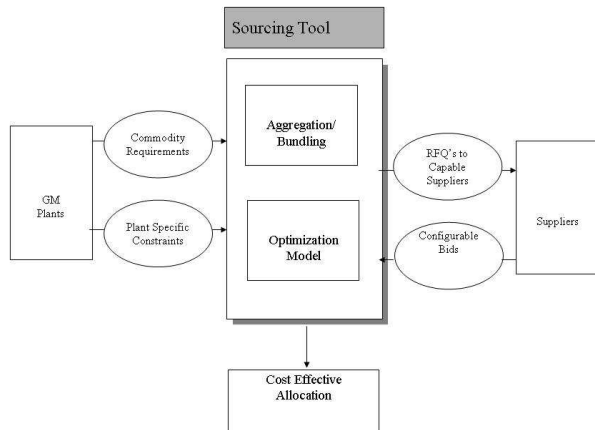


Figure 2: Commodity Sourcing Application

Fig. 2. Commodity Sourcing Application

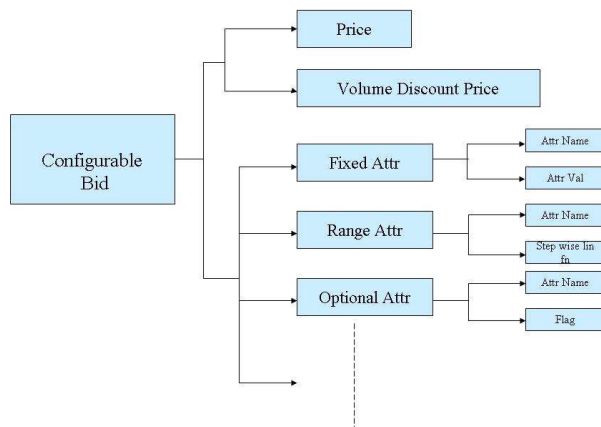


Figure 3: Main Elements of Configurable Bid

Fig. 3. Main Elements of a Configurable Bid

The mathematical programming model used combines several features of the formulations discussed in Sections III.B and IV.B and extends those formulations with business constraints unique to GM.

C. Results and Conclusions

Using the above approach, the team was able to determine a cost minimizing potential re-allocation of awards that could be used for negotiations with the current suppliers (A and B) regarding the status of expiring contracts. The set of specialty commodities that the team considered were C1, C2, C3, and C4. The solution obtained by the conventional method (using current suppliers A and B) gave an Annual Purchase Value (APV) of \$ 31.11 M while that obtained using our optimization approach gave an APV of \$ 30.4M. The results are shown in Table XI and Table XII respectively. These preliminary results gave significant savings of about \$ 0.71M (2.3% of APV) in a solution time in the single digit seconds range.

Also, the team considered how the optimization model might be used to help determine an optimal supply re-allocation strategy in the event of a supplier disruption event. For this, the team considered the reallocation of a portion of the commodity from one of the suppliers to two other suppliers. Three scenarios were investigated:

- 1) Reallocating only the highest volume parts
- 2) Reallocating so as to minimize the total cost impact
- 3) Same as Scenario 1 above, but excluding certain parts for reallocation

The results are shown in Table XII. So, if the number of reallocations (Scenario 1) is sought to be minimized, the cost impact is high (\$ 3.2M). On the other hand, trying to minimize the total cost impact increases the number of reallocations to 280. Assuming a switching cost of approximately \$10,000 for reallocating a part to a new supplier, the cost minimizing reallocation has an impact of \$ 2.7M with 146 reallocations.

In conclusion, the team demonstrated that significant cost savings are possible when applying auctions and optimization to a real-world procurement problem. The savings could be as high as 3%. Of course, given that the analysis was limited to a particular commodities group, there is reason to believe that this estimate is only a lower bound on what is possible using this approach. It was also shown that using this type of modeling approach allows business analysts to explore different constraints on the award and reallocation process. This leads to a much better understanding of the sensitivity of the business constraints to optimal sourcing decisions than was previously possible using a more traditional manual process.

In the following section, we summarize the discussion in this paper by providing a few concluding remarks and pointers to potential research opportunities.

VII. CONCLUSIONS AND FUTURE WORK

In this paper, we have surveyed the state-of-the-art in auction-based mechanisms for automating negotiations in electronic procurement. We have discussed these mechanisms under three categories:

- *Single Item Auctions*: Procurement of a single unit or multiple units of a single item based on a single attribute
- *Multi-Item Auctions*: Procurement of a single unit or multiple units of multiple items based on a single attribute
- *Multi-Attribute Auctions*: Procurement of a single unit or multiple units of a single item based on multiple attributes.

A fourth category would be: procurement of multiple units of multiple items based on multiple attributes. Given that the first three categories are areas of active research with unresolved issues, this fourth category would be even more challenging. In each of the three categories of procurement, we discussed several procurement scenarios, provided problem formulations, and discussed issues of computational complexity related to the mechanism and the bidding process. We have also provided a description of several case studies, including a detailed one from General Motors.

Currently, procurement auctions is an active area of research with many interesting problems which merit further study. Here, we provide pointers to a few selected categories:

Mill/Commodity	C1	C2	C3	C4	Grand Total
A	\$107,010	\$ 2,863,928	\$ 7,069,270	\$1,502,302	\$ 11, 542, 510
B	\$303,039	\$ 1, 739, 721	\$ 10, 153, 908	\$ 7, 367, 029	\$ 19, 563, 697
Grand Total	\$ 410, 049	\$ 4, 603, 649	\$ 17, 223, 178	\$ 8, 869, 331	\$ 31, 106, 207

TABLE XI
CURRENT COMMODITY ALLOCATION (APV: \$ 31.22M)

Mill/Commodity	C1	C2	C3	C4	Grand Total
A	-	-	-	\$5, 117, 764	\$ 5, 117, 764
B	\$ 71, 708	\$ 1, 629, 282	\$ 7,600, 918	\$1, 276, 440	\$ 10, 578, 348
C	\$ 5, 981	\$ 1, 763, 698	\$ 6, 970, 686	-	\$ 8, 739, 365
D	\$ 277, 243	\$ 1, 007, 565	\$ 775, 921	\$ 2, 182, 372	\$ 4, 243, 101
E	\$ 27, 945	\$ 176, 621	\$ 1, 513, 435	-	\$ 1, 718, 001
Grand Total	\$ 382, 877	\$ 4, 577, 166	\$ 16, 860, 960	\$ 8, 576, 576	\$ 30, 396, 579
Savings (\$)	\$ 27, 172	\$ 27, 483	\$ 362, 218	\$ 292, 755	\$ 709, 628
Savings (%)	6.627 %	0.597%	2.103 %	3.301%	2.2813 %

TABLE XII
OPTIMAL COMMODITY ALLOCATION (APV: \$ 30.40M)

Scenario	Cost Impact (Increase)	No. of Reallocations
1	\$ 3.2M	77
2	\$ 2.5 M	280
3	\$ 3.8M	81

TABLE XIII
REALLOCATION OF MATERIAL

Winner Determination Problem in Procurement Auctions

Though extensive work has been done on solving the winner determination problem in different types of procurement auctions, there is still wide scope for future work here. One of the promising directions is to come up with efficient approximation algorithms with provable bounds for solving the winner determination problem. There are many efforts in this direction already, see for example, [54], [65]. The rich body of literature available in combinatorial optimization and approximation and randomized algorithms will be extremely useful here. Another key opportunity here is in the area of on-line algorithms. In many situations, it becomes imperative to quickly evaluate the bids received and allocate quantities/orders to suppliers. In such situations, on-line algorithms would be useful. As of the present art, there is not much work in this direction and this would be a valuable topic for further investigation.

Iterative Procurement Auctions

As already stated in the paper, iterative mechanisms have several advantages over one shot mechanisms. Also, there have

been several iterative mechanisms developed for procurement [41], [74], [79], [90], [103]. Iterative mechanisms are appealing to the buyer and suppliers because they enable the bidders to apply corrections to their bids in a continuous way. More work is required in ensuring that iterative mechanisms satisfy desirable economic properties also. The use of on-line algorithms for rapid winner determination in successive rounds of bidding is an important direction for future work.

Design of Incentive Compatible Mechanisms

Inducing truth revelation will continue to be a major issue in the design of procurement mechanisms. Designing such mechanisms has intrinsic difficulties such as very high computational complexity and loss of efficiency. It would be interesting to study how the use of approximate algorithms for winner determination and payment computation would affect incentive compatibility. There is already a fair amount of work in this direction [65], [84], [56].

Optimal Procurement Auctions

Based on the work of Myerson [48], it would be interesting to investigate cost minimizing procurement auctions subject to individual rationality and Bayesian incentive compatibility constraints. Myerson's work considers forward auction (selling) of a single indivisible item. Extending the work to procurement auction of multiple units and items would be quite challenging.

Multi-Attribute Procurement

Multi-attribute procurement is an intrinsically difficult problem, but at the same time an important problem that needs immediate attention. There are a few results available [92], [98], [103] and there are a few promising approaches such as goal programming [69], but much more needs to be researched in this area.

Use of Learning in Procurement Auctions

Procurement auctions provide an ideal platform for use of machine learning techniques in improving the efficiency of the process. The history of bidding by a supplier is an important parameter for winner determination. To incorporate history into procurement decision making will call for use of appropriate machine learning techniques such as reinforcement learning. Learning based models would be useful in iterative procurement auctions to help the buyer estimate the cost functions of the suppliers and in optimally incrementing the procurement budgets. One such application is discussed in [80]. Another interesting application is discussed in [104]. We believe machine learning techniques have powerful applications in procurement auctions.

Procurement Auctions from a Total Supply Chain Perspective

It is important to design procurement mechanisms based on a total cost approach where the total cost captures all aspects of the entire supply chain. This has been explored in a few papers already [55] but a deeper understanding and a more systematic approach is required here.

Deployment Issues

Practical deployment of procurement auctions will throw up numerous challenges. Several authors have addressed these issues: security of transactions [105]; collusion of suppliers [57]; user interface issues [101]; fairness issues [33]; failure freeness and robustness against failures [33], [105], [60]. These issues need immediate attention for successful adoption of auctions by purchasing departments. Designing software implementation frameworks so as to allow sensitivity analysis of procurement decisions in complex supply chain environments is also an important issue. Use of multi-agent agent technology in automating standard electronic procurement problems is one more issue. Using emerging Internet technology standards such as ebXML in implementing e-procurement solutions is an immediate practical issue.

Procurement Exchanges

Procurement exchanges are those where there are multiple buyers and multiple suppliers and the exchange facilitates matching of buyers with suppliers. All the issues become more complex with exchanges because of the presence of multiple buyers. There is a large body of literature on exchanges; for example, see [106], [24], [107].

ACKNOWLEDGMENT

The first two authors would like to acknowledge the support of General Motors Research and Development Center, Warren, Michigan, USA for carrying out this research under the IMPROVE research grant.

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