DETERMINATION OF THE ELASTIC CONSTANTS OF ISOTROPIC MEDIA: A NEW METHOD

By S. Bhagavantam and B. Ramachandra Rao

(*From the Department of Physics, Andhra University, Guntur*)

Received March 5, 1946

1. Introduction

The determination of the elastic constants of isotropic substances by dynamical methods has been the subject of investigation by several workers. In most of such methods, the material was used in the form of a rod and ultrasonic waves were transmitted through it by attaching a piezoelectric quartz to one of the ends. Rohrich[1] made a complete determination of the velocity of ultrasonic waves in steel, brass, copper, aluminium and glass. His experiments were continued by Schoeneck[2] who investigated the elastic longitudinal vibrations in single crystals.

More exact determinations of the elastic constants of transparent bodies were carried out by Schæfer and Bergmann.[3] In this method, both longitudinal and shear waves are excited and the corresponding diffraction patterns observed. The method has been extended also to opaque substances.

It was recently observed[4] in this laboratory that characteristic thickness shear modes could also be transmitted through the crystal plates and communicated to liquids in the form of consequential longitudinal strains. This has suggested the possibility of exciting shear modes in thin plates of isotropic substances as well and detecting them by optical methods if they could similarly be communicated to a liquid.

2. Experimental Method and Observations

Using a tourmaline wedge with a frequency range of 3 to 16 megacycles per second, the characteristic transmission frequencies of several plates of glass, steel, brass and platinum have been studied by the method of ultrasonic diffraction, the details of which have been described in earlier papers.[5] Plates of different sizes and of the same thickness have been examined in each case with a view to see if the size has any effect on the intensity of the shear modes. In order to avoid errors due to the mounting, the wedge is chosen to be a size smaller than the smallest of the specimens used. Both the longitudinal and shear fundamental frequencies could easily be detected and measured in transmission if the plates chosen are sufficiently thin and small. The elastic constants C_{11} and C_{44} are then evaluated. Using the well-known relations of transformation, the Young's modulus *y* and the rigidity modulus *n* may be obtained for each material. In the case of glass the values thus obtained are compared with the values obtained by separate static experiments on the same specimen in this laboratory. Comparison for the rest is effected by taking the values from standard tables. It may be noted here that practically all the substances showed an increase in intensity of shear modes with smaller areas of the specimen plates. This supports our view that shear modes are communicated as corresponding longitudinal strains to the adjoining liquid due to a coupling effect arising in these cases from the finite size of the plates. The shear modes are comparatively weak in soft metals like brass whereas they are very bright in glass and steel, sometimes being equal in intensity to the longitudinal ones. The fundamental frequencies of the longitudinal and shear modes and the calculated values of C_{11} and C_{44} are given in Table I for different materials.

TABLE I

Material	Thickness in mm.	Fundamental frequency of longitudinal mode. Megacycles per second	Fundamental frequency of torsion mode. Megacycles per second	Density	$C_{11} \times 10^{-10}$ dynes/cm.^{2}	$C_{44} \times 10^{-10}$ dynes/cm.^{2}
Glass ..	0·93	3·22	1·40	26·02	93·2	17·6
Steel ..	0·64	4·80	2·60	7·502	287·0	84·0
Brass ..	0·58	4·20	1·745	8·56	203·0	35·0
Platinum ..	0·16	13·30	5·335	20·99	380·2	61·1

The values of *y* and *n* deduced from the above data along with the experimental static values of glass and the standard values taken from tables in the other cases are given in Table II.

TABLE II

Material	Authors' results		Static values	
	y	*n*	*y*	*n*
Glass ..	4·80	1·76	4·60	1·80
Steel ..	21·71	8·40	19 to 21	7·7 to 8·3
Brass ..	9·77	3·50	9·7 to 10·2	3·5
Platinum ..	17·16	6·11	16·80	6·10

The unit is 10^{11} dynes per cm.^{2}

3. Discussion

The smallest size of the plate used in the above investigation is 6 mm. square. The method is simple and sufficiently accurate, being particularly suitable for substances available only as small bits. The possibility of investigating the elastic properties of precious metals, alloys and other such materials under varying physical conditions is obvious. The exact mechanism by which the shear mode in the plate is communicated as a longitudinal wave to the liquid medium is of theoretical interest and requires to be further investigated. Examination of plates of different sizes has shown that edge coupling of the plates is probably the cause.

4. Summary

A new method of determining the Young's modulus and the rigidity modulus of isotropic materials using ultrasonic frequencies has been described. Results obtained with glass, steel, brass and platinum by such a method compare well with the standard values. Only a small plate of about 6 mm. square of the material is all that is required and hence the method is capable of being utilized under varying physical conditions in respect of rare and precious specimens.

5. References

1. Rohrich, K. .. *Zeits. f. Physik.*, 1934, 73, 11–12, 813–32.
2. Schoeneck, H. .. *Ibid.*, 1934, **92**, 390.
3. Schæfer, Cl., and L. Bergmann .. *Sitz-Ber-Berliner Akad.*, p. 155, 1934. *Naturwiss.*, 1934, **22**, 685.
4. Bhagavantam, S., and Bhimasenachar, J. .. *Nature*, 1945, **156**, July 7.
5. ——— .. *Proc. Ind. Acad. Sci.*, (A), 1944, **20**, 298.