DETERMINATION OF THE ELASTIC CONSTANTS
OF ISOTROPIC MEDIA: A NEW METHOD

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1. INTRODUCTION

The determination of the elastic constants of isotropic substances by
dynamical methods has been the subject of investigation by several workers.
In most of such methods, the material was used in the form of a rod and
ultrasonic waves were transmitted through it by attaching a piezoelectric
quartz to one of the ends. Rohrich\textsuperscript{1} made a complete determination of the
velocity of ultrasonic waves in steel, brass, copper, aluminium and glass.
His experiments were continued by Schoeneck\textsuperscript{2} who investigated the elastic
longitudinal vibrations in single crystals.

More exact determinations of the elastic constants of transparent bodies
were carried out by Schæfer and Bergmann.\textsuperscript{3} In this method, both longi-
tudinal and shear waves are excited and the corresponding diffraction patterns
observed. The method has been extended also to opaque substances.

It was recently observed\textsuperscript{4} in this laboratory that characteristic thickness
shear modes could also be transmitted through the crystal plates and com-
municated to liquids in the form of consequential longitudinal strains. This
has suggested the possibility of exciting shear modes in thin plates of isotropic
substances as well and detecting them by optical methods if they could simi-
larly be communicated to a liquid.

2. EXPERIMENTAL METHOD AND OBSERVATIONS

Using a tourmaline wedge with a frequency range of 3 to 16 megacycles
per second, the characteristic transmission frequencies of several plates of
glass, steel, brass and platinum have been studied by the method of ultrasonic
diffraction, the details of which have been described in earlier papers.\textsuperscript{5}
Plates of different sizes and of the same thickness have been examined in
each case with a view to see if the size has any effect on the intensity of the
shear modes. In order to avoid errors due to the mounting, the wedge is
chosen to be a size smaller than the smallest of the specimens used. Both the
longitudinal and shear fundamental frequencies could easily be detected and
measured in transmission if the plates chosen are sufficiently thin and small. The elastic constants $C_{11}$ and $C_{44}$ are then evaluated. Using the well-known relations of transformation, the Young’s modulus $Y$ and the rigidity modulus $n$ may be obtained for each material. In the case of glass the values thus obtained are compared with the values obtained by separate static experiments on the same specimen in this laboratory. Comparison for the rest is effected by taking the values from standard tables. It may be noted here that practically all the substances showed an increase in intensity of shear modes with smaller areas of the specimen plates. This supports our view that shear modes are communicated as corresponding longitudinal strains to the adjoining liquid due to a coupling effect arising in these cases from the finite size of the plates. The shear modes are comparatively weak in soft metals like brass whereas they are very bright in glass and steel, sometimes being equal in intensity to the longitudinal ones. The fundamental frequencies of the longitudinal and shear modes and the calculated values of $C_{11}$ and $C_{44}$ are given in Table I for different materials.

**Table I**

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness in mm.</th>
<th>Fundamental frequency of longitudinal mode. Megacycles per second</th>
<th>Fundamental frequency of torsion mode. Megacycles per second</th>
<th>Density</th>
<th>$C_{11} \times 10^{-10}$ dynes/cm.²</th>
<th>$C_{44} \times 10^{-10}$ dynes/cm.²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>0.93</td>
<td>3.22</td>
<td>1.40</td>
<td>26.02</td>
<td>93.2</td>
<td>17.6</td>
</tr>
<tr>
<td>Steel</td>
<td>0.64</td>
<td>4.80</td>
<td>2.60</td>
<td>7.502</td>
<td>287.0</td>
<td>54.0</td>
</tr>
<tr>
<td>Brass</td>
<td>0.58</td>
<td>4.20</td>
<td>1.745</td>
<td>8.56</td>
<td>208.0</td>
<td>35.0</td>
</tr>
<tr>
<td>Platinum</td>
<td>0.16</td>
<td>13.30</td>
<td>5.335</td>
<td>20.99</td>
<td>380.2</td>
<td>61.1</td>
</tr>
</tbody>
</table>

The values of $Y$ and $n$ deduced from the above data along with the experimental static values of glass and the standard values taken from tables in the other cases are given in Table II.

**Table II**

<table>
<thead>
<tr>
<th>Material</th>
<th>Authors' results</th>
<th>Static values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y$</td>
<td>$n$</td>
</tr>
<tr>
<td>Glass</td>
<td>4.80</td>
<td>1.76</td>
</tr>
<tr>
<td>Steel</td>
<td>21.71</td>
<td>8.40</td>
</tr>
<tr>
<td>Brass</td>
<td>9.77</td>
<td>3.50</td>
</tr>
<tr>
<td>Platinum</td>
<td>17.16</td>
<td>6.11</td>
</tr>
</tbody>
</table>

The unit is $10^{11}$ dynes per cm.²
3. DISCUSSION

The smallest size of the plate used in the above investigation is 6 mm. square. The method is simple and sufficiently accurate, being particularly suitable for substances available only as small bits. The possibility of investigating the elastic properties of precious metals, alloys and other such materials under varying physical conditions is obvious. The exact mechanism by which the shear mode in the plate is communicated as a longitudinal wave to the liquid medium is of theoretical interest and requires to be further investigated. Examination of plates of different sizes has shown that edge coupling of the plates is probably the cause.

4. SUMMARY

A new method of determining the Young's modulus and the rigidity modulus of isotropic materials using ultrasonic frequencies has been described. Results obtained with glass, steel, brass and platinum by such a method compare well with the standard values. Only a small plate of about 6 mm. square of the material is all that is required and hence the method is capable of being utilized under varying physical conditions in respect of rare and precious specimens.

5. REFERENCES

1. Rohrich, K. \( \ldots \) Zeits. f. Physik., 1934, 73, 11-12, 813-32.
2. Schoeneck, H. \( \ldots \) Ibid., 1934, 92, 390.