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True and apparent spectra of buried polarizable targets

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ABSTRACT

If the chargeability of a buried target is not infinitesimal, the popularly used low chargeability approximation formulated by Seigel (1959) can produce large errors in the computation of apparent polarizability spectra. A more accurate alternative approximation, based on a complex, frequency dependent "dilution factor" is presented. It turns out that for dispersions of the minimum phase shift type this approximation can be somewhat simplified and that for targets with such a dispersion, buried in a nondispersive host rock, the apparent log-phase spectrum is only slightly different from a vertically shifted version of the true phase spectrum of the target.

These results should be useful for the computation of apparent polarizabilities in numerical modeling for IP, and in attempts for mineral discrimination through field measurements of phase spectra.

INTRODUCTION

The apparent polarizability of a slightly polarizable target buried in a nonpolarizable medium may be approximated as the product of its true polarizability and a factor which has been called the "dilution factor." This simple relationship has been widely used by geophysicists to calculate apparent chargeability m (Seigel, 1959), or apparent PFE (Pelton et al, 1978), or the time-domain voltage response of a ground to an infinitely long charging current switched off at $t = 0$.

LOW CHARGEABILITIES

In reviewing the basic macroscopic theory of induced polarization, Wait (1981) dealt with this approximation and made the point that this is a first-order theory valid only for small chargeabilities.

Following Wait (1981), let the complex, frequency dependent apparent resistivity $\rho_a(\omega)$ as a function of the angular frequency ω be written

$$\rho_a(\omega) = \rho_a(0)[1 + \delta_a(\omega)], \tag{1}$$

where $\rho_a(0)$ is the dc apparent resistivity and $\delta_a(\omega)$ is its complex departure from the ac value, normalized by the ac value.

Considering a single target buried in a homogeneous surrounding medium for simplicity, let the complex target resistivity be

$$\rho_2(\omega) = \rho_2(0)[1 + \delta_2(\omega)] \tag{2}$$

and the surrounding medium resistivity be

$$\rho_1(\omega) = \rho_1(0)[1 + \delta_1(\omega)], \tag{3}$$

where $\rho_2(0)$, $\rho_1(0)$, $\delta_2(\omega)$, and $\delta_1(\omega)$ are quantities corresponding to $\rho_a(0)$ and $\delta_a(\omega)$.

Expanding ρ_a in a Maclaurin series around $\rho_a(0)$ and comparing with equation (1), one obtains

$$\begin{aligned}\delta_a(\omega) = {} & \delta_1(\omega)\frac{\rho_1(0)}{\rho_a(0)}\frac{\partial\rho_a}{\partial\rho_1} + \delta_2(\omega)\frac{\rho_2(0)}{\rho_a(0)}\frac{\partial\rho_a}{\partial\rho_2} \\ & + \frac{1}{2}\Bigg[\delta_1^2(\omega)\frac{\rho_1^2(0)}{\rho_a(0)}\frac{\partial^2\rho_a}{\partial\rho_1^2} + 2\delta_1(\omega)\delta_2(\omega)\cdot \\ & \cdot\frac{\rho_1(0)\rho_2(0)}{\rho_a(0)}\frac{\partial^2\rho_a}{\partial\rho_1\partial\rho_2} \\ & + \delta_2^2(\omega)\frac{\rho_2^2(0)}{\rho_a(0)}\frac{\partial^2\rho_a}{\partial\rho_2^2}\Bigg] + \cdots\end{aligned} \tag{4}$$

where

$$\frac{\partial\rho_a}{\partial\rho_1} = \left[\frac{\partial\rho_a\{\rho_1(0)[1+\delta_1(\omega)],\ \rho_2(0)[1+\delta_2(\omega)]\}}{\partial\rho_1(0)[1+\delta_1(\omega)]}\right]\lim_{\delta_1=\delta_2=0} \tag{5}$$

and so on for the other derivatives, all of these being evaluated around $\omega = 0$.

If the δs are so small that terms containing their products and higher powers may be neglected, equation (4) becomes

$$\delta_a(\omega) = \delta_1(\omega)B_1 + \delta_2(\omega)B_2, \tag{6}$$

where

$$B_1 = \frac{\partial \ln \rho_a}{\partial \ln \rho_1}, \qquad B_2 = \frac{\partial \ln \rho_a}{\partial \ln \rho_2} \tag{7}$$

are the familiar dilution factors. Using the scaling property of ρ_a expressed by

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$$\lambda\rho_a = \rho_a(\lambda\rho_1, \lambda\rho_2), \tag{8}$$

it is easily shown that $B_1 + B_2 = 1$. The formulation is easily extended to the case involving more than two regions.

I shall refer to the standard approximation represented by equation (6) as Approximation I.

Pelton et al (1978) used this approximation in a somewhat different form. For a nonpolarizable surrounding medium, they stated that the slope of the plot of log (modulus) of apparent resistivity is, at every frequency, B_2 times the corresponding plot for the true resistivity of the target. This result may be obtained, for low vaues of $\delta_2(\omega)$, as follows.

From equation (6), with $\delta_1(\omega) = 0$,

$$\delta_a(\omega) = \delta_2(\omega)B_2 \tag{9}$$

so that

$$\ln \frac{\rho_a(\omega)}{\rho_a(0)} = \ln\,[1 + B_2\,\delta_2(\omega)]$$

$$\approx B_2\,\delta_2(\omega) \tag{10}$$

and

$$\ln \frac{\rho_2(\omega)}{\rho_2(0)} = \ln\,[1 + \delta_2(\omega)]$$

$$\approx \delta_2(\omega) \tag{11}$$

if $\delta_2(\omega)$ is very small.

Thus

$$\ln \frac{\rho_a(\omega)}{\rho_a(0)} = B_2 \ln \frac{\rho_2(\omega)}{\rho_2(0)}. \tag{12}$$

Equating real and imaginary parts of equation (12),

$$\ln \left|\frac{\rho_a(\omega)}{\rho_a(0)}\right| = B_2 \ln \left|\frac{\rho_2(\omega)}{\rho_2(0)}\right| \tag{13}$$

and

$$\phi_a(\omega) = B_2\,\phi_2(\omega), \tag{14}$$

where the sign | | is used to represent the modulus of the quantity within, and ϕ_a and ϕ_2 are the phase angles of the apparent and true target resistivities, respectively. Equation (13) leads to the approximation used by Pelton et al (1978)

$$\left|\frac{\rho_a(\omega)}{\rho_a(0)}\right| = \left|\frac{\rho_2(\omega)}{\rho_2(0)}\right|^{B_2}. \tag{15}$$

I refer to equations (13) and (14) as Approximation II.

Approximations I and II are identical for infinitesimal values of target polarizabilities. For somewhat larger values of polarizability, however, the consequences of these two approximations are quite different.

Approximation I for a nonpolarizable surrounding medium demands that the out-of-phase, i.e., quadrature, component of the normalized apparent resistivity is B_2 times the quadrature component of the true resistivity. In other words, the "peaking frequency" of the out-of-phase component remains invariant with dilution. Approximation II requires that peaking frequency of the phase angle, rather than the quadrature component, remains invariant with dilution.

Approximation I leads to the conclusion that the apparent step response to an infinitely long charging current switched off at time $t = 0$ is, for all values of $t > 0$, equal to B_2 times the corresponding response of the target material alone. Approximation II leads to this relationship being valid for $t = 0$ after switch off and for no other later value of t.

MODERATE AND HIGH CHARGEABILITIES: APPARENT SPECTRA AND SPECTRAL SLOPES

To examine what happens with moderate polarizabilities, one can take higher order terms of equation (4) and, using the mixed and higher order derivatives obtained from the dc solution, compute the apparent ρ_a.

The coefficients involving mixed and higher order derivatives were called "distortion factors" by Wait (1981). These are real constants which need to be evaluated afresh for every pair of $\rho_1(0)$ and $\rho_2(0)$ along with the dilution factors.

In the following I present a somewhat different formulation in which the complex resistivity of the target is normalized by dividing by the complex resistivity of the host. I use the scaling law expressed by equation (8), which means that all EM inductive effects are overlooked.

Such a formulation allows one to see some interesting results applicable to complex resistivities of the minimum phase shift type. Furthermore, for a given target geometry, the dc solution needs to be worked out only once over a relevant range of the ratio of target-to-host resistivities in order to compute the complex apparent resistivity quickly for any given combination of complex host and target resistivities. This also produces more accurate results for comparable computation efforts using equation (4).

Using $1/\rho_1$ for λ in equation (8), ρ_a may always be written as

$$\rho_a = \rho_1 S(r), \tag{16}$$

where S is some function of the complex ratio $r = \rho_2(\omega)/\rho_1(\omega) = |r|e^{j\phi_r}$. S is determined by the geometry and is real for real values of r. With

$$\ln r = u = u_R + ju_I = \ln|r| + j\phi_r, \tag{17}$$

then

$$\ln(\rho_a/\rho_1) = \ln S = T. \tag{18}$$

Expanding T as a Taylor series around u_R,

$$T = T(u_R) + j\phi_r T'(u_R) - \frac{\phi_r^2}{2} T''(u_R) + \cdots \tag{19}$$

Since T is real for real r, $T(u_R)$ and its derivatives $T'(u_R)$, $T''(u_R)$, etc. are real for all real u, and are obtained from the real function $T(u_R)$ at any specified value of u_R by using the dc solution for the given geometry. Set $\rho_1 = 1$ and compute $T = \ln(\rho_a)$ for a suitable set of real values of $\ln \rho_2 = u$. The derivatives of T at any real u can then be computed from this set. Since $\phi_r = \phi_2 - \phi_1$ and $u_r = \ln|\rho_2/\rho_1|$ at any frequency, $T(u)$ can be computed by using equation (19).

Equating real and imaginary parts of equation (18), and using the symbols Re [ ] and Im [ ] for the real and imaginary parts, respectively, of the quantity within the brackets,

$$\ln|\rho_a| = \ln|\rho_1| + \mathrm{Re}\,[T]$$

$$= \ln|\rho_1| + T(u_R) - \frac{\phi_r^2}{2} T''(u_R) + \cdots \tag{20}$$

and

$$\phi_a = \phi_1 + \mathrm{Im}\,[T]$$

$$= \phi_1 + \phi_r T'(u_R) - \frac{\phi_r^3}{6} T^{(3)}(u_R) + \cdots. \tag{21}$$

If $\phi_r = \phi_2 - \phi_1$ is not too large, a good approximation is obtained by taking

$$\ln |\rho_a| \approx \ln |\rho_1| + T(u_R) - \frac{\phi_r^2}{2} T''(u_R) \tag{22}$$

and

$$\phi_a \approx \phi_1[1 - T'(u_R)] + \phi_2 T'(u_R). \tag{23}$$

The last equation shows that the apparent phase may be approximated as a weighted sum of the true phase angles, the weights being functions of u_R and, consequently, of frequency ω, instead of being constants. Furthermore, the weights for ϕ_1 and ϕ_2 add up to unity.

For the spectral slopes, I consider the complex number

$$B = \frac{\partial \ln \rho_a}{\partial \ln \rho_2}. \tag{24}$$

One can see from equation (16) that it is a function of u, real for real u, and

$$B = \frac{d \ln S}{d \ln r} = \frac{dT}{du} = \frac{\partial \operatorname{Re}[T]}{\partial u_R} + j \frac{\partial \operatorname{Im}[T]}{\partial u_R}, \tag{25}$$

which may also be expanded around u_R as

$$B = B(u_R) + j\phi_r B'(u_R) - \frac{\phi_r^2}{2} B''(u_R) - \cdots, \tag{26}$$

and comparing with dT/du,

$$B(u_R) = T'(u_R), \qquad B'(u_R) = T''(u_R), \tag{27}$$

and so on. Using B, one can write expressions for the spectral slopes of $|\rho_a|$ and ϕ_a. Thus, from equation (16),

$$\frac{d \ln \rho_a}{d \ln \omega} = \frac{d \ln \rho_1}{d \ln \omega} + \frac{d \ln S}{d \ln \omega} = \frac{d \ln \rho_1}{d \ln \omega} + \frac{d \ln r}{d \ln \omega} B. \tag{28}$$

Equating real and imaginary parts,

$$\frac{d \ln |\rho_a|}{d \ln \omega} = \frac{d \ln |\rho_1|}{d \ln \omega} + \operatorname{Re}\left[B \frac{d \ln r}{d \ln \omega}\right] \tag{29}$$

and

$$\frac{d\phi_a}{d \ln \omega} = \frac{d\phi_1}{d \ln \omega} + \operatorname{Im}\left[B \frac{d \ln r}{d \ln \omega}\right], \tag{30}$$

where

$$\operatorname{Re}\left[B \frac{d \ln r}{d \ln \omega}\right] = \operatorname{Re}[B] \frac{d \ln |r|}{d \ln \omega} - \operatorname{Im}[B] \frac{d\phi_r}{d \ln \omega} \tag{31}$$

and

$$\operatorname{Im}\left[B \frac{d \ln r}{d \ln \omega}\right] = \operatorname{Re}[B] \frac{d\phi_r}{d \ln \omega} + \operatorname{Im}[B] \frac{d \ln |r|}{d \ln \omega}. \tag{32}$$

However,

$$\frac{d \ln |r|}{d \ln \omega} = \frac{d \ln |\rho_2|}{d \ln \omega} - \frac{d \ln |\rho_1|}{d \ln \omega} \tag{33}$$

and

$$\frac{d \ln |\rho_a|}{d \ln \omega} = \{1 - \operatorname{Re}[B]\} \frac{d \ln |\rho_1|}{d \ln \omega} + \operatorname{Re}[B] \frac{d \ln |\rho_2|}{d \ln \omega} - \operatorname{Im}[B] \frac{d\phi_r}{d \ln \omega}. \tag{34}$$

Since $\phi_r = \phi_2 - \phi_1$

$$\frac{d\phi_a}{d \ln \omega} = \{1 - \operatorname{Re}[B]\} \frac{d\phi_1}{d \ln \omega} + \operatorname{Re}[B] \frac{d\phi_2}{d \ln \omega} + \operatorname{Im}[B] \frac{d \ln |r|}{d \ln \omega}. \tag{35}$$

Equations (34) and (35) show that if Im $|B|$ is small, the spectral slopes $d \ln |\rho_a|/d \ln \omega$ and $d\phi_a/d \ln \omega$ are well approximated as weighted sums of the corresponding slopes for the two regions. B may thus be regarded as a complex, frequency-dependent dilution factor.

SIMPLIFICATION FOR MINIMUM PHASE TYPE DISPERSIONS

Equations (20), (21), (34), and (35) are exact expressions for the apparent spectra and the apparent spectral slopes. B or T need be computed only once, over an adequate range of u_R, for a given geometry, and these expressions allow the computation of the spectra for any given pair of dispersions for the two regions.

If, however, the dispersions are of the minimum phase type, further simplification is possible. The examination of this type of dispersion becomes important because most polarizable materials show a minimum phase type dispersion in practice (Zonge et al, 1972).

For a complex function of frequency which asymptotically attains constant values of modulus at zero and infinite real frequencies, a necessary and sufficient condition for its being of the minimum phase type is that the net phase change from zero to infinity is zero (Bode, 1957). If ρ_1 and ρ_2 are of the minimum phase type, approaching constant moduli at $\omega = 0$ and $\omega = \infty$, the complex dilution factor B also approaches real constant values at $\omega = 0$ and $\omega = \infty$. As a function of frequency, $B(\omega)$ is therefore of the minimum phase type. So are $\rho_a(\omega)$ and $T(\omega)$.

In practice, induction and propagation prevent any polarizable ground from behaving like a truly minimum-phase system. The assumption of minimum-phase ρ_1, ρ_2, ρ_a, etc. implies that all phase angles over the frequency range of our concern are due to polarization and are undisturbed by induction or propagation effects. I tacitly assume this to be true.

The phase angle and imaginary part of a minimum-phase function are uniquely related to the frequency dependence of its modulus and real part, respectively. For slowly varying functions of frequency very good approximations are (Zonge et al, 1972; Bode, 1957)

$$\text{phase} \approx \frac{\pi}{2} \frac{d \ln (\text{modulus})}{d \ln \omega} \tag{36}$$

and

$$\text{imaginary part} \approx \frac{\pi}{2} \frac{d\,(\text{Real part})}{d \ln \omega}. \tag{37}$$

Applying equations (36) and (37) to the right-hand member of equation (35),

$$\frac{d\phi_a}{d \ln \omega} \approx (1 - \operatorname{Re}[B]) \frac{d\phi_1}{d \ln \omega}$$

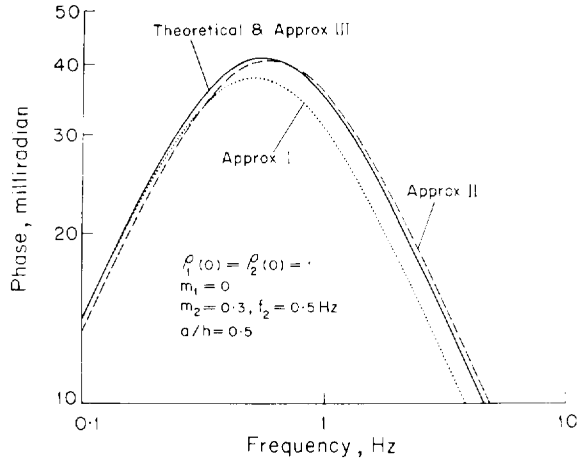

FIG. 1. The apparent phase spectrum is very well approximated by Approximation-III. This figure compares the theoretical apparent spectrum with the spectra calculated by Approximations-I, II, and III. The model is a two-layer ground under a two-electrode array, with array spacing *a* equal to half of the upper layer thickness *h*. The upper layer is nondispersive. The lower has chargeability $m_2 = 0.3$ and characteristic frequency $f_2 = 0.5$ Hz.

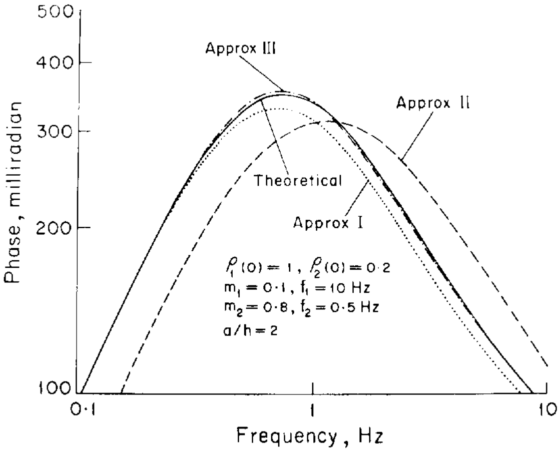

FIG. 2. With both layers polarizable, Approximation-I can be superior to Approximation-II as in this case with the lower layer less resistive than the upper.

$$+ \operatorname{Re}[B] \frac{d\phi_2}{d \ln \omega} + \frac{d \operatorname{Re}[B]}{d \ln \omega} (\phi_2 - \phi_1), \tag{38}$$

and hence, integrating both sides,

$$\phi_a \approx (1 - \operatorname{Re}[B])\phi_1 + \operatorname{Re}[B]\phi_2. \tag{39}$$

Applying equation (36) to both members of equation (39),

$$\frac{d \ln |\rho_a|}{d \ln \omega} \approx (1 - \operatorname{Re}[B]) \frac{d \ln |\rho_1|}{d \ln \omega} + \operatorname{Re}[B] \frac{d \ln |\rho_2|}{d \ln \omega}. \tag{40}$$

Extension of these results to more than one target region is straightforward. For example, with a host region 1 and target regions 2 and 3,

$$\rho_a = \rho_1 S(r_2, r_3), \tag{41}$$

where

$$r_2 = \rho_2/\rho_1 \quad \text{and} \quad r_3 = \rho_3/\rho_1.$$

Hence

$$\frac{d \ln \rho_a}{d \ln \omega} = \frac{d \ln \rho_1}{d \ln \omega} + B_{(2)} \frac{d \ln r_2}{d \ln \omega} + B_{(3)} \frac{d \ln r_3}{d \ln \omega}. \tag{42}$$

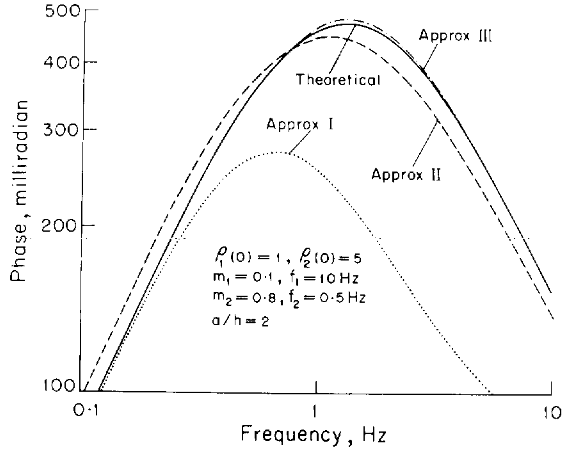

FIG. 3. Same as the case of Figure 2, except that the lower layer is more resistive than upper. Approximation-I is seen to be very poor in this case.

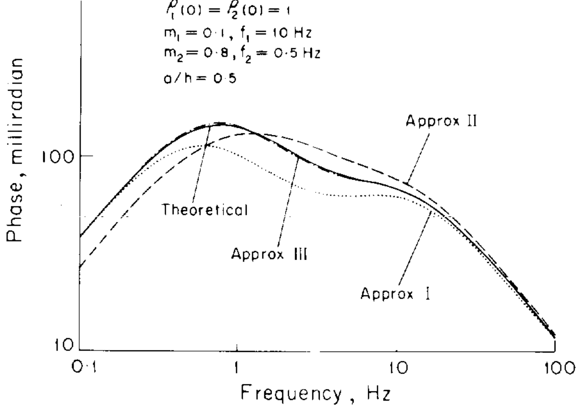

FIG. 4. Both Approximations-I and II may produce large errors as in this case. The layer polarizabilities are the same as in Figures 2 and 3, but in this figure *a/h* is 0.5 and both layers are equally resistive.

where $B_{(2)} = \partial \ln S/\partial \ln r_2$ and $B_{(3)} = \partial \ln S/\partial \ln r_3$.

Equating imaginary parts of equation (42), applying the minimum phase condition and integrating,

$$\phi_a \approx (1 - \mathrm{Re}\,[B_{(2)}] - \mathrm{Re}\,[B_{(3)}])\phi_1 + \mathrm{Re}\,[B_{(2)}]\phi_2 + \mathrm{Re}\,[B_{(3)}]\phi_3. \tag{43}$$

Hence, using equation (36),

$$\frac{d \ln |\rho_a|}{d \ln \omega} \approx (1 - \mathrm{Re}\,[B_{(2)}] - \mathrm{Re}\,[B_{(3)}]) \frac{d \ln |\rho_1|}{d \ln \omega} + \mathrm{Re}\,[B_{(2)}] \frac{d \ln |\rho_2|}{d \ln \omega} + \mathrm{Re}\,[B_{(3)}] \frac{d \ln |\rho_3|}{d \ln \omega}. \tag{44}$$

Equations (39) and (40) represent approximations which are far more accurate than Approximations I and II when the dispersions are of the minimum phase type even if the phases ϕ_1 and ϕ_2 are not small. I shall refer to approximations (39) and (40) as Approximation III.

In most modeling situations, Re [B] changes only by a small amount over the range $\omega = 0$ to $\omega = \infty$. Treating Re [B] as a constant leads to Approximation II, and this explains why in many situations Approximation II gives a more nearly correct result when compared with Approximation I. It turns out that a compromise value to use for Re [B], as a constant, is the geometric mean of its values at dc and at high frequency, rather than using its dc value as in Pelton et al (1978).

Figures 1 to 4 show the phase spectra obtained by applying all three approximations to the case of a two-layer ground under a two-electrode array. Approximation III is seen to be much better than the other two in all cases. The dispersions are assumed to be of the Debye minimum-phase type represented by

$$\rho(\omega) = \frac{1 + j\omega(1 - m)/\omega_0}{1 + j\omega/\omega_0}\,\rho(0), \tag{45}$$

where $\omega_0 = 2\pi f_0$ is the characteristic angular frequency of the dispersion. A Debye type dispersion is assumed only for simplicity. Any other minimum-phase type dispersion, e.g. Cole-Cole, could have been used for such a comparison.

Computation of the theoretical apparent phase angle at each frequency was done by using the process of summation of all images of the source (Wait, 1958), with complex values of resistivity of the layers.

In Figure 1 the upper layer is nondispersive, and the lower has a moderate chargeability of $m = 0.3$. The dc resistivities of both layers are the same. The electrode separation a is half the upper layer thickness h. Approximation II is seen to be superior to Approximation I, and Approximation III produces practically the same result as the correct apparent theoretical spectrum.

Figures 2, 3, and 4 are similar plots of calculated and theoretical phase spectra with both layers polarizable. The upper layer has $m = 0.1$ and a characteristic frequency of 10 Hz; the lower has a high value of $m = 0.8$ and a characteristic frequency 0.5 Hz. The a/h ratio and the relative dc resistivities of the two layers have different values. Approximation-II is seen to be inferior to Approximation-I in the case of Figure 2, and Approximation-I is seen to be quite wrong for the case of Figure 3. In Figure 4 both Approximation-I and Approximation-II are seen to produce large errors in the spectral shape.

Figures 5 and 6 are spectral plots of the target phase, apparent phase, and imaginary parts of the target and apparent resistivities. The upper layer is made nondispersive and the lower has $m = 0.6$. The dc resistivity contrast is made different

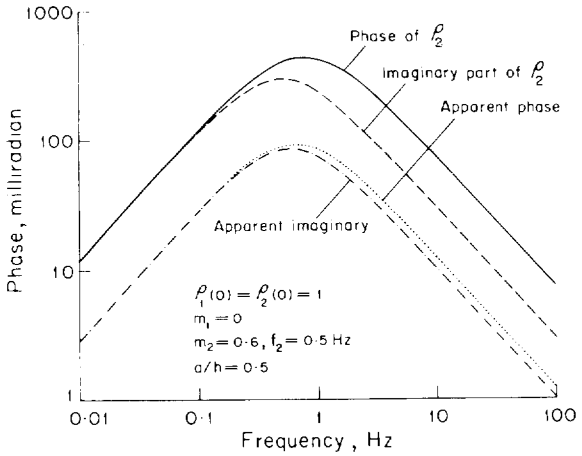

FIG. 5. With a minimum phase type target in a nondispersive host, the apparent phase spectrum is only slightly different from a vertically shifted version of the true phase spectrum. The shape of the apparent imaginary part spectrum, however, is not as close to the true imaginary part spectrum as one would expect from Approximation-I.

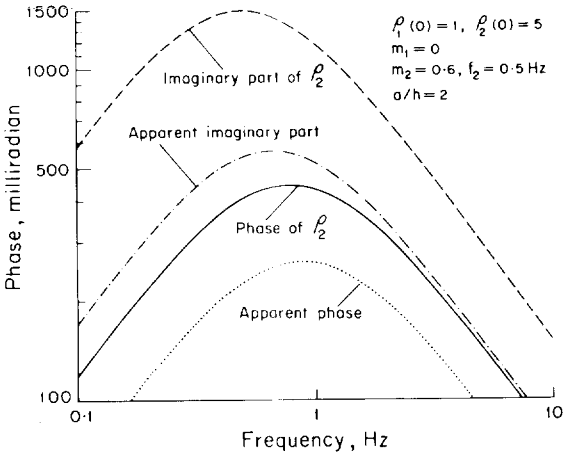

FIG. 6. This demonstrates the same result as in Figure 5 more clearly. The peaking frequency of the apparent imaginary part is shifted much more by dilution than the peaking frequency of the phase angle.

for the two cases. It is clearly seen that dilution affects the peaking frequency of the imaginary part spectrum much more than that of the phase spectrum. This is true, in spite of the large value of m, because the assumed dispersion is of the minimum phase type. However, this is just the opposite of what Approximation-I leads one to believe.

CONCLUSIONS

In numerical modeling of IP for a buried target in an otherwise homogeneous medium, one usually computes just one dilution factor using dc values of the resistivities. The above analysis suggests that it may be very useful to calculate Re $[B]$ for a suitable set of values of real ratios ρ_2/ρ_1, to enable more accurate estimates of ρ_a for moderate chargeabilities. Alternatively, values of real ρ_a for a set of values of real ρ_2/ρ_1 may be computed. Re $[B]$ or T may then be computed therefrom when needed. In most situations B and T are very well approximated by taking only the first two terms in equation (26) and the first three terms in equation (19), respectively. This involves the computation of first and second derivatives of log ρ_a with respect to log r, for a set of real r, and is easily done by using the dc solution for the given geometry.

If, as usual, the dispersion characteristics of the target are of the minimum phase type, one can expect that with a nondispersive host ground the observed log-phase spectrum is nearly the same as a vertically shifted version of the true phase spectrum even for moderately large polarizabilities. If the host is also dispersive, a weighted summation of the two-phase characteristics is a fair approximation.

Since dilution is seen to affect the shape of the log-phase spectra only slightly for minimum phase type target dispersions, the apparent phase spectrum should prove to be useful in attempts at mineral discrimination.

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