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A micro-convection model for thermal conductivity of nanofluids

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Abstract. Increase in the specific surface area as well as Brownian motion are supposed to be the most significant reasons for the anomalous enhancement in thermal conductivity of nanofluids. This work presents a semi-empirical approach for the same by emphasizing the above two effects through micro-convection. A new way of modeling thermal conductivity of nanofluids has been explored which is found to agree excellently with a wide range of experimental data obtained by the present authors as well as the data published in literature.

Keywords. Micro-convection; nanofluid; thermal conductivity; Brownian motion.

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1. Introduction

Miniaturization of components which accelerated the growth of electronic industry has brought new challenges in thermal management. Nanofluids, which are suspensions of nanoparticles in fluids, have attracted the attention of researchers in the recent past [1,2] due to their high thermal conductivity values even at low-particle concentrations with tremendous potential in electronic package cooling applications. Usual slurries with suspended particle sizes of the order of micro to millimeters suffer from a number of drawbacks such as particle sedimentation, erosion of components, particle clogging and excessive pressure drop. These drawbacks can be eliminated by using nanofluids which have higher heat transfer capability, higher stability, lower pressure drops and lower erosion of components, compared to slurries. Subsequently, a plethora of studies on thermal conductivity of nanofluids has been conducted by researchers all over the world [3–6].

The theoretical modeling of thermal conductivity of suspensions was initiated more than a century back by Maxwell [7]. Several authors extended the above theory (see, Hamilton and Crosser [8]) to provide comprehensive explanation for the thermal conductivity enhancement of usual slurries and suspensions; however,

these theories failed to predict the anomalous thermal conductivity variation of nanofluids. Keblinski *et al* [9] presented four possible mechanisms for the high conductivity enhancement but they themselves indicated that none of these mechanisms could really explain the thermal behaviour of nanofluids. Xuan *et al* [10] presented a model which combined the concept of fractals and Brownian motion. Jang and Choi [11] have modeled nanofluids based on kinetics, Kapitza resistance and convection. Recently, Hemanth *et al* [12] gave a model which accounts for the dependence of particle size, concentration and temperature on thermal conductivity. The strong effect of particle size on thermal conductivity was attributed to the increase in surface area and this was found with the help of a stationary particle model. In addition, a moving particle model was also developed from the Stokes–Einstein formula to explain the temperature effect on thermal conductivity. All the above models use one or more adjustable fitting parameters, whose values vary with each set of experiment. Hence, the aim of the present paper is to present a model which explains the thermal transport phenomena at nanoscale without changing the adjustable constants for each set of experimental data and yet preserving the physical features.

2. Experiments

The transient hot-wire (THW) method is used to measure the thermal conductivity of liquids [1]. The measurement cell of the experimental set-up consists of a 15 cm long platinum wire which acts as both a heater as well as a thermometer is kept in a glass container and forms an arm of a Wheatstone bridge. When calibrated against water, the equipment is found to be working with an accuracy of 1% in the chosen range of temperature, as shown in figure 1a.

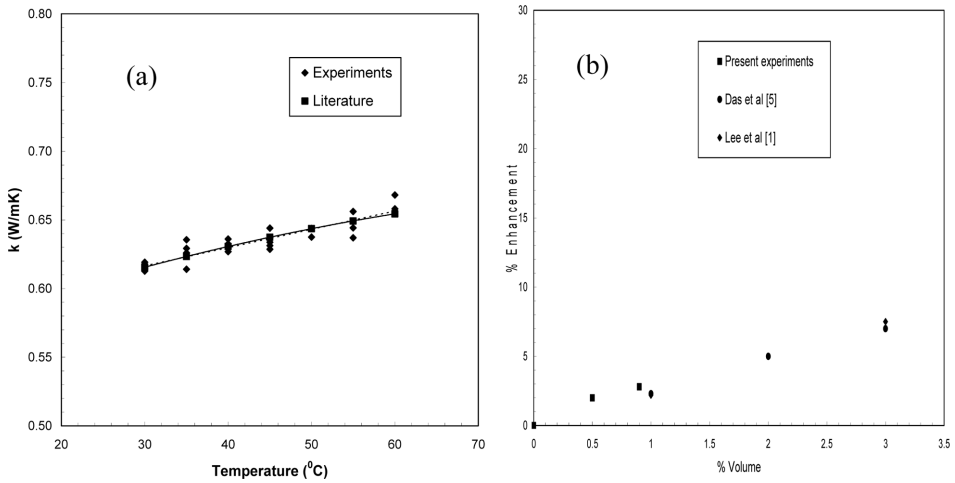

Figure 1. (**a**) Calibration of THW with water. (**b**) Comparison of the thermal conductivity measurement of alumina–water nanofluid with the existing data.

In the present experimental study, thermal conductivity of 11 nm-sized alumina particles suspended in water is measured. The nanofluid is stabilized by ultrasonic vibration for 6 h. The measured thermal conductivity conforms to the experiments by Das *et al* [5] and Lee *et al* [1] as shown in figure 1b.

3. Micro-convection model

In the present model, effort has been made to improve the model by Hemanth *et al* [12] by introducing the concept of micro-convection and thus get rid of adjustable constant. Here it is assumed to have three contributions for heat flow through the nanofluids, viz. conduction through liquid, conduction through solid and advection because of the Brownian motion of the particles. It may be noted that both the liquid and particle phases are considered as continua. The motion of particles inside the liquid can be modeled as flow over a sphere. The overall heat transfer can be written as

$$q = q_{\mathrm{m}} + q_{\mathrm{p}} + q_{\mathrm{adv}} = \left(-k_{\mathrm{m}} A_{\mathrm{m}} \left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)_{\mathrm{m}}\right) + \left(-k_{\mathrm{p}} A_{\mathrm{p}} \left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)_{\mathrm{p}}\right) + \left(\frac{1}{3} h A_{\mathrm{p}} \Delta T\right). \tag{1}$$

Here A, k, T, $(\mathrm{d}T/\mathrm{d}x)$ denote the heat transfer area, thermal conductivity, temperature and temperature gradient of the respective media, x denotes the special coordinate while the subscripts m and p denote quantities corresponding to the liquid medium and the particle, respectively. h is the convective heat transfer coefficient.

Here, we assume that the liquid medium and the nanoparticles are in local thermal equilibrium at each location, and so the temperature gradients in liquid and solid phases are the same. Only one-directional heat transfer is considered here for the analysis. So particle movement in one direction only will be helpful in increasing the heat transfer. Hence, h is divided by 3 in eq. (1). The Brownian velocity of the particles, which is causing the convection, is calculated from the diffusion coefficient equation given by the Stokes–Einstein formula, using particle diameter d_{p} as the characteristic length [9]. So the effective temperature difference for convective heat transfer can be taken as the product of the temperature gradient in liquid and d_{p}. Thus, the heat transfer enhancement in nanofluid over base liquid can be given as

$$\frac{q}{(-k_{\mathrm{m}} A_{\mathrm{m}} (\mathrm{d}T/\mathrm{d}x))} = 1 + \frac{k_{\mathrm{p}} A_{\mathrm{p}}}{k_{\mathrm{m}} A_{\mathrm{m}}} + \frac{1}{3} h d_{\mathrm{p}} \frac{A_{\mathrm{p}}}{k_{\mathrm{m}} A_{\mathrm{m}}}. \tag{2}$$

The Nusselt number for the situation of flow over a sphere is given as [13]

$$\mathrm{Nu} = 2 + 0.5\ \mathrm{Pe} + O(\mathrm{Pe}^2). \tag{3}$$

The Peclet number $\mathrm{Pe} = u_{\mathrm{p}} d_{\mathrm{p}} / \alpha_m$, where α_{m} is the thermal diffusivity of liquid and u_{p} is the Brownian motion velocity of particles given by [9]

$$u_{\mathrm{p}} = (2k_{\mathrm{B}}T/\pi\eta d_{\mathrm{p}}^2), \tag{4}$$

where k_{B} is the Boltzmann constant and η is the dynamic viscosity of the liquid.

The size of the particles is of the order 10^{-7} to 10^{-8} m and the Brownian motion velocity is also very low; hence Pe^2 is negligible compared to Pe. As the conduction through liquid as well as the particle conduction are separately accounted in q_{m} and q_{p} respectively, only the advection part needs to be considered. Thus eq. (3) can be further simplified in the form

$$\mathrm{Nu} \ = 0.5 \ \mathrm{Pe}. \tag{5}$$

At this point, an empiricism has been introduced in the model looking at the experimental data. It is found that if thermal conductivity of liquid is replaced by thermal conductivity of solid phase with a constant c', the model predictions fit to the experimental data quite well. The Nusselt number here is defined as

$$\mathrm{Nu} = hd_{\mathrm{p}}/(c'k_{\mathrm{p}}). \tag{6}$$

From eqs (5) and (6),

$$hd_{\mathrm{p}} = c'k_{\mathrm{p}}\frac{\mathrm{Pe}}{2}. \tag{7}$$

Thus, the enhancement is given as

$$\frac{q}{(-k_{\mathrm{m}}A_{\mathrm{m}}\,(\mathrm{d}T/\mathrm{d}x))} = 1 + \frac{k_{\mathrm{p}}A_{\mathrm{p}}}{k_{\mathrm{m}}A_{\mathrm{m}}} + ck_{\mathrm{p}}\ \mathrm{Pe}\ \frac{A_{\mathrm{p}}}{k_{\mathrm{m}}A_{\mathrm{m}}}. \tag{8}$$

From [12], $A_{\mathrm{p}}/A_{\mathrm{m}}$ can be deduced as

$$\frac{A_{\mathrm{p}}}{A_{\mathrm{m}}} = \frac{d_{\mathrm{m}}}{d_{\mathrm{p}}}\frac{\varepsilon}{1-\varepsilon}, \tag{9}$$

where d_{p} is the particle diameter, d_{m} is the molecular size of liquid and ε is the volume fraction of particles in liquid. Putting eq. (9) in eq. (8), we get

$$\%\ \mathrm{Enhancement} \ = \frac{k_{\mathrm{p}}}{k_{\mathrm{m}}}\left(1 + c\frac{u_{\mathrm{p}}d_{\mathrm{p}}}{\alpha_{\mathrm{m}}}\right)\frac{d_{\mathrm{m}}}{d_{\mathrm{p}}}\frac{\varepsilon}{1-\varepsilon} \times 100, \tag{10}$$

where the constant c is to be determined experimentally. In the present work, it has been found that even a fixed value of $c = 25{,}000$ gives very accurate predictions for a wide range of experiments.

4. Results and discussion

The experimental results available in literature and the experiments conducted by the authors are compared to the predictions of the model mentioned above. Figure 2 shows comparison of the present model with the experimental data of Lee *et al* [1] and Wang *et al* [2] as well as with Hamilton–Crosser model. The

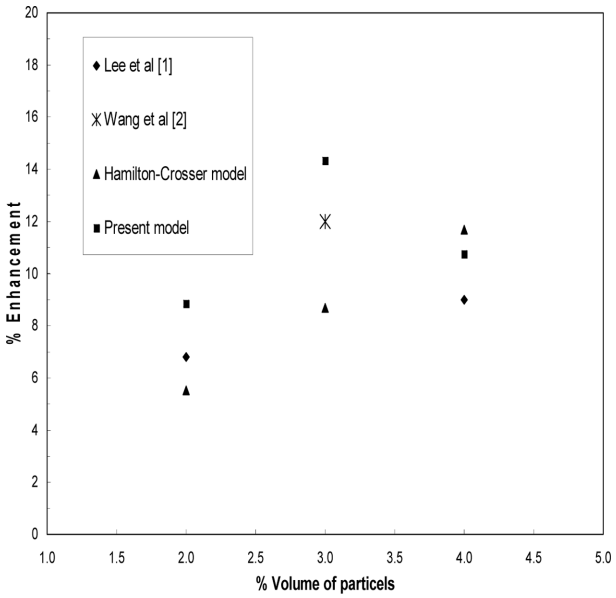

Figure 2. Comparison of the model predictions for oxide nanoparticle systems.

experiment referring to particle volume of 2% is CuO nanoparticles of 23 nm size, suspended in water. The experiments referring to 3% and 4% particle volume are alumina nanoparticles of 28 nm and 38 nm size respectively, suspended in water. It can be seen that the model predicts the conductivity values quite close to that of experimental data.

Comparison of the model predictions with the experiments of Das *et al* [5] with 38 nm-sized alumina particles suspended in water with 4% volume is shown in figure 3a. The figure shows that model successfully predicts the increase in thermal conductivity with temperature; whereas Hamilton–Crosser model fails to do it.

Comparison of the model with the experiments of Xuan and Li [3] and Eastman *et al* [4] are shown in figure 3b. It can be seen that even with 10 nm-sized and 100 nm-sized copper particles, which are almost on the lower and upper limit of nanoregime, the model predicts the enhancement in thermal conductivity quite closely and much better than the predictions by Hamilton–Crosser model.

The liquid particle size, d_{m} is taken as the molecular size of the liquid (2 Å for water and 3.1 Å for ethylene glycol). The model uses only one empirical constant whose value is fixed and equal to 25,000. The model is found to be working well in predicting the thermal conductivity of nanofluids over a wide range of data.

5. Conclusions

The nanofluids of metallic nanoparticles have shown tremendous enhancement in heat transfer [3,4] whereas our experiments with alumina particles of 11 nm size,

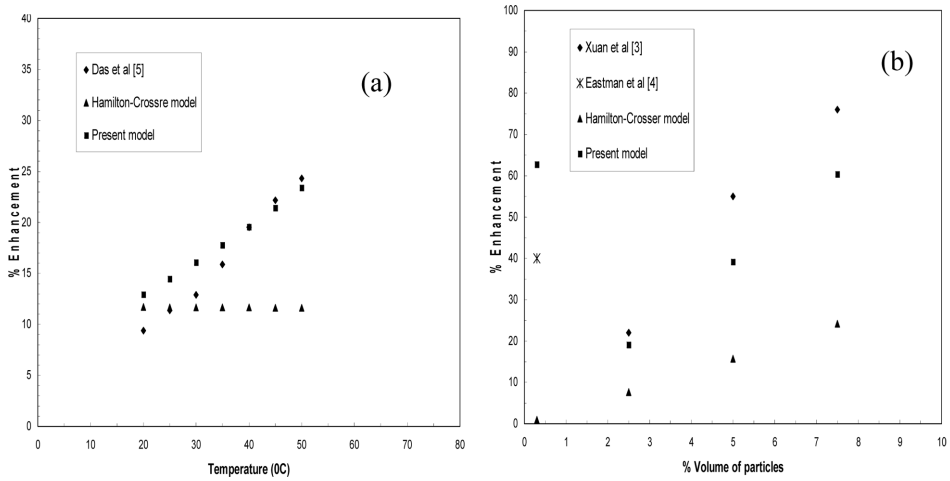

Figure 3. (**a**) Comparison of the model predictions with temperature variation. (**b**) Comparison of the model predictions for copper nanoparticle systems.

suspended in water with 0.8% volume did not show significant enhancement over that of the predictions of Hamilton–Crosser model. Other experiments with alumina–water suspension [1,5] also showed that there is no significant increase in the thermal conductivity of oxide-based nanofluids.

A micro-convection based model for the prediction of thermal conductivity of nanofluids is presented. The model is found to be predicting the thermal conductivity accurately over a wide range of particle sizes (10–100 nm), particle concentrations (1–8%), particle materials (metal particles as well as metal oxides), different base fluids (water, ethylene glycol) and temperature (20–50°C). It gives a good insight to the physical phenomena which are happening at the nanoscale. The modeling of micro-convection is an important outcome of this work. More efforts are required to bridge the semi-empiricism in the correlation developed here with physical reasoning.

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