Side-stepping of axial surface traces in superposed folding

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Abstract. During the refolding of an early non-isoclinal fold (say, $F_1$) we may find an offset or side-stepping of the axial surfaces of the later folds (say, $F_2$). The offsets can be seen in both type 2 and type 3 interference patterns. An analysis of the shear fold model shows that there is a maximum limit for the magnitude of side-stepping. The side-stepping is larger for larger interlimb angles of $F_1$. It decreases with progressive tightening of $F_2$. By recognizing such side-stepping we can predict on which side the $F_1$ hinge should lie even if the hinge is unexposed or lies outside the domain of observation. The general rule for the sense of side-stepping is the same for shear folds, flexural slip folds and buckling folds. However, the side-stepping in buckling folds should be used with caution, since $F_2$ folds on buckled single-layers may show an offset whose sense is opposite to that predicted by the general rule.

Keywords. Superposed folding; axial surfaces; side-stepping.

1. Introduction

During the coaxial folding of an early non-isoclinal fold (say, $F_1$) we may find an offset or side-stepping of the axial surfaces of the late folds (say, $F_2$). This feature, first described by Ramsay (1967, p. 509) and Ramsay and Huber (1987) is readily explained by the model of shear folding (Ramsay 1967). In the following analysis this model is discussed in detail with a view to determine the magnitude and sense of side-stepping of axial surfaces of $F_2$ when the initial shape of the $F_1$ fold and the nature of variation of the heterogeneous shear across it are given. The sense of side-stepping of flexural slip folds can also be determined by geometrical constructions or by paper stack models. For buckle folds, however, a kinematic model is inadequate, and the sense of side-stepping can only be determined from physical models.

2. Model of shear folding

Let us assume that the limbs of $F_1$ are straight and that the shear direction for the second deformation is perpendicular to the axial trace of $F_1$ on its profile plane. In the following analysis it has been further assumed that the magnitude of heterogeneous simple shear varies sinusoidally to generate the $F_2$ fold (figure 1). The sinusoidal variation of simple shear is shown in figure 1 by the curved axial surface (dashed line) of the deformed $F_1$ fold. Consider the transverse profile of an early fold ($F_1$) with axial surface trace parallel to the $y$ co-ordinate axis, with its amplitude $a$ and with two limbs $A$ and $B$ at angles of $\theta_1$ and $\theta_2$ with the $x$ axis (figure 2). The variation of the heterogeneous simple shear of the second deformation can be represented by the
Figure 1. Development of $F_2$ by shear folding of $F_1$. The axial trace of $F_1$ is deformed (dashed line) by sinusoidally varying simple shear. The axial surface traces of the $F_2$ folds show a left-stepping or sinistral offset if we look across the axial trace of $F_1$.

equation

$$\gamma = \gamma_0 \sin \bar{y},$$

(1)

where $\bar{y} = 2\pi/\lambda$. For a simple situation it is assumed that $\lambda = 4a$ (figure 2). The $F_1$ axial surface trace can be represented by the equation

$$x = 0.$$

By simple shear, with the particle-path equation

$$x' = x + \gamma y$$

$$y' = y,$$
the axial trace of $F_1$ is deformed to a curve
\[ x' = y' \gamma_0 \sin ly'. \] (2)

Similarly, if the limb $A$ of $F_1$ is represented by the equation
\[ y = x \tan \theta_1 - a, \]
by simple shear it is deformed to the curve
\[ x' = y' \cot \theta_1 + \gamma_0 y' \sin ly' + a \cot \theta_1. \] (3)

The $F_2$ hinge is located where $dx'/dy' = 0$, or
\[ ly' \cos ly' + \sin ly' = -\frac{\cot \theta_1}{\gamma_0}. \]

Since,
\[ \theta_1 = 90^\circ - \alpha_1 \]
\[ \theta_2 = 90^\circ + \alpha_2 \] (figure 2), we have
\[ ly' \cos ly' + \sin ly' = -\frac{\tan \alpha_1}{\gamma_0}, \] (4)
\[ ly' \cos ly' + \sin ly' = \frac{\tan \alpha_2}{\gamma_0}. \] (5)
For specific values of $\alpha_1$ and $\gamma_0$, each of these equations is satisfied for a particular value of $y'$ (say, $y'_0$). This is the $y$-coordinate of the hinge of $F_2$. For $\alpha_1 = \alpha_2 = 20^\circ$, the $y'$-coordinate of the hinge is represented (figure 3) in non-dimensional form by the ratio

$$f(y')$$

$\gamma_0 = 0.36$

$\gamma_0 = 0.5$

$\gamma_0 = 1$

$\gamma_0 = 2$

$\gamma_0 = 2.0$

$\gamma_0 = 0.5$

$\gamma_0 = 0.36$

Figure 3. Variation of $f(y')$ with $y'/\alpha$ for $\alpha = 20^\circ$ for different values of $\gamma_0$. The total offsets for $\gamma_0 = 0.36, 0.5, 1.0$ and $2.0$ are $AA', BB', CC'$ and $DD'$. 
$y'/a$. Figure 3 shows the variation of $f(ly')$ with $y'/a$, where $f(ly')$ represents the function on the left hand side of equation (4) or (5). $OA$ and $OA'$ or $OB$ or $OB'$ are the offsets of the $F_2$ axial surface traces on either side. $y'_c$ has opposite signs for the two limbs because the positions of the hinge points of $F_2$ are shifted towards opposite directions. The difference between the $y'_c$ values of the two limbs gives us the magnitude of offset. Thus, for example, the total offsets in figure 3 are $AA'$, $BB'$, $CC'$ and $DD'$ with increasing $\gamma_0$.

Say, $\alpha_1 = \alpha_2 = \alpha$, so that the interlimb angle of the early fold is $2\alpha$. The total offset of the axial surface trace, in nondimensional form, is then $2y'_c/a$. Equations (4) and (5) show that for the same value of $\gamma_0$, the magnitude of the total offset increases with an increase of the initial interlimb angle of the early fold (figure 4). For the same value of the interlimb angle of $F_1$, the offset of axial surface traces of $F_2$ decreases as these folds are tightened with progressive simple shear (figure 5). In this model of development of $F_2$ by heterogeneous simple shear the axial surface traces of the $F_2$ folds are parallel to the direction of simple shear. At the hinge point of $F_2$ the trace of the limb of $F_1$ must be rotated to become perpendicular to the direction of simple shear (figures 1, 2 and 5). This may not be possible if the magnitude of simple shear is too small (figure 5a). In other words, if the simple shear is too small the axial surfaces of $F_2$ do not become well-defined.

There is a maximum limit of the magnitude of side-stepping ($y'_c$). This limit is obtained from the condition

$$\frac{df(ly')}{d(ly')} = 0,$$

Figure 4. Variation of total offsets $2y'_c/a$ with increasing $\gamma_0$ for three interlimb angles, $40^\circ$, $60^\circ$ and $80^\circ$, of the initial $F_1$ fold.
where $f(l_y')$ represents the left hand side of equation (4) or (5). From this condition we obtain the relation

$$
tan \, l_y' = \frac{2}{l_y'}. \tag{6}$$

The solution of this equation can be obtained graphically from the intersection of the curves

$$
z = tan \, l_y', \text{ and}$$

$$
z = 2/(l_y'). \tag{7}$$

The maximum value of $y'/a$ is 0.685 (figure 6). In other words, the largest possible total offset ($2y'/a$) of the axial traces of $F_2$ is 1.37, irrespective of the interlimb angle of $F_1$ (dotted line in figure 4).

The idealized model of heterogeneous simple shear gives us the following rule for the sense of side-stepping of the $F_2$ axial traces. If we move across the axial trace of $F_1$, with the $F_1$ hinge on the left hand side, an upward closing $F_2$ fold will show a right-stepping or a dextral shift (figure 7a) and a downward closing $F_2$ fold will show a left-stepping (figure 7c). If we look across the axial trace of $F_1$, with the $F_1$ hinge on our right, an
Figure 6. Determination of maximum value of $y'/a$ from intersection of two curves.

Figure 7. Rule of axial trace offset of $F_2$ folds produced by heterogeneous shear.
upward closing \( F_2 \) will show a left-stepping (figure 7b) and a downward closing \( F_2 \) will show a right-stepping (figure 7d).

3. Side-stepping of axial surfaces in other types of folds

Natural folds are mostly buckling folds or flexural slip folds. From both paper stack models and graphical construction (figure 8), it is found that there is a side-stepping of axial surfaces of second generation flexural slip folds if the first generation fold is non-isoclinal. Figure 8a shows the transverse profile of an initial \( F_1 \) fold with horizontal traces of the axial plane cleavage. In figure 8b a parallel fold has been geometrically constructed on the cleavage traces, with the assumption that the deformation was by the ideal model of flexural flow (Ramsay and Huber 1987). The shape of the deformed bedding is constructed by measuring the arc-length distances of points on the bedding traces from the hinge point along several cleavage traces parallel to \( AB, CD \) or \( EF \) (figure 8b). Evidently, these arc-length distances remain the same in the deformed and undeformed models (figures 8a and 8b). The rule concerning the sense of side-stepping is the same as in the case of shear folding. The magnitude of side-stepping increases with an increase in the initial interlimb angle of \( F_1 \). It decreases with progressive tightening of \( F_2 \).

Unlike the idealized models of shear folding and flexural slip folding, the modification of earlier folds by buckling (Ghosh 1993, p. 337) cannot be analysed by a kinematic model. To get an insight into this problem, experiments on coaxial refolding were carried out with soft models of multilayers and single layers of the same types as described by Ghosh et al (1992, 1993). The first generation folds in these models were tight but not isoclinal. The models were deformed by pure shear, with the direction of

![Figure 8](image_url)
Figure 9. Sections of models showing superposed buckling. $F_1$ and $F_2$ are coaxial. (a) Offset of axial surface traces of $F_2$ folds in experiment of superposed buckling of a multilayer. Dotted layers and the intervening blank layers are modelling clay with greased interfaces. The rest of the blank areas are of painter's putty. (b) Superposed buckling in single layer of modelling clay embedded in painter's putty. Note that the side-stepping of axial surface traces has a sense opposite to that shown by the shear fold models. (c) Superposed buckling in a thin multilayer of a few layers of modelling clay. The embedding medium is painter's putty. Note the sense of axial trace offsets of $F_2$ folds. The hinge of each $F_2$ fold tends to move along a direction at a high angle to the trends of the $F_1$ limbs (scale bars 1 cm).

shortening parallel to the axial planes of $F_1$ and perpendicular to the $F_1$ axis. The second generation buckling folds in the thick multilayered models showed on the profile plane a side-stepping of axial surface traces (figure 9a) in the same sense as in the shear fold model. In the other set of experiments the $F_1$ folds were produced on single
layers of modelling clay or on thin multilayers embedded in painter's putty. During the second stage of deformation, with the direction of shortening parallel to the axial plane of \( F_1 \) and perpendicular to the \( F_1 \) axis, the axial planes of \( F_2 \) were not at a right angle to the direction of the second shortening. In the initial stage of the second deformation, the axial surface trace of an \( F_2 \) fold on the profile plane was at a high angle to the traces of the \( F_1 \) fold limbs so that the axial surfaces of \( F_2 \) folds, produced on the two limbs of an \( F_1 \) fold, were not parallel (figures 9b, 9c). With progressive deformation, the angle between the axial surfaces of the two \( F_2 \) folds was reduced and both surfaces made lower angles with the \( XY \) plane of the second deformation. Yet, the two \( F_2 \) axial surfaces remained non-parallel even at a large value of bulk shortening. If we look along the trace of the \( XZ \) plane on the fold profile, the hinges of the two \( F_2 \) folds show sidewise shifting with respect to each other, and the sense of this shifting is opposite to that of the model of shear folding.

Side-stepping of \( F_2 \) axial surface traces is not restricted to the type 3 interference. In experiments of superposed buckling of multilayers, offsets of \( F_2 \) axial traces are often seen in model sections of a type 2 fold interference when the traces of limbs of \( F_1 \) folds are not parallel (figure 10). Figure 10 shows the outcrop pattern on the horizontal section of a part of a multilayered model which had undergone superposed buckling, with the direction of second shortening parallel to the axis of the first generation folds.

The sense of side-stepping of the \( F_2 \) axial traces is in accordance with the general rule as given above, although an opposite sense of offset can be seen in some places. Thus, in the outcrop of the thick layer of modelling clay (dotted) the axial trace offsets of \( F_2 \) are in agreement with the general rule. However, in the thin layer (black) the offset of \( F_2 \) axial traces is opposite to the general rule on the right side of the figure near the downward closing nose of the \( F_1 \) fold. It is suggested that during disharmonic buckle folding a competent layer within a multilayer may deform in certain segments more or less independently as a single unit. The sense of side-stepping of axial traces of buckle folds should therefore be used with some caution.

![Figure 10](image-url) **Figure 10.** Horizontal section of multilayered model showing two generations of disharmonic buckle folds. Only a part of the model is shown. The fold interference pattern is of type 2. Note the sense of offset of axial traces of \( F_2 \) folds (scale bar 1 cm).
4. Conclusion

The offset of axial surface traces is a useful feature for structural analysis of both type 3 and type 2 fold interferences. In areas of superposed folding it is often difficult to locate or identify the hinges of early folds. Moreover, when the early fold is tight and has a large amplitude it may not be possible to distinguish its different limbs if later folds are superimposed on them. From the axial trace offsets we may then identify the presence of two generations of folds even if the early fold hinge is not located in the outcrop. It may also be possible to locate the axial surface trace of the early fold either in outcrop scale or in the map scale, even if the early fold closure lies outside the observed domain. The analysis given above shows that, from the magnitude of offset and the tightness of the $F_2$ folds, we may obtain an approximate idea of the initial tightness of $F_1$. Large offsets are expected when $F_2$ is fairly open and $F_1$ is moderately tight. Evidently, offsets of $F_2$ axial traces should not occur if $F_1$ was strictly isoclinal. From the sense of offset we can determine on which side of the $F_2$ axial surface the $F_1$ hinge is situated. The model of shear folding shows that there is a maximum limit for the value of side-stepping. The sense of side-stepping for flexural slip folds and buckle-folded multilayers is, in general, the same as in the model of shear folding; the sense of side-stepping in single-layer buckle folds may, however, be the opposite.

References