DIFFRACTION OF LIGHT BY VERY HIGH FREQUENCY ULTRASONIC WAVES: EFFECT OF TILTING THE WAVE FRONT

By S. Bhagavantam, F.A.Sc., and B. Ramachandra Rao

(From the Department of Physics, Andhra University, Waltair)

Received July 1, 1948

1. Introduction

Asymmetry in the diffraction spectra produced by a tilt in the sound wave front from the position of normal incidence was observed by Debye and Sears[1] and by Lucas and Biquard.[2] The diffraction spectrum of the mth order attains its maximum intensity for a tilt angle given by

$$\theta = \sin^{-1} m\lambda \,/\, 2\,\mu_0 \cdot \lambda^*, \tag{1}$$

where μ_0 is the refractive index of the medium, λ is the wave length of light in vacuum and λ^* is the wave length of sound in the medium. Detailed investigations were made later by Bär[3] and Parthasarathy.[4] Photographs obtained by Bar showed the asymmetry introduced on tilting the sound wave front through an angle of $< 1°$. For a truly normal incidence, a symmetrical diffraction picture and for oblique incidence, a picture with more orders on one side than on the other were obtained. Parthasarathy used progressive sound waves set up in a 12″ column of benzene at a frequency of 7·37 Mcs/sec. and measured the tilt angles in a rough way by attaching a rigid steel rod of 31 cm. to the crystal holder and observing the readings of its end on a vertical scale. Tilting the sound wave front from the normal incidence position, greatest asymmetry in the diffraction spectrum was noticed at an angle of 22′. When the angle of incidence was further increased, the number of diffraction images was found to decrease, and finally disappear almost completely at about 2°. Using a frequency of 20 Mcs./sec., Parthasarathy[5] later showed that intense diffraction spectra on one side of the central image could be obtained for certain angles of incidence given by the Bragg relation. Rytov[6] investigated the effect of tilting the sound wave front on diffraction, both theoretically and experimentally, in the range of 30 to 70 Mcs. His observations showed that at these frequencies, the intensity distribution approximated closely to selective reflection at the Bragg glancing angle. Quantitative measurements of the intensity of first order diffracted light in air for varying angles of incidence were made by Korff.[7] The ratios of the intensities of the first order to the central order were estimated on a microphotometer and compared with the theory of Brillouin. A maximum of diffracted light was found at the Bragg angle of incidence.

The highest frequency at which diffraction effect has so far been studied is thus only 70 Mcs./sec. and this was by Rytov. During the past few years, we have been studying the different aspects of diffraction by very high frequency ultrasonic waves and we have succeeded in generating and using sound waves of frequencies upto 200 Mcs./sec. for this purpose. In this investigation, the effects produced by tilting the wave front are described in some detail.

2. EXPERIMENTAL DETAILS

Four different frequencies, namely, 28, 50·36, 102·6 and 172·2 Mcs./sec. have been used. The wave-length of sound in water corresponding to the highest frequency used here is 0·000849 cm. This is only about 14 times the wave-length of sodium yellow light, which is generally employed by us for obtaining the patterns. The oscillators for generating the 172·2 and 102·6 Mcs. radio waves are specially designed and constructed using a T 55 valve. The circuit for the 50 Mcs. oscillator is the same as that used for 100 Mcs. with slight modifications. A coil of 2" diameter containing two turns of thick tinned copper wire serves as the tank circuit inductance. A Meissner R. F. choke of 2·5 mH with a current carrying capacity of 250 mA is used in the plate circuit. The crystal is connected to a coil of about four turns of copper wire mounted on an amphenol insulator. By varying the distance of this coil from the tank coil of the oscillator the power input to the crystal can be altered. The 28 Mcs. oscillator is of the regular Hartley type using 815 valve and is also coupled to the crystal by a mutual inductance coil, again enabling us to vary the power of excitation of the crystal.

The arrangement for observing and recording the diffraction patterns is the usual one with slight modifications. Water is the medium in which all the effects described in this paper have been studied and 6 mm. square apertures are introduced in the light emerging from the collimator to get uniform light intensity with square cross-section. Except for the low frequency of 28 Mcs., for all the other frequencies a 4" glass cell is found to be sufficient for producing progressive waves, the absorption coefficient then being sufficiently high that the wave gets completely damped before touching the bottom of the cell. For the 28 Mcs. ultrasonic wave, however, a 12" height of glass cell stuffed with glass wool and felt padding at the bottom is used for getting a truly progressive sound wave. Any deviation from the truly progressive nature of the wave can be detected by tilting the cell in a plane at right angles to the direction of the light beam. If any change in the intensity of the diffracted light is noticed it can be taken as an indication that the wave is not a pure progressive wave. This test is necessary and must be applied only when the light beam is incident normally on the faces of the glass cell.

A mirror is rigidly attached to the crystal holder and a telescope and scale placed at a distance of about one metre are used to note the position of the crystal holder. The direction of observation through the telescope is set almost parallel to the direction of the light beam. With this arrangement, very slight variations in the tilting of the crystal holder can be easily detected.

3. RESULTS

Starting from the normal incidence position of the sound wave front, the crystal holder is tilted through known angles and the intensities of both the first order diffraction lines are visually estimated. For all the four frequencies of the sound waves, the maximum intensity of the first order diffraction line is adjusted to be approximately the same and the effect of tilt studied. Visual estimates of the intensities of + 1 and − 1 orders for different angles of tilt θ from the normal incidence position are given in Table I. Table II gives the angles through which the crystal has to be tilted

TABLE I

25 Mcs. ..	θ	−18′	−11·5′	−6′	−4·5′	0	4·5′	6′	11·5′	18′		
	I_{+1}	0	0	0	0	3	8	10	20	12		
	I_{-1}	12	20	10	8	3	0	0	0	0		
50·36 Mcs. ..	θ	−32′	−26′	−21′	−16′	−3′	0	3′	16′	21′	26′	32′
	I_{+1}	0	0	0	0	0	1	1·5	6	10	20	8
	I_{-1}	8	20	10	4	1·5	1	0	0	0	0	0
100·26 Mcs. ..	θ	−56′	−52′	−50′	−46′	−43′	0	43′	46′	50′	52′	56′
	I_{+1}	0	0	0	0	0	0	2	4	10	20	4
	I_{-1}	4	20	10	4	2	0	0	0	0	0	0
177·2 Mcs. ..	θ	−89′	−88′	−84′	0	84′	88′	89′				
	I_{+1}	0	0	0	0	1	10	20				
	I_{-1}	20	10	1	0	0	0	0				

TABLE II

Frequency of sound in Mcs./sec.	θ Calculated	θ Measured
25	12·7′	11·5′
50·36 (+1)	25·5′	26′
(+2)	51′	52′
102·6	51·4′	52′
177·2	90′	89′

from the normal incidence position to get a maximum intensity for the first order. This angle is compared with the angles computed from formula (1). With increased power input, even the second order at 50 Mcs. could be obtained, but only at the proper inclination and the observed angle compared well with that calculated from the formula. In Table III, the value given in the columns designated by $\phi\frac{1}{3}$ and $\phi\frac{1}{2}$ represents the fraction, with reference to the Bragg reflection angle in each case, of the sound wave front tilt from the maximum intensity position required to reduce its intensity to $\frac{1}{2}$ and $\frac{1}{3}$ respectively. ϕ_0 stands for the tilt angle, similarly expressed as a fraction, at which the disappearance of the order takes place. This table brings out the increasing sharpness of the phenomenon with increasing frequency of sound waves.

TABLE III

	25	50	102	177 Mcs.
$\phi_{1/2}$ ..	1/2	1/5	1/26	1/90
$\phi_{1/3}$ ..	2/3	1/3	1/13	1/50
ϕ_0 ..	4/3	9/8	1/4	1/11

The estimates of intensities are plotted in Fig. 1. For the lower frequencies of 25 and 50 Mcs., both I_{+1} and I_{-1} are plotted, while for the two higher frequencies, only I_{+1} is given.

The special features observed in the case of very high frequencies are summarised below.

(*a*) It has not been possible to get the second or higher orders in the diffraction patterns. This is in sharp contrast with the observations at low frequencies where the second and higher orders are easily excited with moderate power.

(*b*) The first order diffraction line appears with maximum intensity not for normal incidence but when the light rays meet the sound wave front at the Bragg angle of reflection. In this position, only a single diffraction line on the appropriate side is obtained, the diffraction line on the other side of the central image having completely disappeared. In no position has it been possible to get both the first order lines on either side of the centre at the same time.

(*c*) The value of the angle for which the first order line attains its maximum intensity is quite critical. Fig. 1 shows that a slight tilt of the crystal holder, about 2′ for the 102 Mcs. and 1′ for the 177 Mcs., will reduce the intensity of the diffraction order by half. Contrary to this, the behaviour of diffraction at 50 and 25 Mcs. shows that in this region the range of reflection is not sharp. Tilting the crystal by 13 for the 100 Mcs. and 8′ for the 177 Mcs. from the maximum intensity position of the first order, makes the diffraction effect completely disappear. Thus for normal incidence, no diffraction lines are obtained at these frequencies whereas for 25 and 50 Mcs., at normal incidence the first order lines on both the sides of the central image are obtained symmetrically though with less intensity. The disappearance of diffraction for normal incidence for the very high frequencies shows that the diffraction of light in the ordinary sense does not exist in this region. On the other hand, the reflection phenomenon predicted by Brillouin becomes conspicuous.

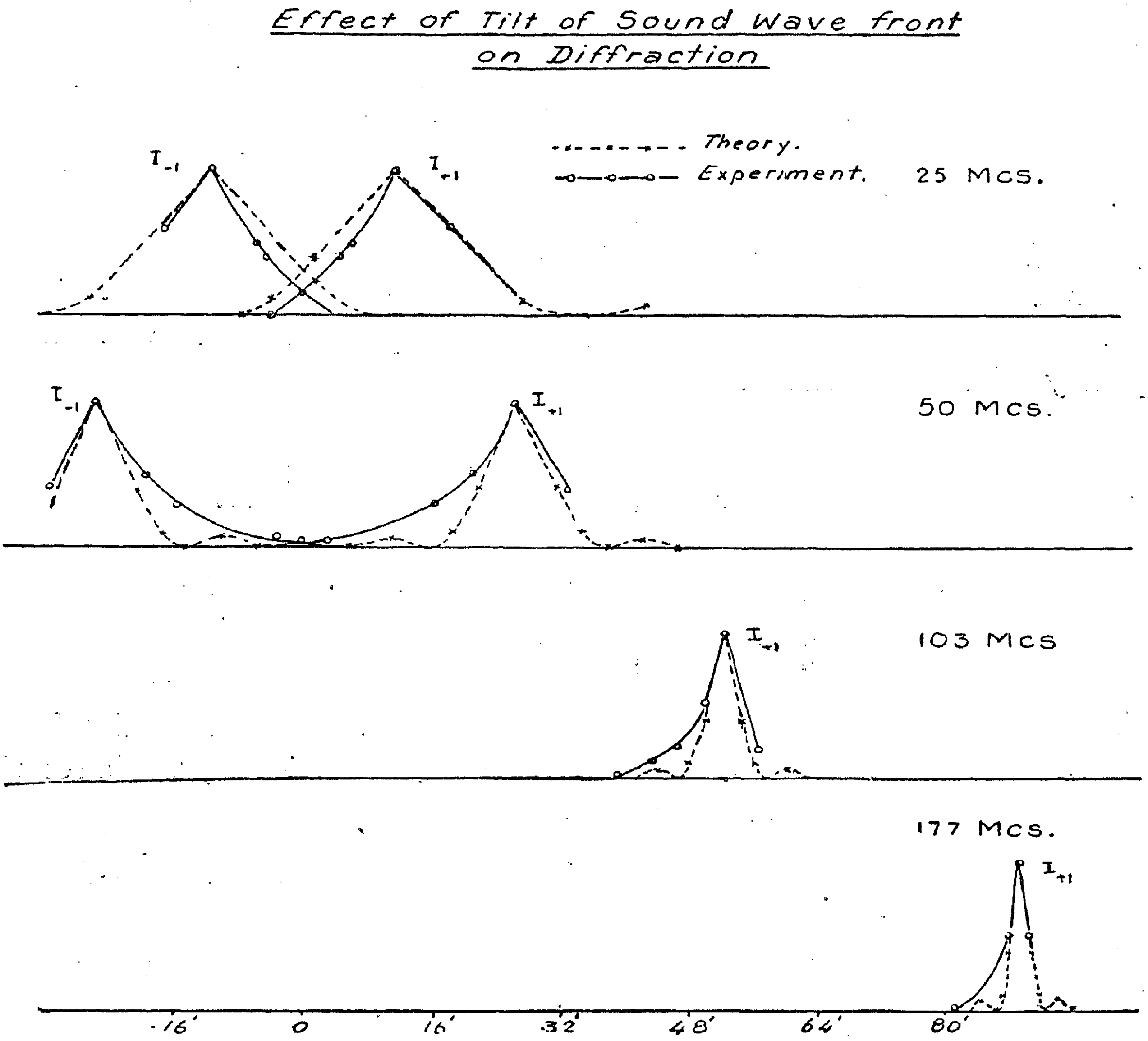

FIG. 1

4. COMPARISON WITH THEORY

Raman and Nath[8] have given a generalised theory of diffraction in which was obtained a difference-differential equation whose solutions correspond to the amplitudes of the diffraction orders. Nath[9] has solved the difference-differential equation by the series method and has shown that the maxima of intensity of first and second orders of diffraction occur at angles of $\lambda/2\mu_0\lambda^*$ and $\lambda/\mu_0\lambda^*$ under certain conditions. This result is in agreement with our observations. An attempt to calculate the intensities using the expressions given by Nath with suitable parameters has not been successful as it is found that the series is not convergent. In the same paper Nath has suggested a method of determining the amplitude of the fluctuation of the refractive index at high supersonic frequencies by determining the ratio of the intensities of central to the first order for normal incidence and $+1$ to -1 order for oblique incidence. Such observations are not possible at the very high frequencies used by us since both the diffraction orders are never present at the same time.

Following the procedure adopted by Brillouin, expressions for intensities of first and second orders of diffraction have been deduced by David,[10] on the assumption that the higher orders are of negligible intensity. To a first approximation the following expression is obtained for I_{+1}, the fraction of the intensity of the first order to the total incident light.

$$I_{+1} = \left(\delta \frac{\sin(\alpha - \frac{1}{2})\,\bar{x}_0}{(\alpha - \frac{1}{2})}\right)^2 \tag{2}$$

where $\delta = \mu\mu_0\,\lambda^{*2}/\lambda^2$

$\alpha = \mu_0\lambda^*\theta/\lambda$

and $\bar{x}_0 = \pi\lambda L/\mu_0\lambda^{*2}$.

L is the length of the sound field traversed by light and θ is the inclination of the sound wave front. It can be easily seen that this expression attains its maximum intensity when $\alpha = \frac{1}{2}$ or $\theta = \lambda/2\mu_0\lambda^*$. This result is the same as (1) and is in agreement with the experimental observations. The maximum value of I_{+1} denoted by I_m comes out as

$$\begin{aligned} I_m &= \delta^2\bar{x}_0^{\,2} \\ &= \frac{\pi^2\mu^2 L^2}{\lambda^2} = \frac{1}{4}\left(\frac{2\pi\mu L}{\lambda}\right)^2 \end{aligned} \tag{3}$$

Using I_m given by equation (3), we get the expression (2) modified as

$$I_{+1} = I_m\left(\frac{\sin(\alpha - \frac{1}{2})\,\bar{x}_0}{(\alpha - \frac{1}{2})\,\bar{x}_0}\right)^2. \tag{4}$$

Substituting

$$\theta = \frac{\lambda}{2\mu_0\lambda^*} + \phi,$$

where ϕ is the deviation of the sound wave front from the maximum intensity position of the first order, we get

$$\bar{x}_0 (a - \tfrac{1}{2}) = \frac{\pi L \phi}{\lambda^*}.$$

The expression for the intensity of the first order becomes

$$I_{+1} = I_m \left(\frac{\sin \frac{\pi L \phi}{\lambda^*}}{\frac{\pi L \phi}{\lambda^*}} \right)^2. \tag{5}$$

This relation is extremely important and contains many noteworthy features which are supported by experiment. These features are described below in some detail. The intensity of a particular order will vanish whenever $L\phi/\lambda^*$ is an integer other than zero. Thus, there are three ways of making a diffraction line periodically vanish. The first is by making the tilt angle ϕ of the sound wave front take successively the values λ^*/L, $2\lambda^*/L$ and so on. This phenomenon was experimentally observed by Debye and Sears and a simple explanation was given by Raman and Nath. The second is by making the sound field length L take successively the values λ^*/ϕ, $2\lambda^*/\phi$ and so on. This phenomenon has not been observed so far by any of the investigators. Korff[7] studied the dependence of intensity on the length of the sound field but his results show only a tendency towards decreasing intensity with increasing length at a certain stage but do not show a passage through successive minima. Such a passage through successive minima has been observed by us using a specially large-sized crystal and results which are in agreement with the above theory obtained. These will be described in a separate communication. The third method is to make λ^* take successively the values $L\phi$, $L\phi/2$ and so on. No one appears to have worked on these lines. For normal incidence, ϕ in respect of the first order line is equal to θ given by (1) with $m = 1$ and substituting this value in (5), we have

$$I_{+1} = I_m \left(\frac{\sin \frac{\pi L \lambda}{2\mu_0 \lambda^{*2}}}{\frac{\pi L \lambda}{2\mu_0 \lambda^{*2}}} \right)^2.$$

As the numerator in the bracket is always less than 1, we cannot have even the first order line appearing with appreciable intensity in normal incidence if $\pi L\lambda/2\mu_0\lambda^{*2} >> 1$. This is equivalent to saying that no diffraction patterns will be observed in normal incidence if $\lambda >> \lambda^{*2}$ because $\pi L/2\mu_0$ is generally of the order of unity. At 100 Mcs., λ^{*2} is 225×10^{-8} and λ is already about 25 times and therefore intense diffraction patterns cannot appear in normal incidence. For higher frequencies, it is still worse and these conclusions are well supported by our observations. It can be easily seen by differentiating (5) that the sharpness of diffraction depends only on the value of $\pi L/\lambda^*$. That is to say, for very high frequencies and large lengths of the sound field, the phenomenon resembles reflection more than diffraction. The sharpness of diffraction does not depend on any other factor like the intensity of the sound field. These conclusions are entirely in conformity with our observations. Using the expression (5), the value of I_{+1}/I_m is calculated for all the four frequencies used and is given in Table IV for a length of sound field of 1 cm. which is the size of the crystal used by us. The intensities are plotted in Fig. 1 keeping I_m constant for all the frequencies. Experimental curves fit extremely well with the theoretical curves. The only point in respect of which there is a discrepancy relates to the gradual fall in intensity as seen in the experimental curves, whereas the theoretical curves show small subsidiary maxima. This is probably due to the fact that the sound field is not the ideal one with undamped amplitude as has been assumed in the theory.

TABLE IV

I_{+1}/I_m	ϕ (177 Mcs)	ϕ102 (Mcs.)	ϕ (50 Mcs.)	ϕ (25 Mcs.)
1	0	0	0	0
·41	1·5′	2·5′	5·1′	10·4′
·09	2·2′	3·8′	7·7′	15·5′
0	2·9′	5·0′	10·3′	20·7′
·045	4·4′	7·5′	15·4′	31′
0	5·8′	10′	20·6′	41·4′
·016	7·3′	12·5′	25·7′	51·3′

It is also seen from equation (5) that the maximum intensity of the first order obtained by tilting the wave front to the appropriate reflection angle is independent of the frequency of the sound wave and is only a function of μ, L and λ.

5. SUMMARY

Diffraction of light by ultrasonic waves of frequencies 25, 50·36, 102·6 and 177·2 Mcs. set up in water has been studied with particular reference to the effect of tilting the wave front. Certain special features which are characteristic of the very high frequencies employed have been observed. The following are the important amongst them. In contrast with the observations at low frequencies, the first order diffraction is predominant at moderately high frequencies like 50 Mcs. and all the higher orders are suppressed. At 100 Mcs. and above, even the first order lines do not appear for normal incidence but when the sound wave front is tilted from this position to the appropriate side by an angle equal to the Bragg reflection angle, the corresponding order makes its appearance with maximum intensity. At these high frequencies, in no position has it been possible to get both the first order lines on either side of the centre at the same time. For very high frequencies and large lengths of the sound field, the angular range during which a particular diffraction line appears is found to be extremely sharp and narrow. With increasing frequency, the sharpness is found to increase. Experimental observations regarding this sharpness of range are made and compared with the result obtained out of an expression derived from the work of David and Brillouin. The agreement is very satisfactory.

REFERENCES

1.	P. Debye and F. Sears	.. *Proc. Nat. Acad. Sci.*, 1932, **18**, 409.
2.	R. Lucas and P. Biquard	.. *C. R. Acad. Paris*, 1932, **194**, 2132.
3.	R. Bär	.. *Helv. Phys. Acta.*, 1936, **9**, 265.
4.	S. Parthasarathy	.. *Proc. Ind. Acad. Sci.*, 1936, **3 A**, 442.
5.	————	.. *Ibid.*, 1936, **3 A**, 594.
6.	S. M. Rytov	.. *Phys. Zeits. d. Sowjetunion*, 1935, **8**, 626.
7.	W. Korff	.. *Phys. Zeitschr*, 1936, **37**, 708.
8.	C. V. Raman and N. S. Nagendra Nath	.. *Proc. Ind. Acad. Sci.*, 1936, **3 A**, 499.
9.	N. S. Nagendra Nath	.. *Ibid.*, 1937, **4 A**, 222.
10.	E. David	.. *Phys. Zeit.*, 1937, **38**, 587.