

# TEMPERATURE DEPENDENCE OF PHOTO-ELASTIC CONSTANTS IN PLASTICS

BY S. BHAGAVANTAM, F.A.SC. AND Y. KRISHNA MURTY

(From the Physical Laboratories, Osmania University, Hyderabad)

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## 1. INTRODUCTION

High polymers generally exhibit a discontinuity in thermal expansion at a characteristic temperature. This temperature also marks a relatively abrupt change in other thermal, mechanical, electrical and optical properties. Ehrenfest<sup>1</sup> has termed these discontinuities as second order transitions. A considerable body of experimental data on high polymers has been reported by a number of investigators in this direction. Recently Subrahmanyam,<sup>2</sup> Krishnamurthi and Sivarama Sastry,<sup>3</sup> during their study of the variation of ultrasonic velocities with temperature in polymerized methyl methacrylate and polystyrene, have reported from this laboratory similar transitions. In the present investigation, these two polymers have been studied by photo-elastic methods.

## 2. DESCRIPTION OF THE APPARATUS AND METHOD OF OBSERVATION

All the specimens needed were cut from cast sheets of polymethyl methacrylate or perspex and polystyrene. The cut surfaces of the samples were filed flat and smooth, and the samples were annealed at 90° C. for about 40 hours. The specimens were then worked out as plane parallel plates and were well polished. None of the specimens showed double refraction under crossed nicols in an unstrained condition.

A parallel beam of light from a sodium-vapour lamp was passed through a Nicol, the vibration direction of which was maintained at 45° to the vertical. The emergent beam then passed through the test-piece into a Babinet compensator, the principal axes of which were vertical and horizontal. After passing through the compensator the light was observed through an eyepiece containing a Nicol crossed with respect to the polarizer, and the usual Babinet fringes were obtained. When the specimen was compressed, the fringes shifted one way or the other and the magnitude of the shift was a measure of the path difference produced between the horizontally and vertically vibrating beams of light passing through the compressed specimen. Compression was produced by a lever arrangement.

Measurements of the fringe shift were made for stresses of about 100 kg./cm.<sup>2</sup> in the case of perspex and 60 kg./cm.<sup>2</sup> in the case of polystyrene. Following Pockels, the displacement of the fringes was measured at the middle of the length of the prisms at three places along its breadth, namely, near the left edge, the middle, and the right edge. The final measurements were made after ascertaining that the distribution of load was fairly uniform. The mean shift of the Babinet fringe in each case was obtained by taking the average of several readings. The path retardation read in terms of the head scale divisions of the Babinet compensator is a measure of the birefringence produced in the samples.

In order to study the variation of the induced birefringence with temperature, a compact electrical heater was built, the specimen being centrally situated within the enclosure. By means of a variac the voltage could be varied slowly and steadily till the requisite temperature was attained and a measurement on the Babinet was made only after ascertaining that the temperature was constant for about two hours. The procedure was repeated till consistent results were obtained at each temperature both while heating and cooling the specimens. Measurements of fringe shifts were made at intervals of 10° C., temperatures being read by a sensitive thermometer. The mean values of the Babinet shifts so obtained at each temperature were then made use of in deducing the relative stress-optical coefficients.

As the refractive index enters in the evaluation of the photoelastic constants, it is found necessary to determine the same for perspex and polystyrene at various temperatures. In the present investigation the usual method<sup>4</sup> in which  $dn/dt$  is directly measured as a function of temperature is employed. The method consists in cutting a plane parallel plate of thickness  $l$  out of the specimen and obtaining localized interference fringes by reflection from the two surfaces of the plate at normal incidence. The specimen was kept in a suitable furnace and the fringes, which consisted of sharp, bright and almost circular rings, were observed by means of a microscope. When heated, the fringes began to expand in both the cases and a continuous observation of noting the temperature corresponding to the passage of successive fringes was carried out. The rate of heating was so slow that it took eight hours to heat the specimen from room temperature to 90° C. The observations were repeated with cooling specimen, this also taking the same time.

The measurement of the expansion gives one the alteration in the path. Knowing  $\Delta t$  corresponding to each wavelength shift,  $dn/dt$  was calculated for the mean temperature making use of the formula

$$dn/dt = \frac{1}{2}l \cdot dx/dt - n\alpha,$$

where  $\alpha$  is the coefficient of thermal expansion at temperature  $t$ . The values of  $\alpha$  at various temperatures were available from the work of Yasaku Wada.<sup>5</sup> The absolute values of indices of refraction ( $n_D$ ) for sodium light at room temperature have been determined to three significant figures using the material in the form of tiny prisms, employing a spectrometer. In both the cases prisms having an angle roughly of about  $60^\circ$  were cut from sheets, 6.5 mm. in thickness, and the two sides of the prisms making the angle were well polished. Making use of these values of ( $n_D$ ) at  $25^\circ$  C., the refractive indices at various temperatures were calculated from the data for  $dn/dt$  by numerical integration.

TABLE I  
Values of  $n_D$  at different temperatures

Temperature ° C.	Perspex	Polystyrene
25	1.49300	1.59500
30	1.49047	1.59328
40	1.48542	1.58986
50	1.48043	1.58646
60	1.47539	1.58308
70	1.45906	1.57973
80	1.45193	1.54927
90	1.44447	1.54113

The calculated values of the refractive indices ( $n_D$ ) at various temperatures for perspex and polystyrene are given in Table I.

### 3. RESULTS AND DISCUSSION

The photo-elastic behaviour of isotropic substances is given by the equations:

$$B_{11} - B = -q_{12} \cdot ZZ; \quad B_{22} - B = -q_{12} \cdot ZZ; \quad B_{33} - B = -q_{11} \cdot ZZ; \text{ and} \\ B_{23} = B_{31} = B_{12} = 0.$$

In these equations  $B = 1/n^2$ ,  $B_{11} = 1/n_{11}^2$ , etc., where  $n$  is the refractive index of the undeformed specimen of an isotropic substance,  $n_{11}$ , etc., are the

components describing the Fresnel ellipsoid in the deformed condition, and ZZ, a simple linear compressional stress along the z-axis.  $q_{11}$  and  $q_{12}$  are the only two independent stress-optical coefficients, ( $q_{11} - q_{12}$ ) being equal to  $q_{44}$ .

The determination of the relative stress-optical coefficients ( $q_{11} - q_{12}$ ) was carried out by employing a Babinet compensator, the observations being given in detail in Tables II and III. 470.5 divisions of the Babinet compensator corresponded to a path retardation of one wave-length of sodium light.

TABLE II  
*Results for perspex*

Temperature °C.	Mean shift in divisions of head-scale	$(q_{11} - q_{12}) \times 10^{13}$	$(p_{11} - p_{12}) \times 10^4$
27	104.0	2.626	1.133
40	110.0	2.846	..
51	116.0	3.031	..
61	121.0	3.271	..
71	130.0	3.572	..
80	139.0	3.922	..
91	150.0	4.321	..

TABLE III  
*Results for polystyrene*

Temp. ° C.	Mean shift in divisions of head-scale	$(q_{11} - q_{12}) \times 10^{13}$	$(c_{11} - c_{12}) \times 10^{-10}$	$(p_{11} - p_{12}) \times 10^4$
32	144.0	4.733	2.782	1.371
42	137.0	4.540	2.722	1.236
52	130.0	4.347	2.672	1.161
62	122.0	4.115	2.614	1.075
72	113.0	3.903	2.552	0.996
81	100.0	3.678	..	..
92	84.0	3.157	..	..

From the mean shift of the Babinet fringe, the path differences corresponding to unit stress (1 dyne cm.<sup>-2</sup>) and unit length (1 cm.) of light beam in the stressed specimen were calculated knowing that the path difference is  $5.893 \cdot 10^{-5}$  cm. for a shift of 470.5 divisions. The stress-optical constant in the corresponding direction of observation is  $2\delta/n^3$ . In the case of polystyrene the shift was in the same direction as for common glass while it was opposite in the case of perspex. Hence, the sign of  $(q_{11} - q_{12})$  is negative for polystyrene and positive for perspex. Also an examination of Tables II and III reveals that the numerical value of the stress-optical constant  $(q_{11} - q_{12})$  for perspex increases with increase of temperature while it decreases in the case of polystyrene. Furthermore the values of the stress-optical constant at the lowest temperature differ by 60 to 70 per cent. from those at about 90° C.

The values of the strain-optical constants  $(p_{11} - p_{12})$  are calculated from the equation connecting  $p$ 's,  $q$ 's and the elastic constants, namely,

$$(p_{11} - p_{12}) = (c_{11} - c_{12}) \cdot (q_{11} - q_{12}).$$

The elasticity data on polystyrene at different temperatures have been obtained by Subrahmanyam.<sup>2</sup> The rigidity modulus of perspex is not available in the literature. Robinson, Ruggy and Elizabeth Slantz<sup>6</sup> have studied the stress-optical properties of polymethyl methacrylate as a function of temperature. However, their results do not give the complete data of the photo-elastic constants.

Temperature variation of the birefringence produced by stress in the above plastics is represented graphically in Fig. 1. A pronounced transition in the birefringence vs. temperature graphs of perspex and polystyrene will be noticed at 62° C. and 70° C. respectively. As has already been mentioned Subrahmanyam,<sup>2</sup> Krishnamurthi and Sastry<sup>3</sup> of this laboratory have studied the variation of ultrasonic velocities in perspex and polystyrene with temperature. They observed that both perspex and polystyrene show a transition at 61° C. and 70° C. respectively. The samples used by the authors and the above workers are taken from the same sheet of material. From the above facts it can be concluded that the present photo-elastic investigations in these two thermoplastics clearly show the existence of the transition points in good agreement with the earlier workers.

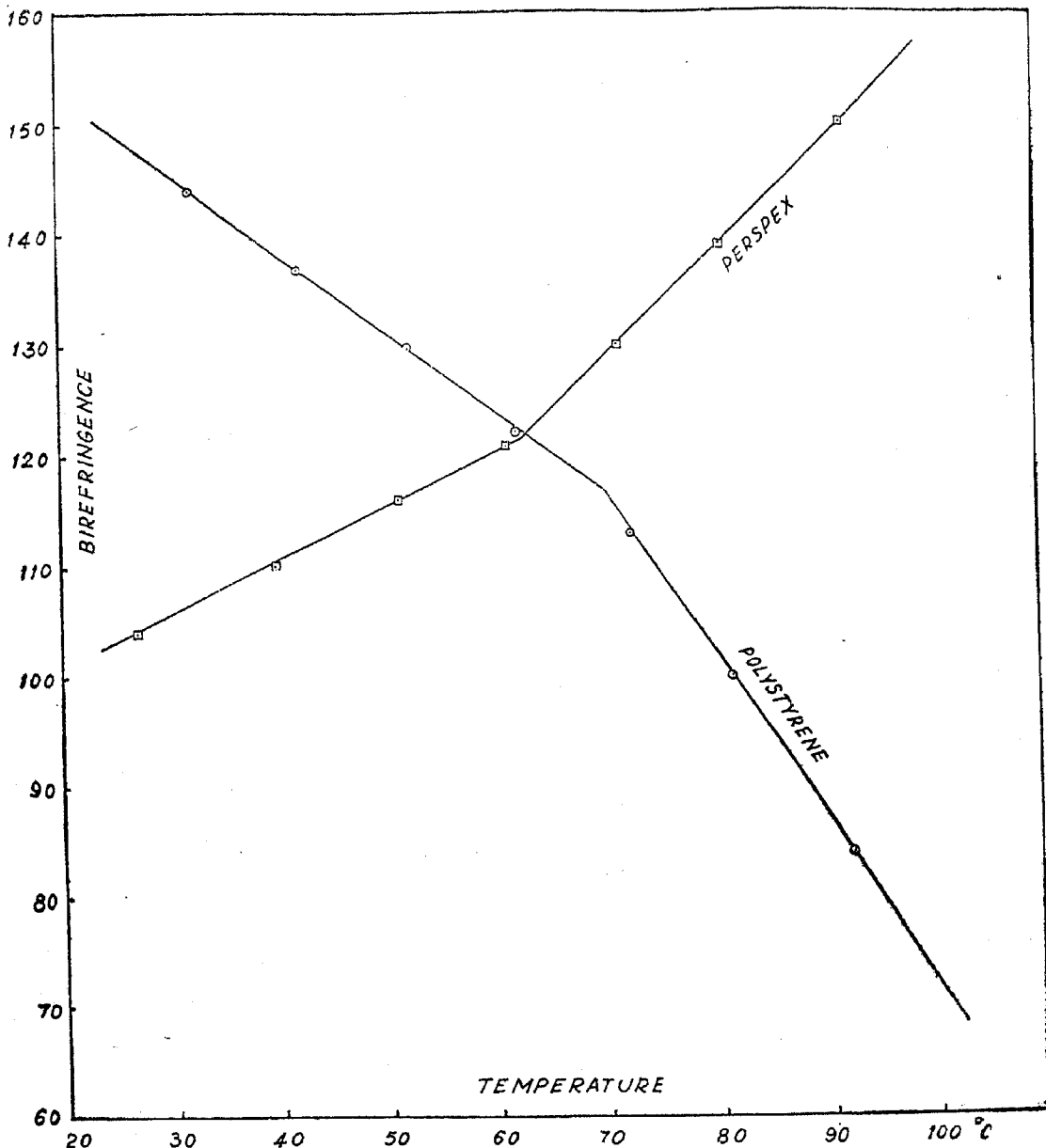


FIG. 1

#### 4. SUMMARY

The birefringence produced in samples of polymethyl methacrylate and polystyrene when subjected to stress is measured at different temperatures employing a Babinet compensator. Graphs of birefringence against temperature are plotted for the temperature range 30 to 90°C. for perspex under 100 kg./cm.<sup>2</sup> stress and for polystyrene under 60 kg./cm.<sup>2</sup> The birefringence of the stressed methacrylate is positive and for polystyrene it is negative. The birefringence in the case of perspex increases with temperature

while in polystyrene it shows a decrease as the temperature increases. There is a transition point at 62° C. for perspex and at 70° C. for polystyrene. The relative stress-optic and strain-optic constants are evaluated at each temperature taking into consideration the temperature variation of the refractive index.

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