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On the Interpretation of the Observed Angular-Size–Flux-Density Relation for Extragalactic Radio Sources

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Abstract. The interpretation of the observed relation between median angular sizes (θ_m) of extragalactic radio sources and flux density at 408 MHz has been examined. The predicted θ_m–S relations based on well-observed strong sources in parent samples selected at 178 and 1400 MHz, and existing models of the evolving radio luminosity function can be made to fit the observed relation only by invoking cosmological evolution in linear sizes even for the $q_0 = 0$ universe. Predictions based on a parent sample at 2.7 GHz are shown to overestimate the contribution of steep-spectrum, compact (SSC) sources in low-frequency samples unless the downward curvature in the spectra of such sources is taken into account. When approximate corrections are made for this effect, predictions based on the 2.7 GHz parent sample cannot obviate the need for linear size evolution as claimed in the literature.

Key words: radio sources, extragalactic—angular-size-flux-density relation—cosmology—radio sources, evolution

1. Introduction

The epoch dependence of the largest physical sizes of extended extragalactic radio source can be investigated using the observed angular-size-redshift (θ–z) or angular-size-flux-density (θ–S) relations. The θ–z relation for radio quasars indicates that angular sizes fall off faster with z than expected in uniform world models in which the maximum physical sizes (l) are independent of epoch (*e.* g. Miley 1971; Wardle & Miley 1974; Ekers & Miley 1977; Wills 1979). The straightforward interpretation of the θ–z relation is that there is cosmological evolution in physical sizes—maximum sizes being smaller at earlier epochs. For simple epoch dependence of the form $l_z \propto l_0(1 + z)^{-n}$, values of the size evolution parameter, $n,$ in the range of 1 to 2 are generally required, depending on the world model ($q_0 = 0$ model requiring the least amount of evolution). But because of the strong correlation between z and radio luminosity, P, in flux-limited samples of quasars, the θ–z relation has also been interpreted to imply an inverse correlation between l and P (e.g. Stannard & Neal 1977; Hooley, Longair & Riley 1978;

Masson 1980). It must be noted, however, that there is no compelling evidence for such an inverse relationship. By considering samples of radio galaxies of similar luminosity at different redshifts, Kapahi (1985; 1986) has in fact shown recently that the sizes of radio galaxies at least are much more likely to be epoch dependent rather than being luminosity dependent in the sense postulated.

The study of the θ–S relation has the advantage that large and complete samples of sources can be used, but the interpretation of the relation requires knowledge of the epoch dependence of the radio luminosity function. Following the first investigation of the θ–S relation (Swarup 1975; Kapahi 1975) based on the comparison of weak sources from the Ooty occultation surveys with stronger known sources, the relation has been confirmed and extended to lower flux levels using data from aperture synthesis observations (*e.* g. Ekers & Miley 1977; Kapahi & Subrahmanya 1982; Fielden *et al.* 1983; Windhorst, van Heerde & Katgert 1984). The θ–S relation has been best determined at 408 MHz where the samples are made up almost exclusively of steep-spectrum extended sources. The median value of angular size, θ_m, appears to fall rather sharply from ~100 arcsec at S_{408} ~ 20Jy to around 15 arcsec at S_{408} ~ 3 Jy and decreases much more gradually at lower flux levels reaching ~ 8 arcsec at S_{408} ~ 0.1 Jy. Early interpretations based on simple density evolution schemes for the luminosity function and a distribution of linear sizes derived from nearby sources in the 3CR sample, indicated (Kapahi 1975, 1977; Swarup & Subrahmanya 1977; Katgert 1977) that the values of θ_m predicted by uniform world models at low flux levels were considerably larger than observed; the data could be fitted only by invoking linear size evolution for all radio sources similar to that inferred from the θ–z relations for quasars.

A somewhat different method was employed by Downes *et al.* (1981) who used the observed properties (S, z,α and θ) of individual sources in the 3CR sample of strong sources selected at 178 MHz to estimate, using a V/V_m type of analysis, the space distributions of such sources and hence the expected distributions of their angular sizes at different flux-density levels. Apart from considering the epoch dependence of the luminosity function according to the evolutionary models successful in explaining the observed source counts, this method has the advantage that it automatically takes into account any correlation between P and l present in the 3CR sample. The method, of course, assumes that all the types of sources seen in low flux-density samples are adequately represented in the strong parent sample, which is a reasonable assumption over the flux-density range of interest. On the basis of such predictions Downes *et al.* (1981) concluded that for the evolutionary models of Wall, Pearson & Longair (1980), no single value of the evolution parameter, n, provided a tolerable fit to the observed median values of θ over the entire available flux-density range. An estimation of the predictions by Kapahi & Subrahmanya (1982) using the same technique and data showed, however, that values of n between 1 and 1.5 provided good fits not only to the observed values of θ_m but also to the θ distributions at different flux levels. The discrepancy in these results was subsequently traced by us to a computational oversight in Downes, Longair & Perryman (1981).

It was further suggested by Downes, Longair & Perryman (1981) that many of the weak sources (0.1 to 1 Jy) at 408 MHz could be similar to the steep-spectrum, compact (SSC) sources that form a substantial fraction of the strong-source surveys at high frequencies (Kapahi 1981; Peacock & Wall 1982) but are poorly represented in surveys at metre wavelengths because of low-frequency turnovers in their spectra. The possibly large redshifts of weak sources at 408 MHz would imply selection at a high emitted frequency so that it may be more appropriate to compare the observed θ distributions at low flux levels at 408 MHz with the predictions made from strong source samples selected at high frequencies. Such predictions using a parent sample selected at 2.7 GHz (Peacock & Wall 1981: PW) and the multifrequency evolutionary models for the luminosity function (Peacock & Gull 1981) have lead Downes (1982), Fielden *et al.* (1983) and Allington-Smith (1984) to conclude that linear size evolution may not be necessary in order to explain the observed θ–S relation.

In this paper we have compared the observed θ–S relation at 408 MHz with predictions made using parent samples of strong sources selected at several frequencies and using several models for the evolution of the radio luminosity function. Several inadequacies associated with the use of a high frequency survey to estimate angular size distributions for sources selected at low frequencies are first pointed out in Section 2. Predictions made after approximately allowing for some of these effects (Sections 3 and 4) show that size evolution is indeed necessary to fit the data. Our conclusions are summarized in Section 5.

2. Problems of using a high-frequency parent sample

There are several problems associated with using a high-frequency parent sample in general, and the 2.7 GHz PW sample in particular, to predict the distributions of θ at different flux levels at 408 MHz, that have either been totally ignored or not paid adequate attention to by earlier workers.

(i) Although the use of the 2.7 GHz parent sample appears to predict values of θ_m similar to those observed at low flux levels (S_{408} < 1 Jy) without introducing size evolution, it fails to predict the fairly rapid fall observed in θ_m between about 20 Jy and 2 Jy at 408 MHz. Rather low values of θ_m are predicted even at high flux levels, the fit with observations getting progressively worse with increasing S_{408} above ~ 2 Jy (*e.g.* Allington-Smith 1984). The reason is easily understood; the presence of a large number of SSC sources in the 2.7 GHz sample, together with the assumption in all evolutionary models that sources have spectral indices independent of frequency, leads to an overestimation of the contribution of such sources in bright-source samples at low frequencies, thereby predicting low values of θ_m. In reality, the spectra of SSC sources must of course turn over at low frequencies since such sources are known to be much less abundant in low-frequency surveys (Kapahi 1981; Peacock & Wall 1982). The overestimate of the number of SSC sources is, in fact, very likely to extend to even the weakest flux levels in the θ–S relation at 408 MHz unless the weak sources have extremely high redshifts of the order of ~ 10. This is because most of the SSC sources are seen at fairly large redshifts (typically z ~ 1) even in the parent PW sample. They are therefore effectively selected at an intrinsic frequency of ~ 5 GHz. Only for redshifts of the order of 10 or 12 would the weak sources at 408 MHz be also selected at this high frequency.

(ii) Because the 2.7 GHz sample is based on finding-surveys with telescopes with rather narrow half-power beamwidths, it is likely to discriminate against large radio sources. A comparison of the sample with the well-studied 3CR-LRL sample (Laing, Riley & Longair 1983, and preprint incorporating revisions upto 1983 October 31) confirms this bias. Although the two samples cover nearly the same area of the sky and have a comparable sensitivity (for steep-spectrum sources) there are 10 sources with $\theta \gtrsim 500$ arcsec in the 3CR-LRL sample but only 3 such sources in the PW sample. From the available spectral information in the literature we find that the 7 large sources listed in Table 1 have flux densities $S_{2.7\ \mathrm{GHz}} \geqslant 1.5$ Jy but are not included in the PW sample, presumably because of their large sizes. All of these are listed in the 3CR-LRL sample.

(iii) Surveys at high frequencies would be expected to discriminate against very-steep-spectrum sources. In the 3CR-LRL sample there are 19 sources with $\alpha_{178}^{750} \geqslant 1.0$, none of which are strong enough at 2.7 GHz to be in the PW sample. These are typically sources of high radio luminosity and have a median linear size of ~ 200 kpc (for $q_0 = 0.5$ and $H_0 = 50 \mathrm{kms^{-1}\ Mpc^{-1}}$).Most of the sources of this type would be seen with $\theta \gtrsim 24$ arcsec even at low flux levels.

(iv) High-frequency surveys would also be expected to discriminate against sources whose spectra steepen considerably at high frequencies. From the known values of S_{178} and α_{178}^{750} we find that 16 3CR-LRL sources should have $S_{2.7} \geqslant 1.5$ Jy, but are not in the PW sample. While 5 of these are large sources that we have already dealt with, the remaining 11 are not in the PW sample because their spectra steepen considerably at high frequencies. These are again mostly of high radio luminosity and have a median linear size of ~ 190 kpc.

All the above-mentioned problems are likely to affect the θ–S predictions from the PW parent sample, with the first two points being the most significant. While it is quite simple to take care of the second effect by adding the large sources to the parent sample, it is difficult to take care of the other effects that arise due to spectral curvature.

The redshift distributions of steep-spectrum ($\alpha_{2.7}^{5} \geqslant 0.5$) radio sources in the PW 2.7 GHz sample are shown in Fig. 1 for the extended as well as the compact sources (angular sizes $\lesssim 2$ arcsec in most cases as defined in PW). A much larger fraction of the SSC sources is seen to be associated with quasars. The median value of redshift for the SSC sources ($z_m \sim 0.83$) is also very significantly higher than that for the extended sources ($z_m \sim 0.23$). The SSC sources in the 2.7 GHz survey are therefore typically seen at an emitted frequency of ~ 5 GHz. The use of a 2.7 GHz parent sample to predict the θ distributions at low flux levels in 408 MHz surveys would therefore be appropriate only if the latter had typical redshifts of ~ 12. Although such large redshifts cannot totally be ruled out, it must be noted that the largest redshift known for a quasar is only 3.8 and several recent investigations have indicated a sharp decline in the space density of quasars beyond $z = 2.5$ or 3 (see Osmer 1986 for a review). Recent work on the space distribution of radio sources using the Leiden–Berkeley deep radio and optical surveys suggests (Windhorst 1984) that there may be a cutoff also in the distribution of radio galaxies at $z \sim 2$. If the typical redshifts of weak SSC sources at 408 MHz are indeed less than 2 or 3, their emission frequencies would match those of SSCs in strong source surveys at frequencies between about 0.7 and 1 GHz. We have therefore also considered a parent sample at 1.4 GHz selected from the BDFL catalogue (Bridle *et al.* 1972) for which reasonably complete data on source structures and redshifts is available in the literature. The details of this sample are given in Section 3.1.

Another point to be noted is that while many SSC sources have curved radio spectra (α decreasing continuously with decreasing frequency) the computations based on the PW parent sample have always used the values of α (assumed independent of

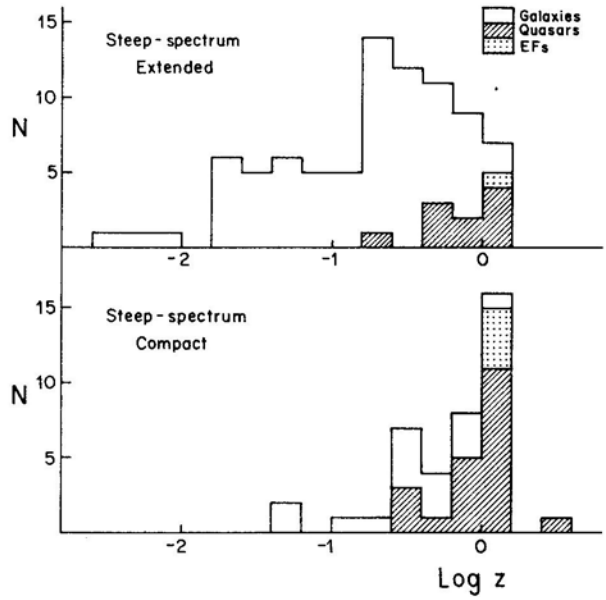

Figure 1. Redshift distributions of extended and compact sources with steep spectrum ($\alpha_{2.7}^{5}$ > 0.5) in the PW 2.7 GHz sample. Redshifts for some galaxies have been estimated from their optical magnitudes. Empty fields (EFs) and quasars with unknown z are assumed to have z >1.

frequency) listed in the PW catalogue, which are obtained from the measured flux densities at 2.7 GHz and the higher frequency of 5 GHz. This results in a further overestimation of the numbers of SSC sources in low frequency samples because the extrapolated flux densities at low frequencies turn out to be too high so that such sources can be seen out to large volumes of space at lower flux levels. To investigate the magnitude of this effect we have carried out the computations using two-point spectral indices between 1.4 and 2.7 GHz for the SSC sources as described in Section 3.1.

3. *θ*–*S* predictions

3.1 *Parent Samples Used*

We have considered three parent samples of strong radio sources selected at frequencies of 178, 1400 and 2700 MHz as follows.

3.1.1 *The* 3*CRL-RL sample* ($S_{178} \geqslant 10$ Jy)

The redshift information for this sample of 174 sources is now extremely good. We use the values of S_{178}, α_{178}^{750} and z listed in LRL. The values of θ have been tabulated by Allington-Smith (1984). The following corrections are however necessary for three sources; 3C16:θ = 60 arcsec (Riley, Longair & Gunn 1980), 3C200: θ = 17 arcsec (Jenkins, Pooley & Riley 1977), 3C457:θ = 190 arcsec (Laing, Riley & Longair 1983).

3.1.2 *The BDFL sample* ($S_{1400} \geqslant 2.5$ Jy)

Although the BDFL catalogue (Bridle *et al.* 1972; Bridle & Fomalont 1974) has a completion limit of S_{1400} = 2 Jy, information on z and θ is not available for many sources, particularly near the lower flux limit. We have therefore restricted the sample to the 151 sources with $S_{1400} \geqslant 2.5$ Jy and $|b| > 20°$. The sources with the relevant information are listed in Table 2. Redshifts are known for ~ 83 per cent of the sources and could be estimated from the magnitudes of the identified galaxies for another 10 per cent (values shown in brackets in Column 7 of Table 2). Only 75 (50 per cent) of the sources in this sample are common to the 3CR-LRL sample and 86 (57 per cent) are common to the PW sample (Columns 2 & 3 of Table 2). The values of θ have been taken from a search of the literature. The values of α for a majority of the sources have been taken from the spectral fits given by Kühr *et al.* (1981). For sources with curved spectra we have used the slope of the best fit log S-log ν curve at 1.4 GHz. This was obtained in most cases from the polynomial fits given by Kühr *et al.* (1981).

3.1.3. *The PW* 2.7 *GHz sample* ($S_{2700} \geqslant 1.5$ Jy)

To the original sample of 168 sources defined by PW we have added the 7 large sources listed in Table 1. The values of θ have been tabulated by Allington-Smith (1984). A comment is necessary about the values of spectral indices.

For the steep-spectrum extended sources and for the flat-spectrum sources we have used the two-point spectral indices between 2.7 and 5 GHz listed in PW, but for SSC sources we have used the values of α between 1.4 and 2.7 GHz (as discussed in Section 2) determined from the best fits to the flux density information at a large number of frequencies tabulated by Kühr *et al* (1981) for most sources. For 21 of the 35 SSC sources in PW, the value of α estimated as above appeared to differ significantly from the values listed in PW; the original and revised values for these sources are shown in Table 3. In several cases the differences are seen to be quite large and the revised values of α are often < 0.5. Not all the differences arise from spectral curvature. An examination of the spectral plots shows that two sources (*viz.* 1600+33 & 1749+70) clearly belong to the flat-spectrum class and have been misclassified due to errors in fluxes at 2.7 or 5 GHz, or due to variability. Three other sources (*viz.* 0223 + 34, 0319 +12 and 2247 +14) have a straight sprectrum with α < 0.5 and should also be classified in the flat-spectrum class. The remaining sources show varying degree of

Table 1. Large sources added to the PW 2.7 GHz sample.

source name	$S_{2.7}$ Jy	α	z	LAS (arcsec)
0745 + 56 (DA 240)	2.84	0.78	0.035	2040
0945 + 73 (4C 73.08)	1.70	0.85	0.0581	1100
1232 + 21 (3C 274.1)	1.55	0.93	0.422	150
1549 + 20 (3C 326)	2.05	0.83	0.0895	1170
1637 + 82 (NGC 6251)	2.17	0.70	0.024	4320
2212 + 13 (3C 442A)	1.67	1.05	0.0263	270
2247 + 11 (NGC 7385)	1.70	0.66	0.0243	720

Table 2. The BDFL source sample at 1.4 GHz.

Source (1)	3CR (2)	PW (3)	$S_{1.4}$ Jy (4)	α (5)	Opt. Id. (6)	z (7)	LAS arcsec (8)
0003 – 00			3.54	0.79	Q	1.037	6
0019 – 00			2.73	0.44	G	(0.6)	< 1
0034 – 01			4.30	0.75	G	0.073	22
0035 – 02			6.25	0.72	G	0.220	10
0038 + 32	√	√	3.12	0.84	G	0.482	9.6
0038 + 09			4.26	0.75	G	0.188	47
0055 – 01			5.22	0.67	G	0.045	150
0104 + 32	√	√	5.22	0.66	G	0.017	3600
0106 + 13	√	√	12.59	0.70	G	0.059	250
0116 + 31		√	2.54	0.37	G	0.059	< 1
0123 – 01			6.42	0.93	G	0.018	1260
0123 + 32	√	√	3.49	0.73	G	0.794	23
0125 + 28	√	√	2.64	0.82	G	0.395	28
0127 + 23	√	√	2.78	0.76	Q	1.459	1.5
0128 + 03			2.52	1.00	G	(0.32)	359
0133 + 20	√	√	3.68	0.97	Q	0.425	69
0134 + 32	√	√	15.29	0.81	Q	0.367	< 1
0138 + 13	√	√	2.69	0.82	G	0.621	1
0154 + 28	√		2.50	0.96	G	0.240	72
0202 + 14		√	3.40	0.37	G?	(1.0)	< 1
0218 – 02			3.32	1.00	G	0.175	< 15
0219 + 08			2.51	0.83	G	(0.4)	134
0221 + 27	√	√	2.94	0.81	G	0.310	2
0240 – 00			4.87	0.75	G	0.003	9
0255 + 05			6.22	0.75	G	0.024	221
0300 + 16	√	√	2.60	0.68	G	0.032	45
0305 + 03			7.24	0.57	G	0.029	110
0307 + 16	√	√	4.59	0.97	G	0.256	87
0316 + 16		√	7.60	0.48	G?	(1.0)	2.5
0320 + 05			2.86	0.83	G	(0.5)	< 15
0325 + 02			4.85	0.67	G	0.030	150
0331 – 01			2.72	1.04	G	0.139	93
0340 + 04			2.84	0.90	G	0.358	25
0347 + 05			3.25	0.71	G	(0.68)	65
0356 + 10	√	√	9.56	0.72	G	0.031	295
0404 + 03			4.93	0.70	G	0.089	265
0410 + 11	√	√	4.09	0.75	G	0.306	90
0430 + 05			5.48	0.0	G	0.032	< 1
0440 – 00			3.18	0.0	Q	0.844	< 1
0511 + 00			2.80	0.83	G	0.127	130
0651 + 54	√	√	3.66	0.90	G	0.238	9
0703 + 42		√	2.82	0.75	G	0.060	360
0744 + 55	√		4.74	0.78	G	0.035	2040
0755 + 37		√	2.54	0.55	G	0.043	140
0802 + 24	√	√	4.89	0.73	G	0.060	190
0809 + 48	√	√	13.85	0.90	Q	0.871	5
0821 + 39			2.65	0.25	Q	1.216	20

Table 2. Continued.

Source (1)	3CR (2)	PW (3)	$S_{1.4}$ Jy (4)	α (5)	Opt. Id. (6)	z (7)	LAS arcsec (8)
0831 + 55		√	8.04	0.0	G	0.242	< 1
0906 + 43	√	√	3.76	0.46	Q	0.668	1
0917 + 45	√	√	8.02	0.93	G	0.174	147
0923 + 39		√	2.52	− 1.0	Q	0.698	< 1
0936 + 36	√	√	3.35	0.77	G	0.137	250
0939 + 14	√	√	3.00	0.94	G	(0.6)	5
0945 + 07			7.40	0.79	G	0.086	178
0947 + 14	√	√	3.47	0.83	G	(0.6)	45
0949 + 00			3.03	0.90	EF	(1.0)	7
0951 + 69	√	√	7.94	0.54	G	0.001	50
0954 + 55		√	3.52	0.23	Q	0.901	< 1
0958 + 29	√	√	5.35	0.96	G	0.185	110
1003 + 35	√	√	3.24	0.66	G	0.099	2340
1005 + 07			6.25	0.88	G	(0.78)	47
1008 + 06			2.93	0.90	G	(1.0)	< 15
1030 + 58	√	√	3.74	0.97	G	0.428	53
1039 + 02			2.84	0.76	EF	(1.0)	5
1040 + 12	√	√	3.07	0.64	Q	1.029	5
1055 + 01			3.10	0.0	Q	0.888	4
1056 + 43	√	√	2.91	0.86	G	0.750	13
1059 − 01			2.56	0.95	EF	(1.0)	18
1106 + 37			3.20	0.40	G	0.346	< 2
1111 + 40	√		3.05	1.00	Q	0.734	13.2
1137 + 66	√	√	2.98	0.85	Q	0.652	44
1140 + 22	√	√	2.96	0.96	G	0.360	5.8
1142 + 19	√	√	5.78	0.83	G	0.021	100
1148 − 00			3.06	0.22	Q	1.982	< 1
1142 + 31	√		2.80	1.00	G	0.811	78
1153 + 31		√	2.77	0.77	Q	1.557	17
1203 + 64	√	√	3.53	0.83	G	0.371	1.3
1213 + 53			2.65	0.78	Q	1.065	33
1216 + 06			17.50	0.53	G	0.007	303
1218 + 33	√	√	2.50	0.82	Q	1.519	12
1222 + 13	√	√	5.80	0.61	G	0.003	120
1226 + 02			38.84	0.0	Q	0.158	21
1228 + 12	√	√	214.0	0.82	G	0.004	460
1232 + 21	√		2.97	0.94	G	0.422	150
1239 − 04			3.34	0.84	G	0.480	< 15
1241 + 16	√	√	2.81	0.76	Q	0.557	14
1251 + 27		√	2.89	0.69	G	0.086	22
1254 + 47	√	√	5.08	0.87	G	0.996	13
1323 + 32		√	4.56	0.50	G	(0.32)	8.4
1328 + 25	√	√	6.72	0.60	Q	1.055	< 1
1328 + 30	√	√	14.78	0.53	Q	0.846	< 1
1330 + 02			3.02	0.59	G	0.216	63

Table 2. Continued.

Source (1)	3CR (2)	PW (3)	$S_{1.4}$ Jy (4)	α (5)	Opt. Id. (6)	z (7)	LAS arcsec (8)
1336 + 39	√	√	3.32	1.00	G	0.246	11.5
1345 + 12		√	5.01	0.48	G	0.122	< 1
1350 + 31	√	√	4.42	0.72	G	0.045	85
1358 + 62		√	4.32	0.70	G	(0.54)	< 1
1409 + 52	√	√	22.18	0.90	G	0.461	4.3
1414 + 11	√	√	4.32	0.65	G	0.024	360
1416 + 06			5.66	1.10	Q	1.439	< 1
1419 + 41	√	√	2.95	0.87	G	0.367	< 1
1420 + 19	√	√	3.44	0.92	G	0.270	96
1425 − 01			3.30	0.90	G	0.308	< 15
1434 + 03			2.86	0.69	EF	(1.0)	< 15
1448 + 63	√	√	2.94	0.88	G	0.041	3.4
1502 + 26	√	√	7.67	1.18	G	0.054	250
1508 + 08			3.65	0.87	G	0.461	134
1511 + 26	√	√	3.87	0.85	G	0.108	200
1514 + 07			5.35	1.30	G	0.035	30
1517 + 20	√		2.50	0.93	G	0.752	< 1
1518 + 04			4.01	0.63	G	(1.0)	< 18
1529 + 24	√	√	3.59	0.88	G	0.090	290
1548 + 05			2.52	0.26	Q	(1.0)	< 2
1549 + 20	√		3.54	0.83	G	0.090	1170
1549 + 62	√	√	3.62	0.94	G	(0.70)	16
1559 + 02			8.95	0.87	G	0.104	209
1602 + 01			4.07	1.01	G	(0.6)	11
1607 + 26		√	4.43	0.35	G	(0.7)	< 1
1609 + 66	√	√	6.98	0.86	G	0.549	56
1611 + 34		√	2.92	0.10	Q	1.401	< 1
1622 + 23	√		2.71	1.00	Q	0.927	22
1626 + 39	√		3.53	1.22	G	0.030	41
1627 + 44	√	√	2.97	0.88	G	0.630	43
1634 + 62	√	√	5.17	0.83	Q	0.988	< 1
1637 + 62	√	√	4.66	0.89	G	0.750	< 1
1641 + 39		√	6.30	0.0	Q	0.594	2.1
1641 + 17	√	√	3.64	0.68	G	0.161	2.3
1648 + 05			44.43	1.06	G	0.154	115
1658 + 47	√	√	3.18	0.79	G	0.205	82
1704 + 60	√	√	3.30	0.79	Q	0.371	58
1726 + 31		√	2.72	0.80	G	0.166	80
1807 + 69		√	2.59	0.0	G	0.050	3.3
1819 + 39		√	3.39	0.75	G	(0.4)	< 2
1828 + 48	√	√	14.11	0.60	Q	0.691	5
1832 + 47	√	√	3.79	0.77	G	0.161	69
1842 + 45	√	√	5.57	0.87	G	0.092	31
2128 + 04			3.98	0.34	EF	(1.0)	< 1
2134 + 00			3.13	−0.76	Q	1.936	< 1

Table 2. Continued.

Source (1)	3CR (2)	PW (3)	$S_{1.4}$ Jy (4)	α (5)	Opt. Id. (6)	z (7)	LAS arcsec (8)
2145 + 15	√	√	2.84	0.88	EF	(1.0)	34
2145 + 06			2.97	−0.21	Q	0.990	< 1
2203 + 29	√	√	2.51	0.85	G	0.707	33
2210 + 01			2.60	0.57	EF	(1.0)	< 1
2212 + 13	√	√	3.29	1.05	G	0.027	270
2221 − 02			5.59	0.81	G	0.057	485
2223 + 21			2.59	0.40	Q	1.959	< 15
2230 + 11		√	6.01	0.36	Q	1.037	< 1
2251 + 15	√	√	11.84	0.0	Q	0.859	5.4
2252 + 12	√	√	2.93	0.91	Q	0.543	3.2
2309 + 09			2.51	0.88	G	0.233	< 15
2310 + 05			2.70	0.81	G	0.290	161
2314 + 03			4.17	1.01	G	0.220	8
2335 + 26	√	√	7.51	0.90	G	0.030	530

Table 3. Revised α for 21 sources considered SSC in PW.

source	$\alpha^{5}_{2.7}$ in PW	α (revised)
0221 + 27	0.99	0.83
0223 + 34	0.53	0.43
0316 + 16	0.89	0.60
0319 + 12	0.59	0.37
0428 + 20	0.50	0.30
1150 + 49	0.55	0.42
1153 + 31	0.92	0.80
1203 + 64	0.97	0.88
1225 + 36	1.17	0.52
1345 + 12	0.51	0.45
1413 + 34	0.60	0.33
1419 + 41	0.92	0.87
1442 + 10	0.68	0.44
1600 + 33	0.63	0.25
1607 + 26	0.89	0.44
1634 + 62	0.95	0.85
1749 + 70	0.89	0.10
1819 + 39	0.98	0.78
1829 + 29	0.80	0.67
2230 + 11	0.50	0.35
2247 + 14	0.62	0.47

spectral curvature at $\nu < 2.7$ GHz, the curvature being strong enough in many cases to 2.7 cause $\alpha^{2.7}_{1.4}$ to become < 0.5. We have, however, retained the steep-spectrum classify-cation for such sources while estimating their space distribution at different flux levels according to the evolutionary models which treat the evolution separately for sources classified as 'steep' and 'flat'. Changing the classification of such sources to 'flat' reduces their contribution at low flux levels even further.

In all the three parent samples, sources without optical identifications or those identified with quasars with unknown redshift were assumed to have $z = 1$. The results are not very sensitive to this assumption because of the small number of sources involved and the nearly flat relation between θ and z at $z \sim 1$ in Friedman cosmologies.

3.2 *Evolutionary Models for the RLF*

We have considered the following three classes of models for the evolution of the radio luminosity function (RLF) available in the literature. All the models allow the RLF to be translated from one frequency to another.

3.2.1 *Peacock & Gull* (1981; *PG*) *models*

In these models the RLF at 2.7 GHz for both steep-spectrum and flat-spectrum sources is expressed as a power series expansion in log $(1+z)$. While the flat-spectrum sources are assumed to have $\alpha = 0$ independent of P, a P–α correlation is assumed for the steep-spectrum sources. The values of q_0 and z_c (redshift cutoff) assumed for the four PG models are as follows:

PG1:	$q_0 = 0.5$,	$z_c = 100$,
PG2:	$q_0 = 0.5$,	$z_c = 5$,
PG3:	$q_0 = 0$,	$z_c = 100$,
PG4:	$q_0 = 0$,	$z_c = 5$.

3.2.2 *Subrahmanya* & *Kapahi* (1983) *model*

This is a parametric model in which the density evolution is assumed to be luminosity dependent. The model assumes $\alpha = 0$ for flat-spectrum sources and a P–α, correlation for the steep spectrum ones. The local RLF is specified at a frequency of 408 MHz. Other assumptions are $q_0 = 0.5$ and $z_c = 3.5$.

3.2.3 *Condon* (1984) *model*

This is also a parametric model that combines both density and luminosity evolution. Two separate Gaussian functions specify the distribution of spectral indices for flat and steep-spectrum sources, the latter distribution being a function of redshift. The local RLF is specified at a frequency of 1.4 GHz and values of $q_0 = 0.5$ and $z_c = 9$ are assumed.

The above models cover a wide range of permitted behaviour of the RLF consistent with the observed counts of radio sources and a variety of other available information on radio sources such as luminosity distributions, optical identification statistics, spectral index distributions *etc.*

4. Results and discussion

Using the different parent samples of bright sources and models for the evolution of the RLF, the predicted distributions of θ at different flux levels were estimated following the method described in detail by Allington-Smith (1984). The calculations were carried out both with and without evolution in the linear sizes. Size evolution was assumed to be of the form $l(z) \propto (1+z)^{-n}$ and values of n = 1,1.5 and 2 were tried out. The results for n = 0 (no size evolution) and a value of n that came closest to fitting the observational data are shown in Figs 2 to 6. The observed θ–S relation shown in the figures is reproduced from Kapahi & Subrahmanya (1982) with the following additions at low flux levels. The point at $S_m = 0.25$ Jy and $\theta_m = 9.5$ arcsec is based on recent VLA observations of 42 5C6 and 5C7 sources with 0.2 Jy $< S_{408} <$ 0.35 Jy (Gopal-Krishna personal communication). The point for 0.055 Jy $< S_{408} <$ 1 Jy from Downes et al. (1981) based on only 24 sources from the 5C6 and 5C7 has been replaced by the point at $S_m \simeq 0.2$ Jy and θ_m = 7.5 arcsec from the observations of 44 sources from the 5C7 and 5C12 surveys in the range of 0.1 to 1 Jy reported by Fielden *et al.* (1983). For clarity, we have not shown in Figs 2 to 6 the range in S covered by each θ_m value shown (see Kapahi & Subrahmanya 1982) and also the errors in the predicted θ_m–S curves due to the finite number of sources in the parent samples. For all the parent samples and evolutionary models used by us (except model PG-2, see discussion below) the estimated uncertainities in the predictions (as defined by Allington-Smith 1984) over the entire flux density range of the data (10 Jy $< S_{408} <$ 0.2 Jy) are considerably smaller than the

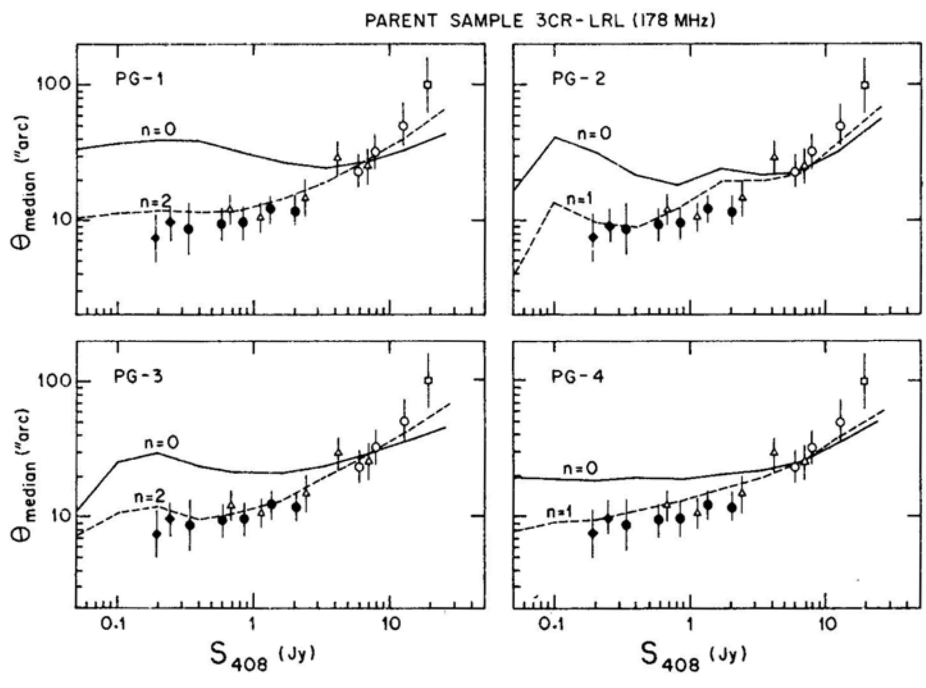

Figure 2. Comparison of the observed θ_m–S relation with predictions based on the 3CR-LRL parent sample for the 4 evolutionary models of Peacock & Gull (1981: PG).

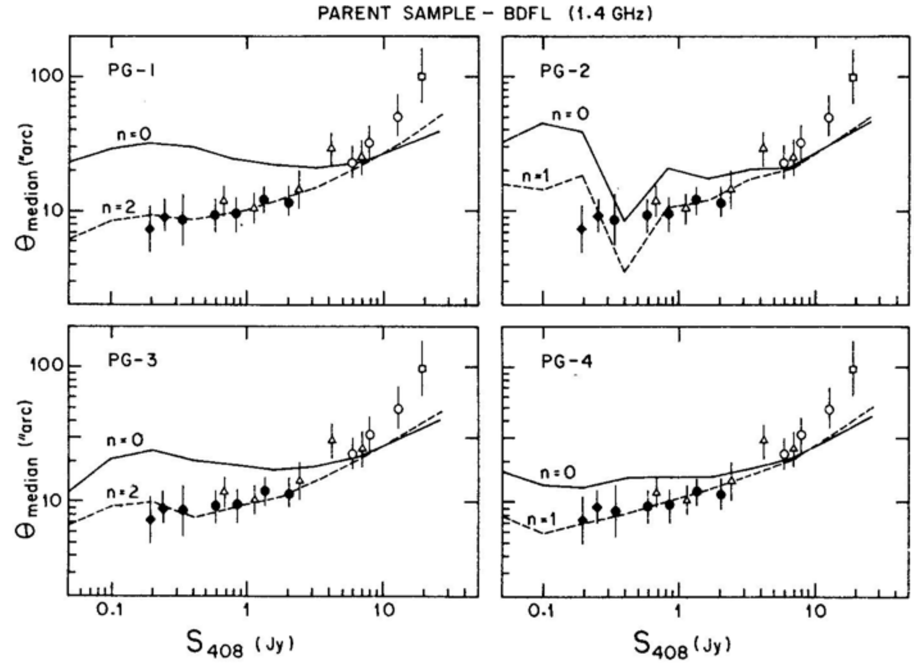

Figure 3. Predictions based on the BDFL parent sample for the PG evolutionary models.

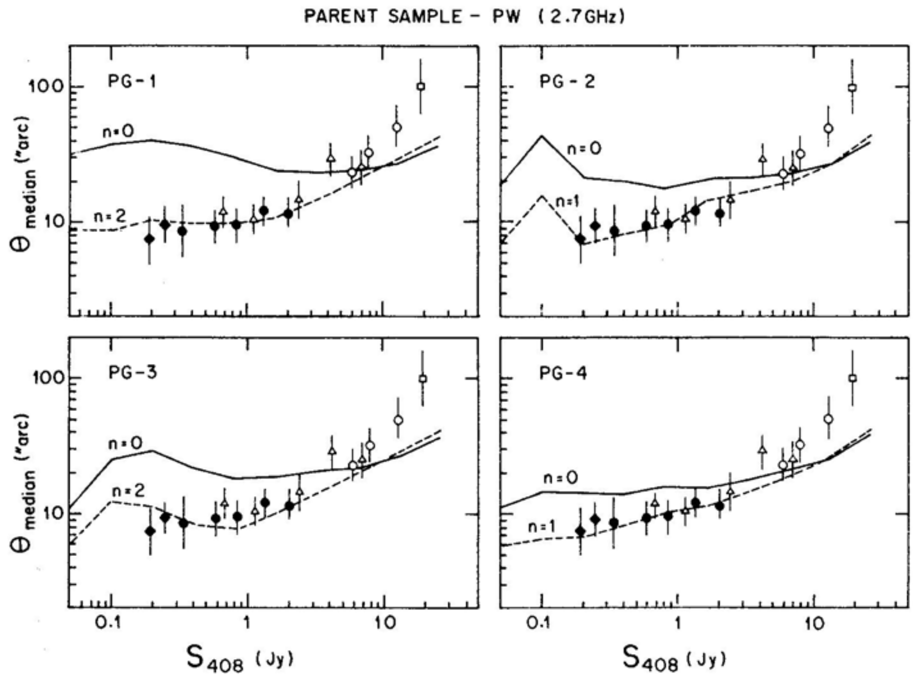

Figure 4. Predictions based on the PW 2.7 GHz parent sample for the PG evolutionary models.

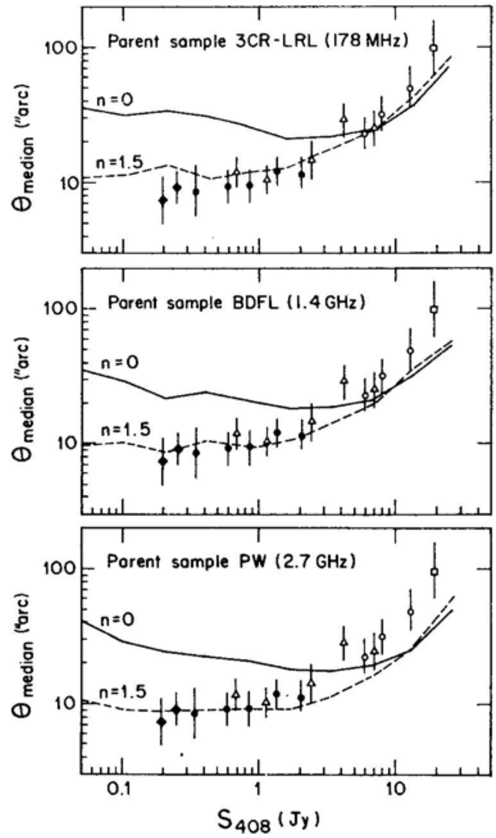

Figure 5. Predictions of the Subrahmanya & Kapahi (1983) evolutionary model using parent samples at different frequencies.

errors in the individual data points. Prediction-errors start becoming increasingly important only at $S_{408} \lesssim 0.1$ Jy where low luminosity sources, not well represented in the parent samples begin to make a significant contribution. It is also worth noting that in view of the good angular resolution of the observations used in determining the θ_m–S relation and the good agreement in the data obtained with different techniques and telescopes, it is very unlikely that the observed character of the θ_m–S relation is seriously affected by instrumental effects.

It is clear from Figs 2 to 6 that in no case do the predictions fit the data without size evolution. While the least amount of evolution ($n \sim 1$) appears to be required for the evolutionary model PG-4 ($q_0 = 0$) used with the PW 2.7 GHz parent sample (Fig. 4), the largest amount of evolution ($n \sim 2.25$) is needed for Condon's (1984) model (Fig. 6). The PG-3 model, which is the only other available model for $q_0 = 0$, requires $n \sim 2$ to fit the observations.

We note from Figs 2 to 4 that compared to other PG models the model PG-2 does not predict a smooth variation of θ_m with S. Closer examination shows that the sharp changes in the θ–S relation for this model arise because of abnormally large contributions made at some flux levels by just one or two sources in the parent samples.

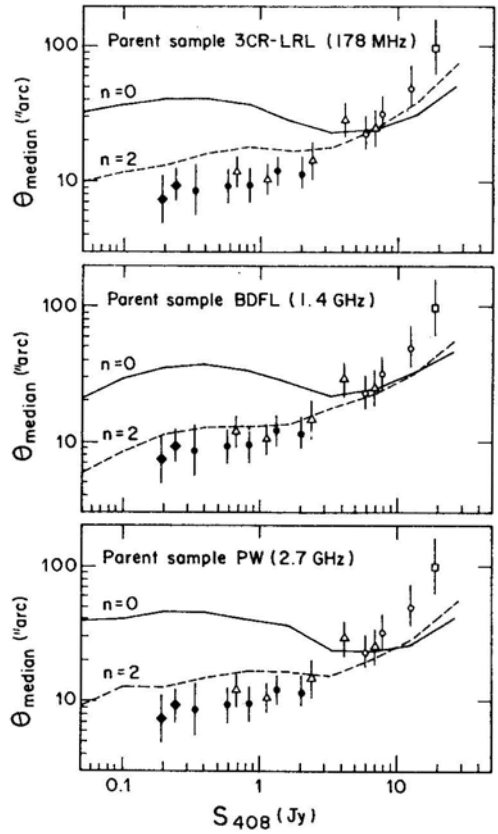

Figure 6. Predictions of the Condon (1984) evolutionary model using different parent samples.

The reason for this is clear from Fig. 7 where we show the evolution functions (ratio of space density at z to the corresponding density at $z = 0$) for different luminosities as a function of redshift in the four PG models. The evolution function for model 2 appears to exhibit a rapid change in slope at high redshifts which causes the contribution of one or two high luminosity sources to dominate at some flux levels. Model 2 of PG does not also appear to fit the observed source counts as noted earlier by Danese, De Zotti & Mandolesi (1984) and Kapahi & Kulkarni (1986).

Although the values of n indicated by predictions based on the 3CR sample and those based on the BDFL and PW samples do not appear to differ significantly, contrary to the conclusions of Fielden *et al.* (1983) and Allington-Smith (1984), the 3CR sample provides a better fit to the data at high flux densities compared to the high-frequency parent samples which predict considerably smaller values of θ_m compared to observations. This is not surprising and is clearly due to the presence of a larger number of SSC sources in the high-frequency samples, whose contribution at 408 MHz is being overestimated by ignoring spectral curvature at low frequencies. Kapahi & Kulkarni (1986) have shown recently that this is likely to be responsible also for the fact that the PG models do not fit the observed flux-density-spectral-index relation for bright sources in the 408 MHz samples.

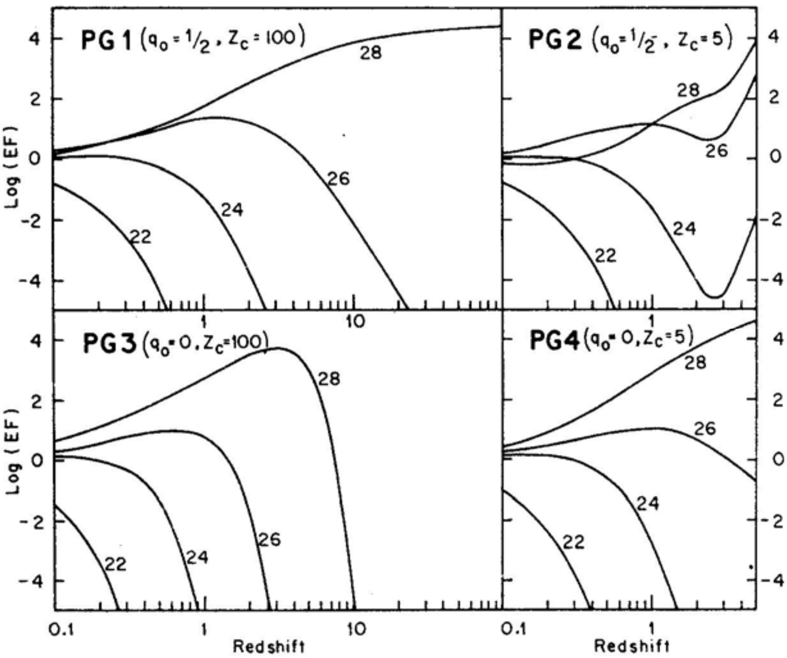

Figure 7. Plots of the evolution functions for steep-spectrum sources as a function of redshift in the 4 evolutionary models of Peacock & Gull (1981) for different values of log *P*.

Most of the difference between the predictions presented here and those reported by earlier workers using the PW parent sample and PG models seems to arise from the use of spectral indices between 1.4 and 2.7 GHz used by us for many of the SSC sources compared to those listed in PW, which are considerably steeper. We have argued in Section 3.1 that the use of the latter spectral indices for extrapolating the flux density of SSC sources from 2.7 GHz to 408 MHz is inappropriate and leads to lowering the values of θ_m at all flux levels. If the weak sources in 408 MHz samples have typical redshifts of say 2.5, then the appropriate spectral index that should be used in translating the flux density of sources from 2.7 GHz to 408 MHz is likely to be the value near an emitted frequency of ~ 1.4 GHz. The value of α between the observed frequencies of 1.4 and 2.7 GHz used by us corresponds typically (for z_m ~ 0.8) to emitted frequencies of 2.5 and 4.9 GHz. Even our revised α values are therefore likely to have overestimated the contribution of SSC sources and hence underestimated the evolution in linear sizes. This is likely to be so also in the case of predictions based on the BDFL sample because here again we have used values of α at the observed frequency of 1.4 GHz.

The omission of large sources (Table 1), which should legitimately be included in the PW sample, has also contributed, but to a much smaller extent, in decreasing the predicted values of θ_m at low flux levels in the earlier work. It thus seems clear that the larger contribution of SSC sources in low flux density samples is unlikely to obviate the need for linear size evolution, as claimed in the literature, unless the weak sources in low-frequency samples have unusually large redshifts of 10 or greater. Finally, we note that one of the important reasons for the suggestion by Downes, Longair & Perryman

(1981) and Fielden *et al.* (1983) that many of the weak sources in the 408 MHz samples may be similar to SSC sources seen in strong source samples at high frequencies was that they found a large fraction (~ 35 per cent) of the 5C sources to be unresolved with the VLA. However, recent high-resolution VLA observations of a comparable sample of 5C6 and 5C7 sources (with 0.2 Jy < S_{408} < 0.35 Jy) at both 1.4 and 5 GHz by Gopal-Krishna *et al.* (1986) show only 13 per cent of the sources to be compact ($\theta \leqslant 3$ arcsec), a fraction similar to that in the 3CR sample. The work of Gopal-Krishna *et al.,* in fact, suggests that the much larger fraction of SSC sources in the earlier study may have resulted from inadequate sensitivity (of the 'A' array observations at 5 GHz) to even moderately extended components.

5. Conclusions

From a comparison of the observed θ–S relation at 408 MHz with the predicted relations based on well-observed parent samples of strong sources at different frequencies and several models of the evolution of the radio luminosity function, we have shown that cosmological evolution in linear sizes of extragalactic radio sources is required in order to fit the data. This result is independent of any correlation between radio luminosity and linear size. We have examined the claim in the literature that the observed θ–S relation may be interpreted without need for size evolution if a substantial fraction of the weak sources in 408 MHz samples are of the steep-spectrum, compact variety that are well represented in bright samples at high frequencies. We find that the available predictions of θ–S relations based on the 2.7 GHz parent sample do not satisfactorily take account of the pronounced curvature in the spectra of SSC sources at low frequencies, thus overestimating their contribution at all flux levels in low-frequency samples. Approximate corrections for spectral curvature, assuming that most radio sources at low flux densities do not have redshifts in excess of 3 or 4, lead us to conclude that size evolution is indeed necessary. The amount of evolution is found to be similar to that inferred from the θ–z relations for quasars and radio galaxies.

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