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O.R. Applications

Multiattribute electronic procurement using goal programming

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Abstract

One of the key challenges of current day electronic procurement systems is to enable procurement decisions transcend beyond a single attribute such as cost. Consequently, *multiattribute procurement* have emerged as an important research direction. In this paper, we develop a multiattribute e-procurement system for procuring large volume of a single item. Our system is motivated by an industrial procurement scenario for procuring raw material. The procurement scenario demands multiattribute bids, volume discount cost functions, inclusion of business constraints, and consideration of multiple criteria in bid evaluation. We develop a generic framework for an e-procurement system that meets the above requirements. The bid evaluation problem is formulated as a mixed linear integer multiple criteria optimization problem and goal programming is used as the solution technique. We present a case study for which we illustrate the proposed approach and a heuristic is proposed to handle the computational complexity arising out of the cost functions used in the bids.

Keywords: Goal programming; e-Procurement; Multiattribute auctions; Multiple criteria optimization; Configurable bids; Business rules; Volume discount cost functions; Bid evaluation problem; Mixed integer linear program

1. Introduction

The Internet and Internet-based technologies are impacting businesses in many ways. With the increasing pressure that companies are experiencing as markets become more global, the Internet continues to play a critical role to speed up operations and to cut costs. By enabling new business processes, Internet also helps organizations to react quickly and efficiently in order to keep up with changing market requirements. One such business process that has gained much attention in recent times is Business-to-Business e-procurement. e-Procurement is an Internet-based business process for obtaining materials and services and managing their inflow into the organization. Procurement is an important part of the more general *supplier selection* or *vendor*

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selection problem [3,48], which is concerned with the selection of candidate suppliers, determining the nature of contracts with them, and then selecting the best set of suppliers among the alternatives.

Initially, in the past few years, naive use of Internet and information technologies saw complex back-end applications supporting supply chains of large companies, with simple front-end e-catalog systems supporting procurement. Recent trends are focusing on user friendly applications that embed sophisticated business logic and algorithms. This involves identifying, evaluating, negotiating, and configuring optimal groupings of suppliers' bids, which are received in response to a buying organization's Request-for-Quote (RFQ). The objective is usually to minimize the total procurement cost subject to various business constraints. This need is present during the initial stage of awarding business to suppliers on new products, and is also present when primary suppliers are unable to deliver supplies (e.g., in the case of a strike, natural disaster, financial default, or other event that causes a work stoppage) to existing products. Such a procurement process, with suppliers bidding in response to buyer's RFQ and the buyer evaluating the bids, borders on the *auction* mechanism. Auction is a market mechanism with well-defined set of rules for determining the terms of an exchange of something for money [36]. Auction mechanisms enable automated negotiation and dynamic pricing, which are not only useful for selling but also in procurement where the buyer is the auctioneer and the sellers are bidders. Numerous major companies have either used or are in the process of using Internet-based automated auction and negotiation mechanisms for their procurement operations. For example, retailers in footwear, home products and fashion are using GlobalNetXchange private auction exchange, auto manufacturers are using Covisint's auctions capabilities, and GE uses its own Global Exchange Services to help procure goods more effectively from suppliers [17]. There are many published case studies of successful deployment of e-auctions in procurement, for example, see [16,18,23,30]. For a more general overview of use of auctions in e-procurement see [9].

1.1. Motivation

The complexity of a procurement process depends primarily on the number and the quantity of the items procured and also the business constraints associated with it. Procuring a single indivisible item is the simplest form of automated negotiation mechanism used in e-procurement. The suppliers respond to the buyer's RFQ with a bid price. The winning supplier is the one who quotes the lowest bid price. Many commercial systems[1] are available with no complex or expensive software and can be deployed within hours of identification of a new procurement opportunity. However, there are certain industrial procurement scenarios which demand more expressive bids and flexible bid evaluation techniques.

A team of researchers from General Motors Research (which included the last three authors of this paper) recently used an approach based on procurement auctions and optimization techniques to solve an industrial procurement problem [9]. This approach, soon to be deployed as a web application within General Motors Corporation (GM Corp), allows business users to determine an optimal allocation of awards to bids using the application over the company's intranet. The procurement corresponds to that of an important raw material for automotive manufacturing. The overall commodity sourcing process is shown in Fig. 1. Within GM, a huge amount of this commodity is sourced every year. To gain maximum cost savings (at a sufficiently high level of desired quality), GM uses a centralized demand aggregation and reselling application for the whole supply chain. This application attempts to combine the individual commodity requirements of its processors and plants, with GM's direct commodity requirements to create large orders. These larger orders often qualify for significant volume discounts with the commodity suppliers. GM then resells a portion of the purchased commodity to its processors to cover their material needs. The overall process is very complex and manual approaches for determining an allocation of awards to the suppliers require enormous effort.

The requirements for the commodity are aggregated within a centralized system. The tool looks at the catalogs of the approved suppliers and sends RFQs. The suppliers submit *configurable bids* [6] to the tool in response to the RFQ. A configurable bid gives either a base price for a bundle and quantity, or a volume discount price, which is a function of quantity. The bid consists of various attributes, with a value specified for each of these attributes. A supplier can also specify some logical rules for assigning some discounts to specific

[1] SpeedBuy from http://www.edeal.com; and many solutions from http://www.freemarkets.com.

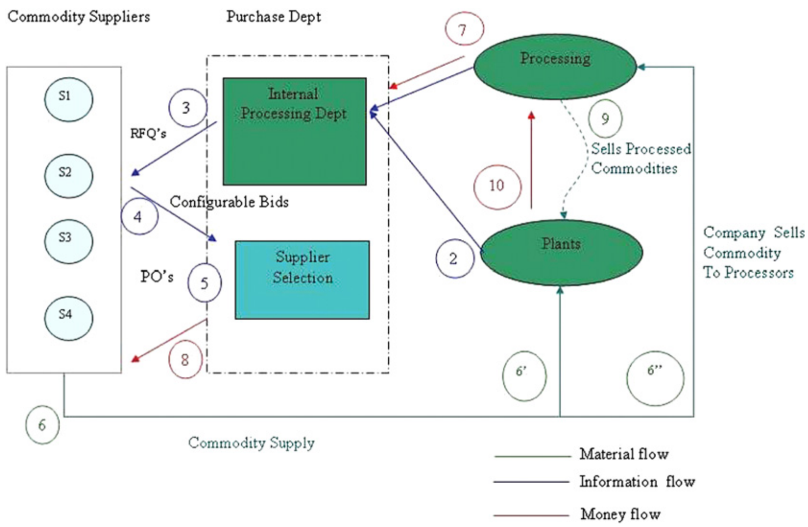

Fig. 1. A typical procurement process.

combination of selected attributes. There are some bounds on the number of units that a supplier can supply to a plant. Also, there are some restrictions on the number of suppliers that can be selected for a plant and also on the number of suppliers that can be awarded business. All of these *business* constraints are determined by taking into consideration the suppliers' capacities, their disruption risk, their overhead costs, etc.

The tool evaluates the bids in order to choose the best configuration and feeds them into the optimization model. The tool outputs a cost effective allocation to the suppliers. The model that the team created to solve the problem determines an efficient allocation of awards to bids so that overall procurement cost is minimized subject to the business constraints. A mathematical programming model used to determine the winners of the procurement auction. It was demonstrated that significant cost savings (as high as 3%) are possible when applying an auctions and optimization based approach to the above problem. It was also shown that using this type of modeling approach allows business analysts to explore multiple different constraints on the award and reallocation process. This leads to a much better understanding of the sensitivity of the optimal sourcing decisions to business constraints than possible using a more traditional manual process.

However, the system is not generic enough to handle volume discount bids for each of the attributes and is not flexible to incorporate multiple objectives in bid evaluation. Often there are multiple criteria in bid evaluation like *maximize on-time delivery*, *minimize part failure rate*, etc., along with the traditional criterion of minimizing procurement spend. Thus a generic system for a large volume industrial procurement demand the following requirements: (1) the bidding language should be expressive enough for the suppliers to express the various attributes of the item (like *quality*) and the procurement process (like *lead time*), (2) the supplier selection decision should be based on various criteria, (3) the process should make best use of the volume discounts offered by the suppliers, and (4) procurement decisions should take into account business constraints of the buyer and the suppliers.

In this paper, we develop a generic framework for an e-procurement system that meets the above requirements, using the state-of-art techniques and the industry best practices. The framework is based on the e-procurement model proposed in [26]. The contributions, along with the outline of this paper can be summarized as follows. The related literature and the industry best practices are briefly reviewed in Section 2. In Section 3, the bid evaluation problem that arises in the above multiattribute procurement is modeled as a mixed linear integer, multiple criteria optimization problem. The structure and complexity of the bid evaluation problem is studied in Section 4, where it is proved that a simple instance of this problem (with a single attribute and with a single value for this attribute) is $\mathcal{NP}$-hard. A goal programming approach for solving the bid evaluation problem is proposed in Section 5. In Section 6 the proposed approach is illustrated with a numerical example for a procurement scenario with realistic constraints and goals. To tackle the computational complexity arising out of the use of piecewise linear cost functions, a linear relaxation based heuristic is proposed in Section 7. Conclusions are drawn in Section 8 with suggestions for future work.

2. Relevant literature and industry best practices

In this section, we provide a brief summary of the techniques from literature and industry best practices, relevant to the requirements stated above. Combinatorial auctions [12] have been found successful in industrial procurement of multiple items [18,37]. In this paper, we consider procurement of large volume of a single item.

2.1. Volume discount bids

The procurement is for multiple units of a single good and the demand needs to be satisfied by procuring from multiple suppliers. Thus, for each winning supplier, the buyer has to decide the winning quantity. This dimension allows the suppliers to submit a *cost function*, instead of a unit price. Quantity discount pricing has been studied by various researchers with varying assumptions on the price structure and the underlying decision problem [31,28,14,19,29]. The commonly used cost function is the piecewise linear function defined over quantity. Procurement auctions with piecewise linear cost curves are common in industry for long-term strategic sourcing [14] and they have proved to be profitable for the buying organization [23]. e-Procurement auctions with such cost functions have been considered in [19,29,27]. The cost function, depending on the mathematical representation, leads to a multiple choice knapsack constraint [19,29] or tree knapsack constraint [27].

2.2. Multiattribute auctions

In industrial procurement, several aspects of the supplier performance, such as quality, lead time, delivery probability, etc. have to be addressed, in addition to the qualitative attributes of the procured item. A multiattribute bid thus has several dimensions and this also allows the suppliers to differentiate themselves, instead of competing only on cost. Multiattribute auctions deal with trading of items which are defined by multiple attributes. They are considered to play significant role in the commerce conducted over the WWW [45,4]. Multiattribute auctions as a model for procurement within the supply chain was studied in [10]. It is a one-shot auction in which the suppliers respond to the *scoring function* provided by the buyer. Multiattribute auction for procurement proposed in [8] has two stages: a supplier is chosen in the first stage and the buyer bargains with the chosen supplier in the second stage to adjust the level of quality. The other approach in designing multiattribute auctions is combining multicriteria decision analysis and single-sided auction mechanisms. Multicriteria decision analysis has been used in supplier selection problems [22,3]. Multiattribute auction based on multiattribute utility theory (MAUT) [41] for e-procurement was proposed in [5]. Multicriteria decision analysis techniques like MAUT are also used in bid analysis products from Frictionless Commerce and Moai Technologies.[2] The bids submitted by the suppliers are in the form of (*attribute*, *value*) pairs. The buyer assigns weights to the attributes indicating their relative importance and has a scoring function for each attribute. The scoring functions essentially convert each attribute value to a *virtual currency*, so that all attribute values can be combined into a single numerical value that quantifies the bid. The combination rule generally used is the weighted additive combination [5]. For a more comprehensive study on the design of multiattribute auctions see [4]. IBM Research's Absolute decision engine [32] provides buyers, in addition to standard scoring mechanisms, an interactive visual analysis capability that enable buyers to view, explore, search, compare, and classify submitted bids. An iterative auction mechanism to support multiattribute procurement was proposed in [2]. The buyer uses an additive scoring function for non-price attributes and he announces a scoring rule at the beginning of each round. Through inverse optimization techniques, the buyer learns his optimal scoring rule from the bids of the suppliers. The mechanism is designed to procure a single indivisible

[2] http://www.frictionless.com and http://www.moai.com.

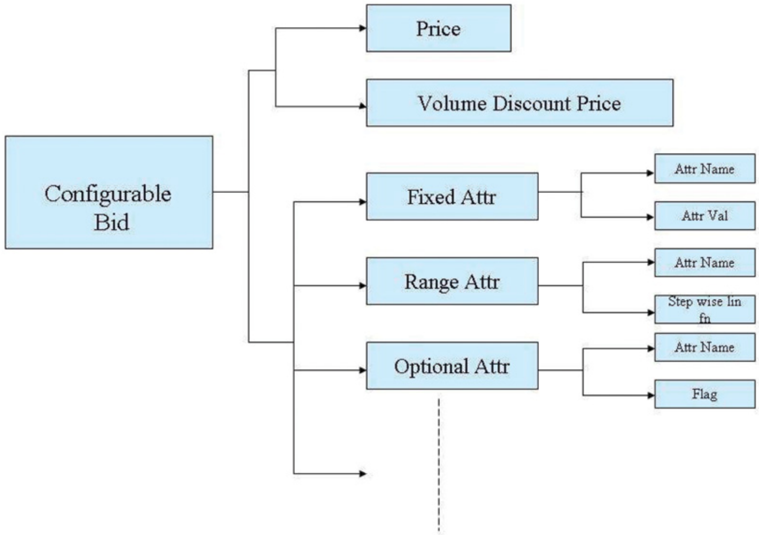

Fig. 2. Main elements of a configurable bid.

item. An English auction protocol for multiattribute items was proposed in [15], which again uses weighted additive scoring function to rank the bids. All the above mechanisms solve the *incomparability* between the bids, due to multiple attributes, by assigning a single numerical value to each bid and then ranking the bids by these values. *Multicriteria auction* proposed in [43] is an iterative auction which allows incomparability between bids and the sellers *increment* their bid value by bidding *more* in at least one attribute. Iterative multiattribute auctions for procurement was proposed in [39] for procuring a single item. The bid consists of a price for each attribute and the iterative format provides feedback to the suppliers to update their bid prices.

2.3. Optimization techniques

e-Procurement systems that promise a productive strategic sourcing should take into account various business rules and constraints, like *exclusion constraints* (goods of supplier X cannot be received from location A), *aggregation constraints* (for example, at least two and at most five winning suppliers), *exposure constraints* (for example, at most forty percent of total business to any supplier), *business objective constraints* (for example, overall quality factor must be at least 7), etc. For such systems, optimization techniques can be used where the business rules are added as side constraints to the optimization problem. The bid evaluation problem is an optimization problem and it is indeed one of the earliest applications of linear programming [20]. Many commercial bid analysis products from companies like Emptoris, Rapt, and Mindflow[3] also use optimization techniques like linear programming and constraint programming.

2.4. Configurable bids

In multiattribute auctions, the bids are generally described as sets of *attribute–value* pairs. To further automate negotiations on complex goods and services, *configurable bids* were proposed in [6]. In configurable bids, bidders can specify multiple values and price mark-ups for each attribute and the buyer can configure the bid optimally by choosing appropriate values for the attributes. But configurable bids exhibit combinatorial features and the total number of possible configurations increases exponentially with the number of attributes and the number of possible values for each attribute. Hence, enumerating and communicating all possible configurations to the buyer is not a viable alternative. The proposal in [6] is to describe configurable bids by closed

[3] http://www.emptoris.com, http://www.rapt.com, and http://www.mindflow.com.

form functions of the configurable bids i.e. the possible configurations are given as a function of price on quantity and qualitative attributes. The total price of a configuration is the sum of the prices of the individual attribute values chosen. The logical constraints between the attribute values are added as linear constraints in the optimization model to select the best configuration for each bid Fig. 2.

The aim is to develop a generic e-procurement system with the following requirements: (1) the supplier should be able to bid on *multiple attributes*, (2) a rich bidding language like *configurable bids* should be available to automate negotiations across multiple bids, (3) should support *volume discount* bids, (4) there should be flexibility for including *business rules* and *purchasing logic* in bid evaluation, and (5) bids should be evaluated using *multiple criteria*. Current solutions satisfy subsets of these requirements, but no single existing solution satisfies all of these requirements at the same time. For example, bid optimization and analysis tool (BOAT) from Perfect[4] provides flexibility for incorporating business rules and uses multicriteria decision analysis technique for handling multiple attributes, but does not allow configurable bids. The multiattribute auctions proposed in [6] and the GM application come closest to satisfying all of these requirements. Our proposal differs from existing approaches in the following ways: (1) the configurable bids *can* possibly be piecewise linear functions of price on quantity for each value of each attribute, (2) the configurable bids on the multiple attributes are directly integrated into the bid evaluation, (3) the bid evaluation problem is modeled as a multiple criteria optimization problem, and (4) goal programming technique is used for bid evaluation.

3. The model

In this section we systematically develop the generic framework for an e-procurement system that meets the requirements of an industrial procurement scenario briefed above. First we note the design issues that are considered in this paper.

3.1. Design issues

The procurement process with RFQ and bidding is inherently based on auctions and hence the design principles generally follow auction design. Auctions can categorized based on the dynamics as: (1) *one-shot* or *single-round* auctions and (2) *progressive* or *iterative* or *multiple-round* auctions. One-shot auctions are sealed bid auctions, which has a single bidding phase, during which all the bidders submit their bids. Progressive auctions can be sealed bid or open bid, but has multiple rounds of bidding phases. At the end of each bidding phase, there will be flow of information from the auctioneer to the bidders. This will help the bidders to prepare their bids for the next bidding phase. The design parameters of one-shot auctions are *bidding language*, *bid evaluation policy*, and *pricing policy*. The bidding language specifies the format of bids, the bid evaluation policy describes the technique to determine the winners, and the pricing policy determines the price of the winning goods. On the other hand, design of progressive auctions is relatively non-trivial, which includes the specification of bidding language, bid evaluation technique at each bidding round, information exchange at the end of each round, termination condition, and the pricing policy. However, the progressive auction has many advantages over its one-shot counterpart [11], especially in procurement [39]. There are many design methodologies for progressive auctions [7,38,25] for procurement, even for multiattribute procurement [39]. However, the extension of the above techniques is not obvious for the procurement scenario considered in this paper. We design here only the one-shot procurement, which has just one bidding phase.

The procurement process with the RFQ and the bidding, only borders on auctions and are indeed less formally structured than auctions. The auction design is generally based on the principles of *mechanism design*. Mechanism design [34] is the sub-field of microeconomics and game theory that considers how to implement good system-wide solutions to problems that involve multiple self-interested agents, each with private information about their preferences. The mechanism design methodology has also been found useful in designing e-markets [46]. One of the main assumptions in mechanism design is that the rules of the auction is a common knowledge to all the participating agents. In procurement, though the rules of bid submission are common

[4] http://www.perfect.com.

knowledge, rules of bid evaluation may not be revealed to the suppliers. In the procurement scenario considered in this paper, the purchasing manager may take into account several business rules and purchasing logic in bid evaluation, which are not generally revealed to the suppliers. Moreover, the criteria and the constraints can be modified by the buying organization, based on the received bids. Hence, we do not follow the mechanism design approach. We consider only the design issues related to bid structure and bid evaluation technique, from the perspective of the buying organization (auctioneer). We use the *pay-as-bid* pricing policy (the suppliers are paid the cost quoted in the bid), which is the commonly used pricing policy in current e-procurement systems. This pricing policy induces strategic behavior in the bid preparation of the suppliers. As the suppliers have to supply the goods at the quoted cost, they do not reveal their true cost and quote a higher cost to obtain profit. This is a liability to both the suppliers (in terms of strategic bid preparation) and the buyer (in terms of less *economically efficient* trade). The bid preparation problem of the supplier is complimentary to the auction design problem of the buyer. This is an equally important design issue (which is implicitly taken care in the mechanism design approach). However, we do not consider this issue in this paper.

Based on the one-shot dynamics, e-procurement system considered in this paper consists of the following phases: (1) *RFQ generation* and *distribution* by the buyer, (2) sealed *bid submission* by the suppliers during a predefined bidding interval, and (3) *bid evaluation* by the buyer (after the expiration of the bidding interval) to determine the winning bids.

3.2. RFQ generation

The RFQ consists of relevant information such as an identifier, product name, issue date, quote due date, the buyer information, and the attributes. The set of attributes is denoted by U and E_u is the admissible domain for each attribute $u \in U$.

3.2.1. RFQ notation

$[\underline{b}, \overline{b}]$	requested quantity range ($\underline{b} \leqslant \overline{b}$)
U	set of attributes
E_u	admissible domain of values for attribute $u \in U$
$u_0 \in U$	is cost attribute

The attributes can consist of product features (like quality), service features (like lead time, warranty, maintenance spares), and supplier features (like stock values, manufacturing capacity). We use u_0 to denote the mandatory attribute cost in U. The domain E_u of attribute u can be either discrete (like lead time) or continuous (like on-time delivery probability).

3.3. Bid submission

Configurable bids proposed in [6] allow bidders to specify multiple values for each attribute and mark-up prices (unit price) for each attribute value. The buyer can configure the product by choosing appropriate attribute values that suit his interests and demand. In this paper we consider configurable bids of the following nature: for each attribute u, the bid j can specify a set of values $W_u \subseteq E_u$ and a piecewise linear price function Q_{juw} defined over quantity range $[\underline{a}_{juw}, \overline{a}_{juw}]$ for each attribute value $w \in W_u$. In [6], the price function of any attribute is linear with quantity, whereas we generalize it to piecewise linear. This generalization allows the bidder to specify piecewise linear cost functions for attributes like lead time (transportation mode), whose cost may depend on quantity. The linear function is a special case of the piecewise linear function, and hence can also be used to represent mark-up prices like in [6] for attributes like warranty. Fig. 3 illustrates sample cost functions for attributes (1) cost (u_0) and (2) delivery lead time. For the cost attribute, the supplier submits a single volume discount bid, whereas for the *lead time* a piecewise linear cost function for each of the attribute value is submitted. The different lead times are due to different transportation modes and hence different cost curves. Depending on the business constraint (with respect to lead time) and budget constraints, the buyer can configure the bid by choosing different quantity for different lead times.

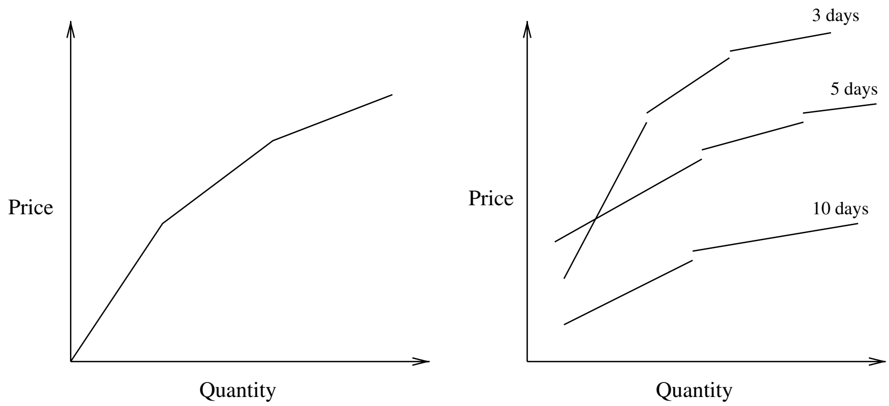

Fig. 3. Configurable bids for attributes cost and *delivery lead time.*

Table 1
Notation for bid j

W_u	Set of values for each attribute $u \in U$
$[\underline{a}_{juw}, \overline{a}_{juw}]$	Supply quantity range available for attribute u with value w
Q_{juw}	Piecewise linear cost function for attribute u with value w defined over $[\underline{a}_{juw}, \overline{a}_{juw}]$
l_{juw}	Number of piecewise linear segments in Q_{juw}
$\tilde{\delta}^{*}_{juw}$	Breakpoints at which the slope of Q_{juw} changes
β^{s}_{juw}	Slope of Q_{juw} on $(\tilde{\delta}^{s-1}_{juw}, \tilde{\delta}^{s}_{juw})$
n^{0}_{juw}	Price at $\underline{a}_{juw}$ $(= \tilde{\delta}^{0}_{juw})$
n^{s}_{juw}	Extra fixed cost at the breakpoint s
$\mathcal{Q}_{juw}$	$\equiv((\tilde{\delta}^{0}_{juw}, \ldots, \tilde{\delta}^{l_{juw}}_{juw}), (\beta^{1}_{juw}, \ldots, \beta^{l_{juw}}_{juw}), (n^{0}_{juw}, \ldots, n^{l_{juw}}_{juw}))$

Table 1 provides the notation for bid j, including the piecewise linear cost function that can be used as the cost curve for each of the attribute values. The price function Q_{juw} shown in Fig. 4 is the total price (not the unit price) at which the bidder is willing to trade as a function of quantity. This piecewise linear cost curve was used in [27] for procurement auctions and similar functions were used in [19]. The function shown in the figure can be compactly represented by tuples of break points and slopes $((\tilde{\delta}^{0}_{juw}, \ldots, \tilde{\delta}^{l_{juw}}_{juw}), (\beta^{1}_{juw}, \ldots, \beta^{l_{juw}}_{juw}), (n^{0}_{juw}, \ldots, n^{l_{juw}}_{juw}))$ where l_{juw} is the number of linear segments and n^{0}_{juw} is the price at $\delta^{0}_{juw} = \underline{a}_{juw}$. The break points $\tilde{\delta}^{0}_{juw}$ $(= \underline{a}_{juw}), \tilde{\delta}^{1}_{juw}, \ldots, \tilde{\delta}^{l_{juw}}_{juw}$ $(= \overline{a}_{juw})$ denote the points where the slope changes and the corresponding

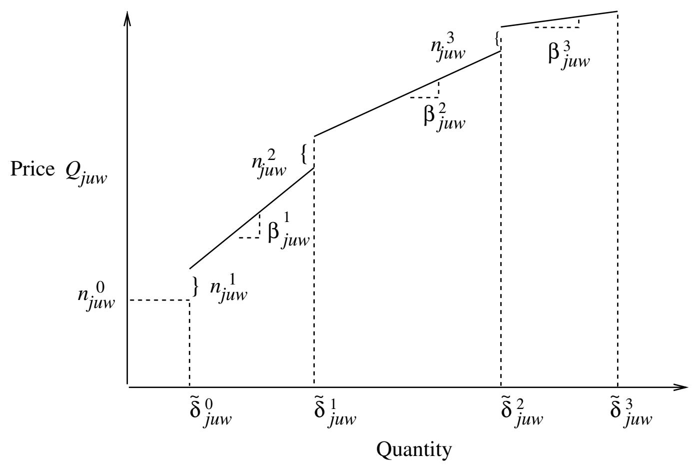

Fig. 4. Piecewise linear price function Q_{juw}.

slopes are $\beta^1_{juw}, \ldots, \beta^{l_{juw}}_{juw}$. The extra fixed costs n^s_{juw} at the breakpoints s introduce the discontinuities in the function. The function is assumed to be non-decreasing, but the slopes β^s_j need not be decreasing as shown in the figure. The assumed cost structure is generic enough to include various special cases: linear, concave, convex, continuous, and $\underline{a}_j = 0$. The cost structure enables the suppliers to express their *volume discount* or *economies of scales* and/or the production and logistics constraints. The set W_u is assumed to be always finite even if the domain E_u is infinite. For example the attribute *delivery probability* can have $E_u = [0.8, 1]$, but the bidder can specify only finite possible values, say, $W_u = \{0.8, 0.85, 0.9\}$. The Q_{juw} shown in Fig. 3 is for only one value w of attribute u. The bidder can specify such function for each $w \in W_u$ as shown in Fig. 1. If the buyer configures the bid j with q_{juw} units for $w \in W_u$ and $u \in U$, then the total cost of procurement from bid j is

$$\text{Cost}_j = \sum_{u \in U} \sum_{w \in W_u} Q_{juw}(q_{juw}). \tag{1}$$

The implicit assumption in the above cost structure is that the attributes are independent of one another except with the quantity procured.

3.4. Bid evaluation

Multiple attributes can be used both in bid definition and bid evaluation. In [2], the attributes are distinguished as endogenous (bidder controllable) and exogenous from the bidders' perspective. Attributes in bid definition (or RFQ) provide a means to specify a complex product or service, whereas in bid evaluation, the buyer can use multiple attributes to select the winning bidders. Therefore in bid definition, all attributes should be endogenous for the bidders, whereas in bid evaluation, the buyer can use some exogenous attributes to select the winners. We will use the words *criteria* for bid evaluation and *attributes* for bid definition. The words *criteria* and *attribute* are used interchangeably in the multiple criteria decision making (MCDM) literature. In MCDM, attributes are defined as descriptors of objective reality which represent values of the decision makers [49]. We associate the word attribute with the RFQ and bids i.e. the buyer declares in the RFQ various attributes of the goods. We use the word *criteria* to indicate the objectives defined by the buyer for evaluating the bids. For example, if the *attributes* defined in the RFQ are cost, delivery lead time, and delivery probability, and then the *criteria* used by the buyer for evaluating the bids can be total cost, delivery lead time, and supplier credibility. With the above norm established, a criterion for evaluating the bids may consist of zero, one, or many attributes defined in the RFQ. For example, the criterion that the winning supplier should have high *credibility*, is not an attribute defined in the RFQ but a private information known to the buyer. On the other hand, minimizing cost of procurement is a function of many attributes defined in the RFQ. Thus *criterion* is used here in the sense of an objective. Multicriteria analysis of supplier selection was discussed in [47].

Evaluating the bids by taking into account different factors is an MCDM problem. MCDM has two parts: *multiattribute decision analysis* and *multiple criteria optimization*. Multiattribute decision analysis techniques like MAUT [41] are often applicable to problems with a small number of alternatives that are to be ordered according to different attributes. In MAUT, a multiattribute utility function representing the preferences of the decision maker is elicited and is used to order the set of feasible alternatives. When the decision space has very large or infinite number of alternatives, the practical possibility of obtaining a reliable representation of the decision maker's multiattribute utility function is limited. Multiple criteria optimization [44] techniques are used in such scenarios where explicit knowledge of the utility function is not available. The bid evaluation problem of [5,15] to rank the bids was solved by MAUT of multiattribute decision analysis. When configurable bids are used, the problem is not just selecting best bid(s) but selecting the best configuration of bids. In [6], the bid evaluation problem is done in two stages: first, the best configuration for each bid is chosen using the scoring function of the buyer and in the second stage, MAUT based techniques are used to rank the bids. With the nature of configurable bids proposed here, and allowing aggregation of goods across the bids, the number of alternatives is very large and hence ranking of alternatives using MAUT is not a viable alternative. In multiple criteria decision making situations with large or infinite number of decision alternatives, where the practical possibility of obtaining a reliable representation of decision maker's utility function is very limited, multiple criteria optimization techniques are useful approaches [42]. We model the bid evaluation problem as a linear integer multiple criteria optimization problem.

3.4.1. Decision variables

$x^s_{juw} \in [0,1]$ fraction of goods bought from bid j with value w for attribute u with unit cost β^s_{juw} $(s = 0,\ldots,l_{juw})$
$d^s_{juw} \in \{0,1\}$ binary variable that assumes value 1 if goods are bought from linear segment s of Q_{juw} $(s = 1,\ldots,l_{juw})$
$X_j \geqslant 0$ amount of goods bought from bid j
$z_j \in \{0,1\}$ binary variable that selects/rejects bid j

We have not included a binary variable for choosing value w for attribute u of bid j, as this decision is implied by the binary variable d^0_{juw} and x^s_{juw} is not defined for $s = 0$ as this refers to the indivisible quantity $\underline{a}_{juw}$ which is again taken care of by d^0_{juw}.

3.4.2. Constraints

We first formulate the constraints in configuring the bids. The amount of goods from bid j with value w for attribute u has to be chosen considering the price function Q_{juw}. As shown in Fig. 1, this function can be non-linear. Using the piecewise linear nature, we can represent them using linear inequalities. Let $\delta^s_{juw} = \tilde{\delta}^s_{juw} - \tilde{\delta}^{s-1}_{juw}$, for $s = 1,\ldots,l_{juw}$ and for all u,w, and j. The quantity range $[\underline{a}_{juw}, \overline{a}_{juw}]$ is split into l_{juw} segments, where the quantity range in segment s is $[0, \delta^s_{juw}]$. For the above conversion to make sense, whenever $x^s_{juw} > 0$, then $d^{s'}_{juw} = 1$ and $x^{s'}_{juw} = \delta^{s'}_{juw}$ for $s' < s$. The following is the set of linear constraints that handle the quantity selection:

$$d^0_{juw} \geqslant d^1_{juw}, \tag{2}$$

$$x^s_{juw} \leqslant d^s_{juw}, \quad s = 1,\ldots,l_{juw}, \tag{3}$$

$$x^s_{juw} \geqslant d^{s+1}_{juw}, \quad s = 1,\ldots,l_{juw} - 1. \tag{4}$$

The above constraints are for each $w \in W_u$ of every attribute $u \in U$ of every bid j. The following constraints handle the consistency and logical relationships among the variables, and the demand requirements:

$$X_j \leqslant z_j \overline{b} \quad \forall j, \tag{5}$$

$$\sum_{w \in W_u} \left(d^0_{juw} \underline{a}_{juw} + \sum_{s=1}^{l_{juw}} \delta^s_{juw} x^s_{juw} \right) = X_j \quad \forall u \in U \;\; \forall j, \tag{6}$$

$$\underline{b} \leqslant \sum_j X_j \leqslant \overline{b}. \tag{7}$$

For each bid, constraint (5) makes sure that X_j is non-zero only if the bid is a winning bid. For each bid, the quantity selected should be the same across all the attributes. This is handled by (6). Finally, (7) constrains the overall procured goods to be within the requested quantity range.

There may be several business rules and purchasing policies like restriction on the number of suppliers, allowable quantity in a single shipment, homogeneity of attributes [6], etc. Such business rules can be added as side constraints. Furthermore, there may be interaction effects between the attributes. These are generally supply side business constraints, where the supplier specifies certain logical restrictions on the allowable combination of the attribute values. The supplier may give special discounts on certain attribute combinations. These logical constraints are modeled as linear constraints in [6], which can be added with the above set of constraints.

3.4.3. Objectives

Let $\mathbf{X}$ denote the vector of decision variables. Then the bid evaluation problem is the following multiple criteria optimization problem with G linear objectives:

$$\min\{\mathbf{c_1X} = f_1\}$$
$$\min\{\mathbf{c_2X} = f_2\}$$
$$\vdots$$
$$\min\{\mathbf{c_GX} = f_G\}$$
$$\text{s.t.}\quad \mathbf{X} \in F,$$

where F is the set of feasible solutions defined by the constraints. Without loss of generality all the objectives considered are of minimization type. The objectives of the buyer can be like *minimize total-cost*, *minimize lead-time*, *maximize on-time delivery probability*, etc. For example, the objective of minimizing total cost is

$$\min \sum_j \sum_{u\in U} \sum_{w\in W_u} \left(n^0_{juw} d^0_{juw} + \sum_{s=1}^{l_{juw}} (n^s_{juw} d^s_{juw} + \beta^s_{juw} \delta^s_{juw} x^s_{juw}) \right).$$

4. Structure and complexity

The knowledge of the structure of an optimization problem helps a great deal in designing heuristics and exact algorithms for solving it. In this section, we investigate the various known structures that arise due to the constraints and also determine the complexity of the bid evaluation problem.

4.1. Structure

The demand constraint (7) with variables X_j is a set of two knapsack constraints [33,40]: one with the lower bound $\underline{b}$ and another with the upper bound $\overline{b}$. Once these X_j's are fixed, then for each bid j, the problem is to optimally configure the bids by choosing appropriate quantity from each attribute value.

Proposition 1. *The constraints (2)–(6) are equivalent to*

$$d^s_{juw} \geqslant d^{s+1}_{juw}, \quad 0 \leqslant s < l_{juw} \ \forall u \in U \ \forall j, \tag{8}$$

$$\sum_{w\in W_u} \left(d^0_{juw} \underline{a}_{juw} + \sum_{s=1}^{l_{juw}} \delta^s_{juw} d^s_{juw} \right) \geqslant X_j \quad \forall u \in U \ \forall j, \tag{9}$$

$$\sum_{w\in W_u} \left(d^0_{juw} \underline{a}_{juw} + \sum_{s=1}^{l_{juw}-1} \delta^s_{juw} d^{s+1}_{juw} \right) \leqslant X_j \quad \forall u \in U \ \forall j. \tag{10}$$

Proof. ($\Rightarrow$) Let $\overline{d}^s_{juw} \in \{0,1\}$ satisfy (3)–(6). If $X_j = 0$, then the implication is trivial. The constraints (2)–(4) imply (8). Substituting (3) in (6), we get (9) and likewise substituting (4) in (6) (ignoring $s = l_{juw}$), we get (10). Thus $\overline{d}^s_{juw}$ that satisfy (3)–(6) also satisfies (8)–(10).

($\Leftarrow$) Let $\overline{d}^s_{juw} \in \{0,1\}$ satisfy (8)–(10). Define $e_{juw} = \max\{s\}$ such that $\{\overline{d}^s_{juw} = 1\}$ if $\overline{d}^0_{juw} = 1$ and $e_{juw} = -1$, otherwise. If $z_j = 0$ and/or $\overline{d}^0_{juw} = 0$, then the implication is trivial. Let $\overline{d}^0_{juw} = 1$. Assign $x^s_{juw} = \overline{d}^s_{juw}$ for $s < e_{juw}$ and $x^s_{juw} = 0$ for $s > e_{juw}$. If $\overline{d}^0_{juw} = 1$, then $X_j > 0$. Now we need to determine $x^{e_{juw}}_{juw}$. Let $X = X_j - \sum_{w\in W_u} (d^0_{juw} \underline{a}_{juw} + \sum_{s=1}^{e_{juw}-1} x^s_{juw} \delta^s_{juw})$. It is easy to see that $X \geqslant 0$. Now determining $x^{e_{juw}}_{juw}$ is a variant of the continuous knapsack [33] problem. The X is the demand in the knapsack and $x^{e_{juw}}_{juw}$ is a continuous variable in $[0,1]$ with weight $\delta^{e_{juw}}_{juw}$. The continuous knapsack problem can be solved in polynomial time [40]. Thus the $\overline{d}^s_{juw}$ and the respective x^s_{juw} satisfy (3)–(6). $\square$

For a fixed X_j, constraints (9) and (10) are knapsack constraints. These constraints with (8) are *precedence knapsack constraints* [24]. The piecewise linear function can also be expressed using *multiple choice knapsack constraints* as in [19,29].

4.2. Complexity

From the above discussion, it is highly likely that the bid evaluation problem (with single objective) would be $\mathcal{NP}$-hard (as knapsack problems are $\mathcal{NP}$-hard [21]). We show here that the problem is $\mathcal{NP}$-hard by showing that the simple special case of the problem with one attribute with one value, and one linear cost segment, is $\mathcal{NP}$-hard. We consider the single objective of minimizing cost (which is a mandatory criterion in any procurement scenario). For this special case, $juw \equiv j$, $l(juw) = 1$, $z_j = d_j$, and $X_j = \underline{a}_j d_j + x_j(\overline{a}_j - \underline{a}_j)$. Hence the decision variables z_j and X_j can be ignored. Any algorithm that can solve the original multiattribute and piecewise linear cost function problem can also solve this special case. Now we show that the decision version of this special case is $\mathcal{NP}$-complete upon reduction from 0–1 knapsack problem (thus the single objective optimization versions of the special and generalized case are $\mathcal{NP}$-hard).

Definition 1 (*DP*). We are given a set of bids $J = \{([\underline{a}_j, \overline{a}_j], \beta_j, n_j)\}$ from the suppliers, the demand requirement of the buyer $[\underline{b}, \overline{b}]$, *and a goal G*. We are asked whether or not there exists $J' \subseteq J$ *and assignment* $x_j \in [0, 1]$, such that

$$\underline{b} \leqslant \sum_{j \in J'} \underline{a}_j + x_j(\overline{a}_j - \underline{a}_j) \leqslant \overline{b} \tag{11}$$

and $\sum_{j \in J'} n_j + \beta_j(\overline{a}_j - \underline{a}_j)x_j < G$.

Definition 2 (*KP*). We are given a finite set $U = \{(s(u), v(u))\}$, $B \in \mathscr{Z}^+$, and a goal $\tilde{G}$. We are asked whether or not there exists $U' \subseteq U$, such that $\sum_{u \in U'} s(u) \geqslant B$ *and* $\sum_{u \in U'} v(u) < \tilde{G}$.

Theorem 1. *DP is* $\mathcal{NP}$*-complete.*

Proof. To show that DP is in $\mathcal{NP}$, we observe that specifying a solution is to choose subset J' and assign nonnegative values to x_j, $j \in J'$. Given such a solution, we can verify whether it meets our requirements in polynomial time. To show $\mathcal{NP}$-hardness, we reduce an arbitrary KP instance to the following DP instance:

- $J = U$, $\underline{a}_j = \overline{a}_j = s(j)$, $n_j = 0$;
- $\underline{b} = B$, $\overline{b} = M$ ($\gg 0$, arbitrarily large number);
- $\beta_j = v(j)/s(j)$ $\forall j$ and set goal $G = \tilde{G}$.

It is obvious that the above reduction can be done in polynomial time. Let us show now that the reduction is valid. Suppose there is a solution to KP instance, i.e. there exists $U' \subseteq U$ such that $\sum_{u \in U'} s(u) \geqslant B$ and $\sum_{u \in U'} v(u) < \tilde{G}$. Choose $J' = U'$ and $x_j = 1$. This solution satisfies (11) and the goal G. So we have a solution to the DP instance. Now suppose the DP instance has a solution, i.e. there exist $J' \subseteq J$ and $x_j \in [0, 1]$ such that (11) is satisfied and $\sum_{j \in J'} n_j + \beta_j x_j(\overline{a}_j - \underline{a}_j) < G$. It can be easily verified that $U' = J'$ is a solution to the KP instance. Thus the above reduction is valid and hence DP is $\mathcal{NP}$-complete. □

If $\underline{a}_j = 0$ for all j, then the above special case is a variation of the continuous knapsack problem and can be solved in polynomial time. If $\underline{a}_j > 0$, even for some j, then the problem is $\mathcal{NP}$-hard. For this special case, if $\overline{b} = \infty$, then the feasible d_j can be found in polynomial time using the greedy approach of the knapsack problem. But for $\overline{b} < \infty$, determining a feasible d_j is equivalent to determining feasible solution in an equality constrained knapsack problem (assign $\overline{b} = B$ in the transformation used in the above theorem). This is $\mathcal{NP}$-hard for equality constrained knapsack problems [40] and hence the problem of finding feasible solutions for our bid evaluation problems is also $\mathcal{NP}$-hard.

5. Bid evaluation using goal programming

Multiple criteria optimization problems can be solved using various techniques like GP, vector maximization, and compromise programming [42,44]. We propose the use of GP to solve the bid evaluation problem. Unlike many multiple criteria optimization techniques which require special software tools, GP can be handled by commercial linear and nonlinear optimization software packages with minimal modifications. In GP, the criteria are given as goals and the technique attempts to simultaneously achieve all the goals as closely as possible. For example, the cost minimization criterion can be converted to the goal: cost $\leqslant$ \$20,000, where \$20,000 is the target or aspiration level. When the target levels are set for all criteria, GP finds a solution that simultaneously satisfies all the goals as *closely* as possible: it is more of a *satisficing* technique than an *optimizing* technique. The goal g can be any of the following types: *greater than or equal to* ($\geqslant t_g$), *less than or equal to* ($\leqslant t_g$), *equality* ($=t_g$), and *range* ($\in [\underline{t}_g, \overline{t}_g]$), where t_g's are the target or aspiration levels. Without loss of generality let us assume the following goal structure for the procurement problem:

$$\begin{aligned}
&\text{goal } \{\mathbf{c_1X} = f_1\} \quad (f_1 \geqslant t_1)\\
&\text{goal } \{\mathbf{c_2X} = f_2\} \quad (f_2 \leqslant t_2)\\
&\text{goal } \{\mathbf{c_3X} = f_3\} \quad (f_3 = t_3)\\
&\vdots\\
&\text{goal } \{\mathbf{c_GX} = f_G\} \quad (f_G \in [\underline{t}_G, \overline{t}_G])\\
&\text{s.t.} \quad \mathbf{X} \in F.
\end{aligned} \tag{12}$$

For each goal, there will be a deviational variable that measures the deviation from the target level and these give rise to new goal constraints:

$$\begin{aligned}
&\mathbf{c_1X} + \gamma_1^+ \geqslant t_1\\
&\mathbf{c_2X} - \gamma_2^- \leqslant t_2\\
&\mathbf{c_3X} + \gamma_3^+ - \gamma_3^- = t_3\\
&\vdots\\
&\mathbf{c_GX} + \gamma_G^+ \geqslant \underline{t}_G\\
&\mathbf{c_GX} - \gamma_G^- \leqslant \overline{t}_G\\
&\text{all } \gamma'\text{s} \geqslant 0.
\end{aligned} \tag{13}$$

The range goal gives rise to two constraints but the other goals lead to only one each. The γ_g^+ measures the deviation away from the goal in the positive direction and γ_g^- is for the negative direction. The above goal constraints do not restrict the original feasible region F. In effect, they augment the feasible region by casting F into a higher dimensional space [44]. The GP techniques vary by the way the deviational variables are used to find the final solution. We present in this paper the weighted GP technique for solving the bid evaluation problem.

Weighted GP (WGP) or *Archimedian* GP uses weights, given by the decision maker, to penalize the *undesirable* deviational variables. The decision maker (in the procurement case, the buyer) specifies the weights κ_g for a goal g. The weights measure the relative importance of satisfying the goals. The GP (12) will then be the following single objective programming problem:

$$\begin{aligned}
&\min \ \kappa_1\gamma_1^+ + \kappa_2\gamma_2^- + \kappa_3^+\gamma_3^+ + \kappa_3^-\gamma_3^- + \cdots + \kappa_G^+\gamma_G^+ + \kappa_G^-\gamma_G^-\\
&\text{s.t.} \quad (13) \text{ and } \mathbf{X} \in F.
\end{aligned} \tag{14}$$

The goals are generally incommensurable (for example, cost minimization is measured in currency whereas minimizing lead time is measured in days) and the above objective function is meaningless as the weighted summation includes different units. The most intuitive and simplest way would be to express γ_g as percentage rather than as absolute value [42]. For e-procurement, the buyer can specify maximum deviation allowed for a goal and then use the percentage of deviation in the objective function.

6. A case study

In this section, we present a procurement scenario, with realistic constraints and goals, as a case study to illustrate the applicability of the proposed model.

6.1. Description of the procurement scenario

Let a purchasing department be interested in procuring multiple units of raw material or accessories like power tools, fixtures, dies, machine tools, etc. The department comes up with the following details based on the requirements.

6.1.1. RFQ

$[\underline{b}, \overline{b}]$ requested quantity range ($\underline{b} \leqslant \overline{b}$)
$U = \{u_0, u_1, u_2\}$ u_0 is the cost, u_1 is lead time, and u_2 is the delivery probability
E_{u_1} $[\underline{w}_1, \overline{w}_1]$ weeks
E_{u_2} $[\underline{w}_2, 1]$

The above RFQ is broadcast to all potential suppliers with a deadline for submission of sealed bids. The following details are used for bid evaluation, which may or may not be disclosed to the suppliers.

6.1.2. Business constraint

[C1] At least c_1% of demand is required within $\hat{w}_1$ weeks.
[C2] Do not procure more than c_2% of demand from risky suppliers (with delivery probability less than $\hat{w}_2$).

6.1.3. Goals

The t_g is the target or aspiration level for goal g with t'_g as the maximum deviation allowed.

[G1] Total procurement cost $\leqslant t_1$ (t'_1).
[G2] Maximum percentage of business from a supplier is t_2 (t'_2).
[G3] Number of winning suppliers $\in [\underline{t}_3, \overline{t}_3]$ ($[\underline{t}'_3, \overline{t}'_3]$).
[G4] At least t_4 (t'_4)% of demand should be procured from suppliers with delivery probability greater than $\tilde{w}_2$.

Some of the above goals could have also been added as business constraints. Though both a goal and a constraint have the same mathematical structure in the form of inequality, they have different influence on a solution. A solution should satisfy a constraint whereas it is not necessary to satisfy a goal. In other words, mandatory conditions are made as constraints and desirable conditions are formulated as goals. The maximum deviation t'_g for a goal is not viewed as a constraint but just as a benchmark to measure the deviation.

After receiving the bids, the bid evaluation is formulated as the following MILP problem. For the sake of notational clarity in the formulation, the c_1, c_2, t_2, t'_2, t_4, and t'_4 denote their respective units of demand, instead of their percentage values.

$$\min \quad \kappa_1 \frac{\gamma_1}{t'_1} + \kappa_2 \frac{(\sum_j \gamma_2^j)}{t'_2} + \kappa_3 \left(\frac{\gamma_3^-}{\underline{t}'_3} + \frac{\gamma_3^+}{\overline{t}'_3} \right) + \kappa_4 \frac{\gamma_4}{t'_4} \tag{15}$$

subject to

$$d^0_{juw} \geqslant d^1_{juw} \quad \forall w, u, j \tag{16}$$

$$x^s_{juw} \leqslant d^s_{juw}, \quad s = 1, \ldots, l_{juw} \ \ \forall w, u, j \tag{17}$$

$$x^s_{juw} \geqslant d^{s+1}_{juw}, \quad s = 1, \ldots, l_{juw} - 1 \ \ \forall w, u, j \tag{18}$$

$$X_j \leqslant z_j \overline{b} \quad \forall j \tag{19}$$

$$\sum_{w \in W_u} \left(d^0_{juw} \underline{a}_{juw} + \sum_{s=1}^{l_{juw}} x^s_{juw} \delta^s_{juw} \right) = X_j \quad \forall u, j \tag{20}$$

$$\underline{b} \leqslant \sum_j X_j \leqslant \overline{b} \tag{21}$$

$$\sum_j \sum_{w \leqslant \hat{w}_1} \left(d^0_{juw} \underline{a}_{juw} + \sum_{s=1}^{l_{juw}} x^s_{juw} \delta^s_{juw} \right) + \gamma_4 \geqslant c_1, \quad u = u_1 \tag{22}$$

$$\sum_{j: u_2 \leqslant \hat{w}_2} X_j \leqslant c_2 \tag{23}$$

$$\sum_j \sum_{u \in U} \sum_{w \in W_u} \left(n^0_{juw} d^0_{juw} + \sum_{s=1}^{l_{juw}} (n^s_{juw} d^s_{juw} + \beta^s_{juw} \delta^s_{juw} x^s_{juw}) \right) - \gamma_1 \leqslant t_1 \tag{24}$$

$$X_j - \gamma_2^j \leqslant t_2 \quad \forall j \tag{25}$$

$$\sum_j z_j + \gamma_3^- \geqslant \underline{t}_3 \tag{26}$$

$$\sum_j z_j - \gamma_3^+ \leqslant \overline{t}_3 \tag{27}$$

$$\sum_{j: u_2 > \tilde{w}_2} X_j + \gamma_4 \geqslant t_4 \tag{28}$$

$$z_j \in \{0, 1\}, \quad X_j \geqslant 0, \quad \gamma \geqslant 0, \quad d^s_{juw} \in \{0, 1\}, \quad x^s_{juw} \in [0, 1].$$

The constraints (16)–(21) handle the demand constraints, (22) and (23) are for the business constraints, and (24)–(28) are goal constraints. In the objective function, the deviational variables γ are normalized using the maximum allowed deviations, thus making γ dimensional and weighted summation is possible.

6.2. A numerical example

We present in this section a numerical example to illustrate the goal programming approach. The example problem consisted of 25 randomly generated bids. The price functions in the bids were correlated with decreasing β^s_{juw} over s. The demand of the buyer was chosen with $\underline{b}$ as 30% of the total supply and $\overline{b} = 1.2 \times \underline{b}$. The number of linear segments for piecewise linear cost curve was randomly chosen between two and five. The number of attribute values for the lead time were randomly chosen in range $[2, 6]$ for each bid and the βs were chosen to be decreasing with these attribute values. The delivery probability for a bid was chosen uniformly in range $[0.7, 1]$ in denominations of 0.05. The parameters related to the constraints and the goals were fixed as follows: $c_1 = 40\%$, $c_2 = 25\%$, $\hat{w}_1 = 2$, $\hat{w}_2 = 0.7$, and $\tilde{w}_2 = 0.9$. With these parameters fixed, the goal parameters were varied to generate different problem instances. The goal parameters and the results are shown in Table 2. The experiment was carried out on a Windows XP based PC equipped with a 2.8GHz Intel P4 processor with 1GB RAM. The algorithms were coded in Java, and for the model building and solving of MILP programs, ILOG Concert Technology of CPLEX 9.0 was used.

The goal values in the *Results* refer to the values of the solution with respect to the goals. The problem was first solved as a traditional single objective optimization problem by minimizing the procurement cost. This provides the buyer with information about the received bids: optimal procurement cost is 2923, number of winners is 11, maximum percentage of demand from a winning supplier is 12.12%, and 30.3% of demand is procured with delivery probability greater than 0.9. If the buyer feels that this allocation is not satisfactory for certain goals, then he can set the goal parameters and solve the problem as multiple criteria optimization. In the example, the buyer wants the G4 to be 40% and the number of winners to be in range [8, 10]. However, he can pay up to 3000 and can accept 15% of demand from a winning supplier. With the above goal parameters set along with the weights and the deviations as given in Table 2 for instance 2, the problem was solved again. The optimal objective value of zero implies that all goals have been satisfied. With further increasing the G4 to 55%, the instance 3 was solved and the optimal objective value is 0.006 (the G1 was violated by three units). The third allocation is preferable to the buyer if he finds the increase in 15% in G4 can compensate the increase of G1 by three units. Thus the buyer can flexibly change the parameters by observing the outcomes of the various instances of the problems. Though from the auction perspective, changing of goal levels would be

Table 2
Numerical example

No.	Parameters				Results					
	Goal	Target	Dev.	Wt.	Obj.	G1	G2	G3	G4	Time
1	*Minimize procurement cost*				2923	2923	12.12	11	30.3	188
2	G1	3000	500	0.4	0	3000	14.54	10	40.60	2515
	G2	15%	5%	0.2						
	G3	8, 10	2, 2	0.2						
	G4	40%	5%	0.2						
3	G1	3000	200	0.4	0.006	3003	14.54	10	57.57	6719
	G2	15%	5%	0.2						
	G3	[8, 10]	(2, 2)	0.2						
	G4	55%	5%	0.2						

seen as unfair (as the bid evaluation criteria are changed after the bidding), this practice is not uncommon in procurement scenarios. This approach is pragmatic in an industrial environment where the buyer can easily set goals and can compromise or improvise the goal values depending on the received bids. This leads to a much better understanding of the sensitivity of the optimal sourcing decisions to business constraints than possible using a more traditional manual process.

7. A heuristic for the bid evaluation problem

The *Time* in the Table 2 is the computational time in milliseconds taken by the CPLEX to solve the problem to optimality. Note that the time increases many-fold as the target goal values are made more tight and conflicting. The solution time also depends on the structure of the business constraints and the piecewise linear cost functions. The business constraints can be broadly categorized as *intra-bid* and *inter-bid*. Restricting the business from a winning supplier is a intra-bid constraint, whereas restricting the number of winning suppliers is inter-bid. The requirement of a certain percentage of demand with a certain attribute and the *homogeneity* constraint [6] of choosing a single attribute value for all the winning bidders are also inter-bid constraints. Inter-bid constraints make the decision variables coupled and interdependent, thus increasing the solution time when solved by generic solvers. On the other hand, the piecewise linear cost functions contribute to the solution time due to the number of binary decision variables. Commercial optimization packages use branching algorithms and with large number of binary variables, the solution times likely tends to increase. In the following we propose a linear programming (LP) relaxation based heuristic to tame the complexity arising out of the piecewise linear cost functions.

Many of the binary variables in the formulation are due to the piecewise linear cost function. Hence by substituting this function with an *approximating* function, which has less number of binary variables, can make the problem easier to solve. However, the solution obtained may not be optimal with respect to the original function. To reduce the optimality gap, one has to choose a tight approximating function. The linear function that joins the end points is an approximating function, but the optimality gap in generally large for such a linear approximation. We use the *convex envelope* as the approximating function. Convex envelope is the best convex function that underestimates the original function. The choice of convex envelope as the approximating function is twofold: (1) it provides good lower bound approximation and (2) it can be directly used with the original formulation by just relaxing the binary variables. It was shown in [13], that the linear relaxation of the piecewise linear function solves the convex envelope of that function. Though it is possible to construct the convex envelope of a piecewise linear function in polynomial time, linear relaxation allows to directly use the mathematical formulation with minimal modifications.

The piecewise linear cost function Q_{juw} for attribute value w of attribute u for bid j is handled by decision variables d^s_{juw} (binary) and x^s_{juw} (linear) through the constraints (2)–(4). We construct the convex envelope of Q_{juw} by relaxing the binary variables d^s_{juw} for $s > 0$. Note that d^0_{juw} is not relaxed to avoid violation on the supply lower bound $\underline{a}_{juw}$. It is worth noting that this relaxation makes the relaxed binary variables redundant as

Table 3
Performance of the heuristic for the bid evaluation problem with 25 bids

Attributes	Time (milliseconds)	Optimality gap (%)
1	172.7	3.16
2	3388.45	3.50
3	24,711.55	3.60
4	318,099.85	4.15
5	1,858,087.7	4.21

the optimal solution will satisfy $d^{s+1}_{juw} \leqslant x^s_{juw} \leqslant d^s_{juw}$, $\forall j, u, w$, and $s = 1, \ldots, l_{juw} - 1$ and hence $d^s_{juw} = x^s_{juw}$ $\forall j, u, w$, and $s = 1, \ldots, l_{juw}$. This further reduces the number of decision variables in the problem. The optimal solution to the above relaxed problem gives a feasible solution to the original problem and the objective value is a lower bound. The original value of the feasible solution is just found by substituting the original cost with the convex cost, thus giving an upper bound.

The experiments were conducted for the bid evaluation problem with the single objective of total cost minimization. Each attribute had three attribute values (three piecewise linear cost functions), in addition to the mandatory cost attribute. The business constraints considered were the restriction on the maximum business from a winning supplier, bounds on number of winning suppliers, and minimum demand for each of the attribute values. The experiments were directly conducted with the relaxed versions to investigate the solution time and the worst case optimality gap. Table 3 shows the results of the experiments with 25 randomly generated bids for different number of attributes. The values are average values computed over 50 problem instances. There is a steep rise in the computational time with the number of attributes. The optimality gap is the worst case optimality gap, as it is measured against the objective value of the relaxed problem and not against the original problem. Our various other experiments by solving the problem with the original piecewise cost functions showed that the optimality gap was less than one percent. However, the solution time were unpredictable and the CPLEX ran out of memory even for problem instances of 25 bids with 10 attributes. It should be noted that the constraints (6) for each of the attributes of every bid strongly couples the decision variables. The proposed heuristic is thus effective in taming the complexity arising out of the piecewise linear cost functions with a marginal loss of optimality.

8. Conclusions and future work

In this paper we proposed a framework for a generic e-procurement system with the following properties: (1) allowing the bidders to bid on multiple attributes, (2) a rich bidding language to automate negotiations across multiple bids, (3) submission of volume discount bids, (4) flexibility for allowing business rules and purchasing logic in bid evaluation, and (5) bid evaluation using multiple criteria. To develop the above proposal into an automated e-procurement system, the following issues need to be addressed: (a) user interface for the bidders to express the piecewise linear cost functions and supply side business constraints, and for the buyer to express the goals and demand side business constraints, (b) design of XML schema for compact representation of the configurable bids, goals and business constraints, and (c) automatic generation of linear and logical constraints for the bid evaluation problem based on the business constraints. Further, conducting laboratory experiments will help in identifying the key issues in bid formats, constraint structures, and goal parameters.

Goal programming was proposed as the solution technique for solving the multiple criteria bid evaluation problem. A numerical example illustrated the flexibility of weighted GP and its effectiveness in obtaining a satisificing solution with respect to various goals. Another prospective research direction is exploring the possibilities of using other multicriteria optimization techniques and other GP techniques for bid evaluation. The interactive sequential GP (ISGP) [35] is a potential candidate which combines and extends the attractive features of both GP and interactive solution approaches. It is based on the implicit assumption that the decision maker can adjust the desired goals through an iterative learning process based on information in a set of solutions. This would enable the buyer to change the aspiration levels of the goal by learning from the bids, similar to the way the buyer learns his scoring rule from the bids in [2].

The computational experiments showed that the bid evaluation problem is inherently complex for the generic commercial solvers like CPLEX even for small instances. We proposed a LP relaxation based heuristic to handle the complexity arising out of piecewise linear cost functions. One interesting research direction in algorithmic and mathematical programming perspective is to design algorithms to handle the *inter bid* business constraints. By exploiting the structure of the constraints and the problem, one could possibly design decomposition algorithms that decouples the decision variables. Such algorithms can possibly have less computational time than the generic branch and bound solvers.

The paper was focused entirely on the perspective of the buying organization. For a successful implementation of an e-procurement system, one also should take into account the participation of the suppliers. The bid preparation problem faced by the supplier is an important decision making problem and the development of decision support systems for bid preparation in another important research problem.

The proposed procurement dynamics borders on the auction dynamics and hence auction theory can contribute significantly in improving the procurement process. Auction literature provides two fundamental prescriptions for designing efficient auctions [1]: (1) to implement Vickrey–Clarke–Grooves (VCG) pricing scheme and (2) to make the auction dynamics open and progressive so that the bidder has sufficient information at the time of bidding. In VCG pricing scheme, the payment is such that the bidders payoff equals their marginal contribution. It provides the bidders with incentive to quote their true values and the allocation is economically efficient. This inherently solves the bid preparation problem, as the bidders just needs to report their costs. For the *pay-as-bid* pricing scheme used in this paper (which is also commonly used in current systems), the bidders have to strategically misreport their costs to gain profit, which is a liability to both the bidders and the auctioneer. The general framework for designing auctions that meets both the above objectives is presented in [7]. It is based on duality theory and the primal–dual algorithm when applied to the bid evaluation problem helps in designing a progressive auction. However, the extension of the above framework to include the business constraints of the procurement and the multiple objectives is non-obvious. It is worth investigating the issues involved in making the procurement dynamics open and progressive.

The paper has discussed the procurement of multiple units of a single good with multiple attributes. Procuring bundles (that is, multiple items) with multiple attributes is a natural but complex generalization of this problem.

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